PART ONE

OVERVIEW

Mechanics

A hiker checks her position using signals from GPS satellites.

A wilderness hiker uses the Global Positioning System to follow her chosen route. A farmer plows a field with centimeter-scale precision, guided by GPS and saving precious fuel as a result. One scientist uses GPS to track endangered elephants, another to study the accelerated flow of glaciers as Earth's climate warms. Our deep understanding of motion is what lets us use a constellation of satellites, 20,000 km up and moving faster than 10,000 km/h, to find positions on Earth so precisely.

Motion occurs at all scales, from the intricate dance of molecules at the heart of life's cellular mechanics, to the everyday motion of cars, baseballs, and our own bodies, to the trajectories of GPS and TV satellites and of spacecraft exploring the distant planets, to the stately motions of the celestial bodies themselves and the overall expansion of the universe. The study of motion is called **mechanics**. The 11 chapters of Part 1 introduce the physics of motion, first for individual bodies and then for complicated systems whose constituent parts move relative to one another.

We explore motion here from the viewpoint of Newtonian mechanics, which applies accurately in all cases except the subatomic realm and when relative speeds approach that of light. The Newtonian mechanics of Part 1 provides the groundwork for much of the material in subsequent parts, until, in the book's final chapters, we extend mechanics into the subatomic and high-speed realms.

Motion in a Straight Line

What You Know

- You've learned the units for basic physical quantities.
- You understand the SI unit system, especially units for length, time, and mass.
- You can express numbers in scientific notation and using SI prefixes.
- You can handle precision and accuracy through significant figures.
- You can make order-of-magnitude estimates.
- You've learned the IDEA problem-solving strategy.

What You're Learning

- You'll learn the fundamental concepts used to describe motion: position, velocity, and acceleration—restricted in this chapter to motion in one dimension.
- You'll learn to distinguish average from instantaneous values.
- You'll see how calculus is used to establish instantaneous values.
- You'll learn to describe motion resulting from constant acceleration, including the important case of objects moving under the influence of gravity near Earth's surface.

How You'll Use It

- One-dimensional motion will be your stepping stone to richer and more complex motion in two and three dimensions, which you'll see in Chapter 3.
- Your understanding of acceleration will help you adopt the Newtonian view of motion, introduced in Chapter 4 and elaborated in Chapter 5.
- You'll encounter analogies to Chapter 2's motion concepts in Chapter 10's treatment of rotational motion.
- You'll apply motion concepts to systems of particles in Chapter 9.
- You'll continue to encounter motion concepts throughout the book, even beyond Part 1.

Electrons swarming around atomic nuclei, cars speeding along a highway, blood coursing through your veins, galaxies rushing apart in the expanding universe—all these are examples of matter in motion. The study of motion without regard to its cause is called **kinematics** (from the Greek "kinema," or motion, as in motion pictures). This chapter deals with the simplest case: a single object moving in a straight line. Later, we generalize to motion in more dimensions and with more complicated objects. But the basic concepts and mathematical techniques we develop here continue to apply.

2.1 Average Motion

You drive 15 minutes to a pizza place 10 km away, grab your pizza, and return home in another 15 minutes. You've traveled a total distance of 20 km, and the trip took half an hour, so your **average speed**—distance divided by time—was 40 kilometers per hour. To describe your motion more precisely, we introduce the quantity x that gives your position at any time t. We then define **displacement**, Δx, as the net change in

The server tosses the tennis ball straight up and hits it on its way down. Right at its peak height, the ball has zero velocity, but what's its acceleration?

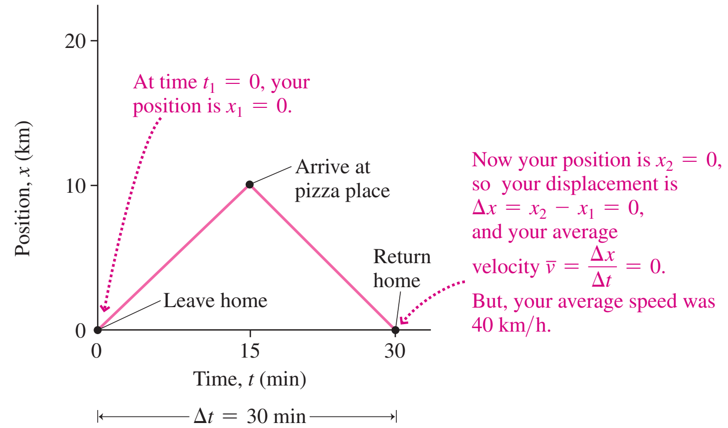

FIGURE 2.1 Position versus time for the pizza trip.

Video Tutor Demo | **Balls Take High and Low Tracks**

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PhET: The Moving Man

position: $\Delta x = x_2 - x_1$, where x_1 and x_2 are your starting and ending positions, respectively. Your **average velocity**, $\bar{v}$, is displacement divided by the time interval:

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \text{(average velocity)} \tag{2.1}$$

where $\Delta t = t_2 - t_1$ is the interval between your ending and starting times. The bar in $\bar{v}$ indicates an average quantity (and is read "v bar"). The symbol Δ (capital Greek delta) stands for "the change in." For the round trip to the pizza place, your overall displacement was zero and therefore your average velocity was also zero—even though your average *speed* was not (Fig. 2.1).

Directions and Coordinate Systems

It matters whether you go north or south, east or west. Displacement therefore includes not only *how far* but also *in what direction*. For motion in a straight line, we can describe both properties by taking position coordinates x to be positive going in one direction from some origin, and negative in the other. This gives us a one-dimensional **coordinate system**. The choice of coordinate system—both of origin and of which direction is positive—is entirely up to you. The coordinate system isn't physically real; it's just a convenience we create to help in the mathematical description of motion.

Figure 2.2 shows some Midwestern cities that lie on a north–south line. We've established a coordinate system with northward direction positive and origin at Kansas City. Arrows show displacements from Houston to Des Moines and from International Falls to Des Moines; the former is approximately +1300 km, and the latter is approximately −750 km, with the minus sign indicating a southward direction. Suppose the Houston-to-Des Moines trip takes 2.6 hours by plane; then the average velocity is $(1300\text{ km})/(2.6\text{ h}) = 500\text{ km/h}$. If the International Falls-to-Des Moines trip takes 10 h by car, then the average velocity is $(-750\text{ km})/(10\text{ h}) = -75\text{ km/h}$; again, the minus sign indicates southward.

In calculating average velocity, all that matters is the overall displacement. Maybe that trip from Houston to Des Moines was a nonstop flight going 500 km/h. Or maybe it involved a faster plane that stopped for half an hour in Kansas City. Maybe the plane even went first to Minneapolis, then backtracked to Des Moines. No matter: The displacement remains 1300 km and, as long as the total time is 2.6 h, the average velocity remains 500 km/h.

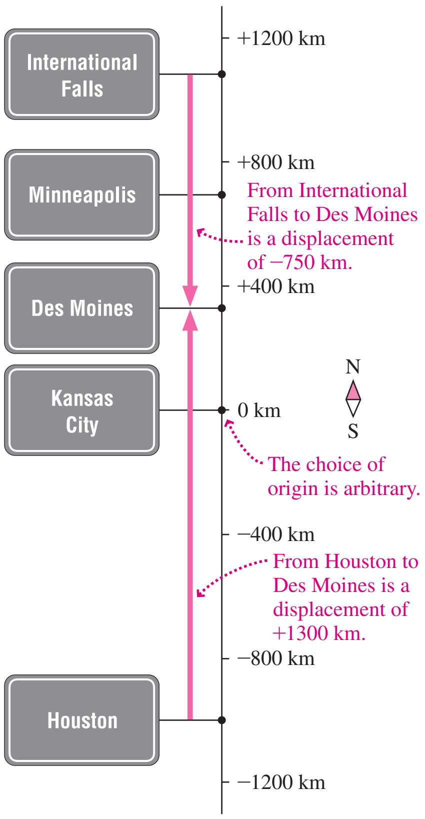


FIGURE 2.2 Describing motion in the central United States.

GOT IT? 2.1 We just described three trips from Houston to Des Moines: (a) direct; (b) with a stop in Kansas City; and (c) via Minneapolis. For which of these trips is the average speed the same as the average velocity? Where the two differ, which is greater?

EXAMPLE 2.1 Speed and Velocity: Flying with a Connection

To get a cheap flight from Houston to Kansas City—a distance of 1000 km—you have to connect in Minneapolis, 700 km north of Kansas City. The flight to Minneapolis takes 2.2 h, then you have a 30-min layover, and then a 1.3-h flight to Kansas City. What are your average velocity and your average speed on this trip?

INTERPRET We interpret this as a one-dimensional kinematics problem involving the distinction between velocity and speed, and we identify three distinct travel segments: the two flights and the layover. We identify the key concepts as speed and velocity; their distinction is clear from our pizza example.

DEVELOP Figure 2.2 is our drawing. We determine that Equation 2.1, $\bar{v} = \Delta x/\Delta t$, will give the average velocity, and that the average speed is the total distance divided by the total time. We develop our plan: Find the displacement and the total time, and use those values to get the average velocity; then find the total distance traveled and use that along with the total time to get the average speed.

EVALUATE You start in Houston and end up in Kansas City, for a displacement of 1000 km—regardless of how far you actually traveled. The total time for the three segments is $\Delta t = 2.2\text{ h} + 0.50\text{ h} + 1.3\text{ h} = 4.0\text{ h}$. Then the average velocity, from Equation 2.1, is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{1000\text{ km}}{4.0\text{ h}} = 250\text{ km/h}$$

However, that Minneapolis connection means you've gone an extra 2×700 km, for a total distance of 2400 km in 4 hours. Thus your average speed is $(2400\text{ km})/(4.0\text{ h}) = 600$ km/h, more than twice your average velocity.

ASSESS Make sense? Average velocity depends only on the net displacement between the starting and ending points. Average speed takes into account the actual distance you travel—which can be a lot longer on a circuitous trip like this one. So it's entirely reasonable that the average speed should be greater. ■

2.2 Instantaneous Velocity

Geologists determine the velocity of a lava flow by dropping a stick into the lava and timing how long it takes the stick to go a known distance (Fig. 2.3*a*). Dividing the distance by the time then gives the average velocity. But did the lava flow faster at the beginning of the interval? Or did it speed up and slow down again? To understand motion fully, including how it changes with time, we need to know the velocity at each instant.

Geologists could explore that detail with a series of observations taken over smaller intervals of time and distance (Fig. 2.3*b*). As the size of the intervals shrinks, a more detailed picture of the motion emerges. In the limit of very small intervals, we're measuring the velocity at a single instant. This is the **instantaneous velocity**, or simply the **velocity**. The magnitude of the instantaneous velocity is the **instantaneous speed**.

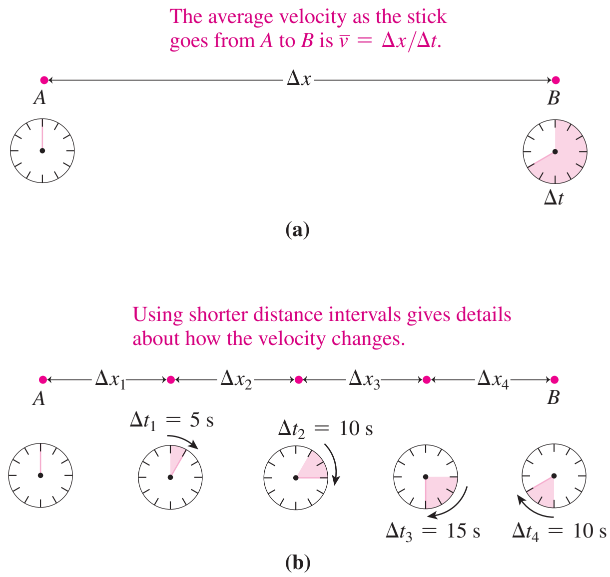


FIGURE 2.3 Determining the velocity of a lava flow.

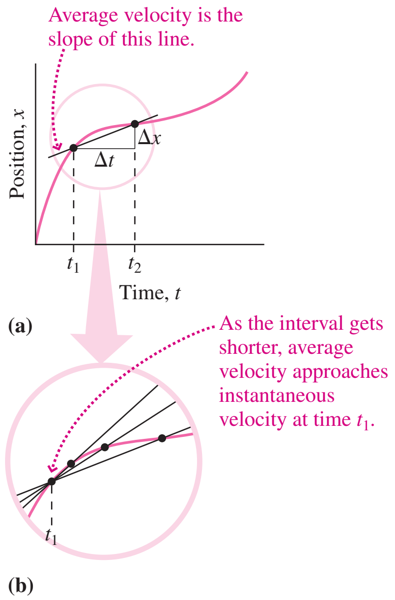


FIGURE 2.4 Position-versus-time graph for the motion in Fig. 2.3.

You might object that it's impossible to achieve that limit of an arbitrarily small time interval. With observational measurements that's true, but calculus lets us go there. Figure 2.4*a* is a plot of position versus time for the stick in the lava flow shown in Fig. 2.3. Where the curve is steep, the position changes rapidly with time—so the velocity is greater. Where the curve is flatter, the velocity is lower. Study the clocks in Fig. 2.3*b* and you'll see that the stick starts out moving rapidly, then slows, and then speeds up a bit at the end. The curve in Fig. 2.4*a* reflects this behavior.

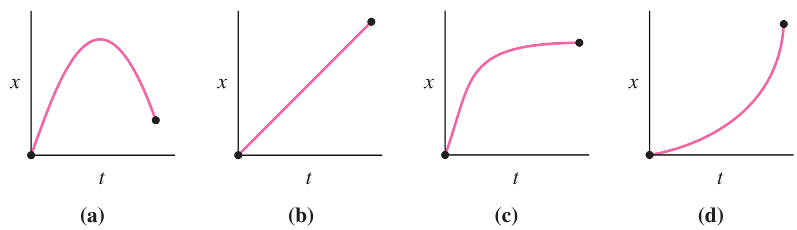
Suppose we want the instantaneous velocity at the time marked t_1 in Fig. 2.4*a*. We can approximate this quantity by measuring the displacement Δx over the interval Δt between t_1 and some later time t_2: the ratio $\Delta x/\Delta t$ is then the average velocity over this interval. Note that this ratio is the slope of a line drawn through points on the curve that mark the ends of the interval.

Figure 2.4*b* shows what happens as we make the time interval Δt arbitrarily small: Eventually, the line between the two points becomes indistinguishable from the tangent line to the curve. That tangent line has the same slope as the curve right at the point we're interested in, and therefore it defines the instantaneous velocity at that point. We write this mathematically by saying that the instantaneous velocity is the limit, as the time interval Δt becomes arbitrarily close to zero, of the ratio of displacement Δx to Δt:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad (2.2a)$$

You can imagine making the interval Δt as close to zero as you like, getting ever better approximations to the instantaneous velocity. Given a graph of position versus time, an easy approach is to "eyeball" the tangent line to the graph at a point you're interested in; its slope is the instantaneous velocity (Fig. 2.5).

GOT IT? 2.2 The figures show position-versus-time graphs for four objects. Which object is moving with constant speed? Which reverses direction? Which starts slowly and then speeds up?



Given position as a mathematical function of time, calculus provides a quick way to find instantaneous velocity. In calculus, the result of the limiting process described in Equation 2.2a is called the **derivative** of x with respect to t and is given the symbol dx/dt:

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

The quantities dx and dt are called **infinitesimals**; they represent vanishingly small quantities that result from the limiting process. We can then write Equation 2.2a as

$$v = \frac{dx}{dt} \quad \text{(instantaneous velocity)} \qquad (2.2b)$$

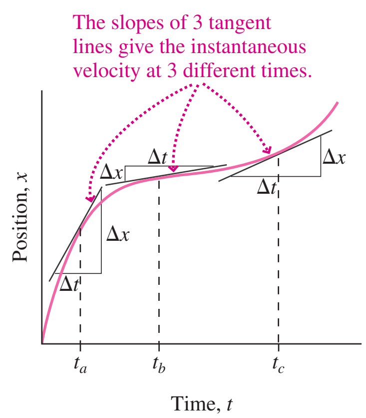


FIGURE 2.5 The instantaneous velocity is the slope of the tangent line.

Given position x as a function of time t, calculus shows how to find the velocity $v = dx/dt$. Consult Tactics 2.1 if you haven't yet seen derivatives in your calculus class or if you need a refresher.

TACTICS 2.1 Taking Derivatives

You don't have to go through an elaborate limiting process every time you want to find an instantaneous velocity. That's because calculus provides formulas for the derivatives of common functions. For example, any function of the form $x = bt^n$, where b and n are constants, has the derivative

$$\frac{dx}{dt} = nbt^{n-1} \qquad (2.3)$$

Appendix A lists derivatives of other common functions.

EXAMPLE 2.2 Instantaneous Velocity: A Rocket Ascends

The altitude of a rocket in the first half-minute of its ascent is given by $x = bt^2$, where the constant b is 2.90 m/s^2. Find a general expression for the rocket's velocity as a function of time and from it the instantaneous velocity at $t = 20$ s. Also find an expression for the average velocity, and compare your two velocity expressions.

INTERPRET We interpret this as a problem involving the comparison of two distinct but related concepts: instantaneous velocity and average velocity. We identify the rocket as the object whose velocities we're interested in.

DEVELOP Equation 2.2b, $v = dx/dt$, gives the instantaneous velocity and Equation 2.1, $\bar{v} = \Delta x/\Delta t$, gives the average velocity. Our plan is to use Equation 2.3, $dx/dt = nbt^{n-1}$, to evaluate the derivative that gives the instantaneous velocity. Then we can use Equation 2.1 for the average velocity, but first we'll need to determine the displacement from the equation we're given for the rocket's position.

EVALUATE Applying Equation 2.2b with position given by $x = bt^2$ and using Equation 2.3 to evaluate the derivative, we have

$$v = \frac{dx}{dt} = \frac{d(bt^2)}{dt} = 2bt$$

for the instantaneous velocity. Evaluating at $t = 20$ s with $b = 2.90$ m/s^2 gives $v = 116$ m/s. For the average velocity we need the total displacement at 20 s. Since $x = bt^2$, Equation 2.1 gives

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{bt^2}{t} = bt$$

where we've used $x = bt^2$ for Δx and t for Δt because both position and time are taken to be zero at liftoff. Comparison with our earlier result shows that the average velocity from liftoff to any particular time is exactly half the instantaneous velocity at that time.

ASSESS Make sense? Yes: The rocket's speed is always increasing, so its velocity at the end of any time interval is greater than the average velocity over that interval. The fact that the average velocity is exactly half the instantaneous velocity results from the quadratic (t^2) dependence of position on time.

✓TIP Language

Language often holds clues to the meaning of physical concepts. In this example we speak of the *instantaneous* velocity *at* a particular time. That wording should remind you of the limiting process that focuses on a single instant. In contrast, we speak of the *average* velocity *over* a time interval, since averaging explicitly involves a range of times.

■

2.3 Acceleration

When velocity changes, as in Example 2.2, an object is said to undergo **acceleration**. Quantitatively, we define acceleration as the rate of change of velocity, just as we defined velocity as the rate of change of position. The **average acceleration** over a time interval Δt is

$$\bar{a} = \frac{\Delta v}{\Delta t} \quad \text{(average acceleration)} \qquad (2.4)$$

where Δv is the change in velocity and the bar on $\bar{a}$ indicates that this is an average value. Just as we defined instantaneous velocity through a limiting procedure, we define **instantaneous acceleration** as

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad \text{(instantaneous acceleration)} \qquad (2.5)$$

As we did with velocity, we also use the term *acceleration* alone to mean instantaneous acceleration.

In one-dimensional motion, acceleration is either in the direction of the velocity or opposite it. In the former case the accelerating object speeds up, whereas in the latter it slows (Fig. 2.6). Although slowing is sometimes called *deceleration*, it's simpler to use

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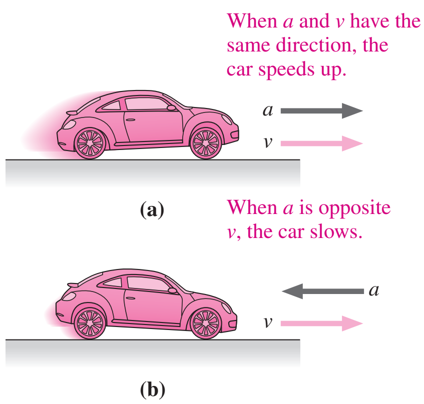


FIGURE 2.6 Acceleration and velocity.

acceleration to describe the time rate of change of velocity no matter what's happening. With two-dimensional motion, we'll find much richer relationships between the directions of velocity and acceleration.

Since acceleration is the rate of change of velocity, its units are (distance per time) per time, or distance/time^2. In SI, that's m/s^2. Sometimes acceleration is given in mixed units; for example, a car going from 0 to 60 mi/h in 10 s has an average acceleration of 6 mi/h/s.

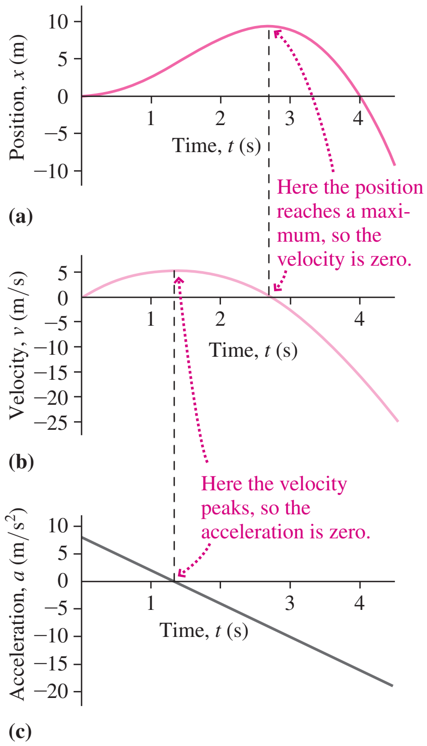


FIGURE 2.7 (a) Position, (b) velocity, and (c) acceleration versus time.

Position, Velocity, and Acceleration

Figure 2.7 shows graphs of position, velocity, and acceleration for an object undergoing one-dimensional motion. In Fig. 2.7*a*, the rise and fall of the position-versus-time curve shows that the object first moves away from the origin, reverses, then reaches the origin again at $t = 4$ s. It then continues moving into the region $x < 0$. Velocity, shown in Fig. 2.7*b*, is the slope of the position-versus-time curve in Fig. 2.7*a*. Note that the magnitude of the velocity (that is, the speed) is large where the curve in Fig. 2.7*a* is steep—that is, where position is changing most rapidly. At the peak of the position curve, the object is momentarily at rest as it reverses, so there the position curve is flat and the velocity is zero. After the object reverses, at about 2.7 s, it's heading in the negative *x*-direction and so its velocity is negative.

Just as velocity is the slope of the position-versus-time curve, acceleration is the slope of the velocity-versus-time curve. Initially that slope is positive—velocity is increasing—but eventually it peaks at the point of maximum velocity and zero acceleration and then it decreases. That velocity decrease corresponds to a negative acceleration, as shown clearly in the region of Fig. 2.7*c* beyond about 1.3 s.

CONCEPTUAL EXAMPLE 2.1 Acceleration Without Velocity?

Can an object be accelerating even though it's not moving?

EVALUATE Figure 2.7 shows that velocity is the *slope* of the position curve—and the slope depends on how the position is *changing*, not on its actual value. Similarly, acceleration depends only on the *rate of change* of velocity, not on velocity itself. So there's no intrinsic reason why there can't be acceleration at an instant when velocity is zero.

ASSESS Figure 2.8, which shows a ball thrown straight up, is a case in point. Right at the peak of its flight, the ball's velocity is instantaneously zero. But just before the peak it's moving upward, and just after it's moving downward. No matter how small a time interval you consider, the velocity is always changing. Therefore, the ball is accelerating, even right at the instant its velocity is zero.

MAKING THE CONNECTION Just 0.010 s before it peaks, the ball in Fig. 2.8 is moving upward at 0.098 m/s; 0.010 s after it peaks, it's moving downward with the same speed. What's its average acceleration over this 0.02-s interval?

EVALUATE Equation 2.4 gives the average acceleration: $\bar{a} = \Delta v/\Delta t = (-0.098\text{ m/s} - 0.098\text{ m/s})/(0.020\text{ s}) = -9.8\text{ m/s}^2$. Here we've implicitly chosen a coordinate system with a positive upward direction, so both the final velocity and the acceleration are negative. The time interval is so small that our result must be close to the instantaneous acceleration right at the peak—when the velocity is zero. You might recognize 9.8 m/s^2 as the acceleration due to the Earth's gravity.

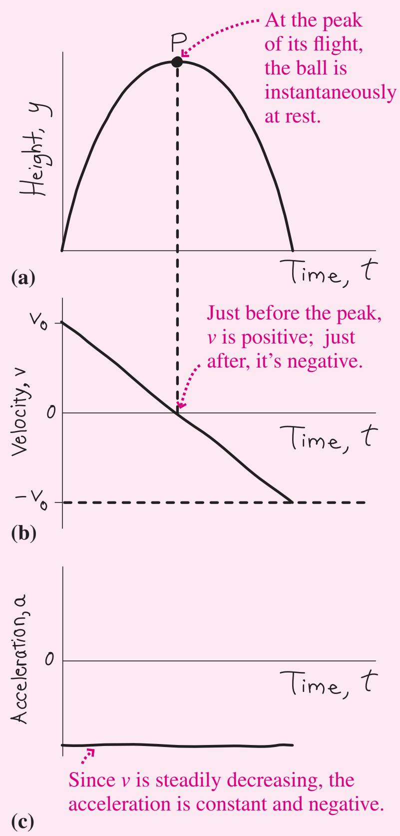


FIGURE 2.8 Our sketch for Conceptual Example 2.1.

Acceleration is the rate of change of velocity, and velocity is the rate of change of position. That makes acceleration the rate of change of the rate of change of position. Mathematically, acceleration is the **second derivative** of position with respect to time. Symbolically, we write the second derivative as d^2x/dt^2; this is just a symbol and doesn't mean that anything is actually squared. Then the relationship among acceleration, velocity, and position can be written

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.6}$$

Equation 2.6 expresses acceleration in terms of position through the calculus operation of taking the second derivative. If you've studied integrals in calculus, you can see that it should be possible to go the opposite way, finding position as a function of time given acceleration as a function of time. In Section 2.4 we'll do this for the special case of constant acceleration, although there we'll take an algebra-based approach; Problem 87 obtains the same results using calculus. We'll take a quick look at nonconstant acceleration in Section 2.6. The Application on this page provides an important technology that finds an object's position from its acceleration.

GOT IT? 2.3 An elevator is going up at constant speed, slows to a stop, then starts down and soon reaches the same constant speed it had going up. Is the elevator's average acceleration between its upward and downward constant-speed motions (a) zero, (b) downward, (c) first upward and then downward, or (d) first downward and then upward?

2.4 Constant Acceleration

The description of motion has an especially simple form when acceleration is constant. Suppose an object starts at time $t = 0$ with some initial velocity v_0 and constant acceleration a. Later, at some time t, it has velocity v. Because the acceleration doesn't change, its average and instantaneous values are identical, so we can write

$$a = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$$

or, rearranging,

$$v = v_0 + at \qquad \text{(for constant acceleration only)} \tag{2.7}$$

This equation says that the velocity changes from its initial value by an amount that is the product of acceleration and time.

✓TIP Know Your Limits

Many equations we develop are special cases of more general laws, and they're limited to special circumstances. Equation 2.7 is a case in point: It applies *only when acceleration is constant*.

Having determined velocity as a function of time, we now consider position. With constant acceleration, velocity increases steadily—and thus the average velocity over an interval is the average of the velocities at the beginning and the end of that interval. So we can write

$$\bar{v} = \tfrac{1}{2}(v_0 + v) \tag{2.8}$$

for the average velocity over the interval from $t = 0$ to some later time when the velocity is v. We can also write the average velocity as the change in position divided by the time interval. Suppose that at time 0 our object was at position x_0. Then its average velocity over a time interval from 0 to time t is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0}$$

APPLICATION **Inertial Guidance**

Given an object's initial position and velocity, and its subsequent acceleration—which may vary with time—it's possible to invert Equation 2.6 and solve for position (more on the mathematics of this inversion in Section 2.6). *Inertial guidance systems*, also called *inertial navigation systems*, exploit this principle to allow submarines, ships, and airplanes to keep track of their locations based solely on internal measurements of their own acceleration. This frees them from the need for external positioning references such as the Global Positioning System (GPS), radar, or direct observation. Inertial guidance is especially important for submarines, which usually can't access external sources for information about their positions. In the one-dimensional motion of this chapter, an inertial guidance system would consist of a single accelerometer whose reading is tracked continually. In practical systems, three accelerometers at right angles track acceleration in all three dimensions. Information from on-board gyroscopes registers orientation, so the system "knows" the changing directions of the three accelerations.

Early inertial guidance systems were heavy and expensive, but the miniaturization of accelerometers and gyroscopes—so that they're now in every smartphone—has enabled smaller and less expensive inertial guidance systems. The photo shows a complete inertial navigation system developed by the U.S. Defense Advanced Research Projects Agency (DARPA) for use in locations where GPS signals aren't available; it's so small that it fits within the Lincoln Memorial on a penny!

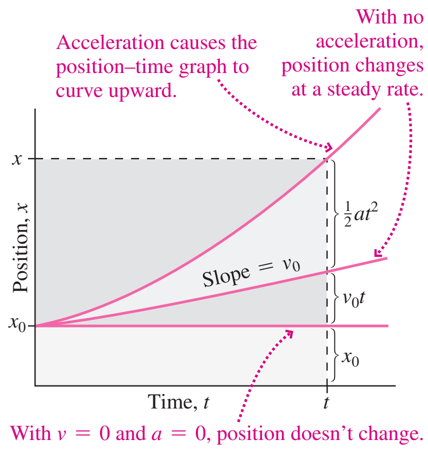


FIGURE 2.9 Meaning of the terms in Equation 2.10.

where x is the object's position at time t. Equating this expression for $\bar{v}$ with the expression in Equation 2.8 gives

$$x = x_0 + \bar{v}t = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad (2.9)$$

But we already found the instantaneous velocity v that appears in this expression; it's given by Equation 2.7. Substituting and simplifying then give the position as a function of time:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad \text{(for constant acceleration only)} \qquad (2.10)$$

Does Equation 2.10 make sense? With no acceleration ($a = 0$), position would increase linearly with time, at a rate given by the initial velocity v_0. With constant acceleration, the additional term $\frac{1}{2}at^2$ describes the effect of the ever-changing velocity; time is squared because the longer the object travels, the faster it moves, so the more distance it covers in a given time. Figure 2.9 shows the meaning of the terms in Equation 2.10.

How much runway do I need to land a jetliner, given touchdown speed and a constant acceleration? A question like this involves position, velocity, and acceleration without explicit mention of time. So we solve Equation 2.7 for time, $t = (v - v_0)/a$, and substitute this expression for t in Equation 2.9 to write

$$x - x_0 = \frac{1}{2}\frac{(v_0 + v)(v - v_0)}{a}$$

or, since $(a + b)(a - b) = a^2 - b^2$,

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (2.11)$$

Table 2.1 Equations of Motion for Constant Acceleration

Equation	Contains	Number
$v = v_0 + at$	v, a, t; no x	2.7
$x = x_0 + \frac{1}{2}(v_0 + v)t$	x, v, t; no a	2.9
$x = x_0 + v_0t + \frac{1}{2}at^2$	x, a, t; no v	2.10
$v^2 = v_0^2 + 2a(x - x_0)$	x, v, a; no t	2.11

Equations 2.7, 2.9, 2.10, and 2.11 link all possible combinations of position, velocity, and acceleration for motion with constant acceleration. We summarize them in Table 2.1, and remind you that they apply *only* in the case of constant acceleration.

Although we derived these equations algebraically, we could instead have used calculus. Problem 87 takes this approach in getting from Equation 2.7 to Equation 2.10.

Using the Equations of Motion

The equations in Table 2.1 fully describe motion under constant acceleration. Don't regard them as separate laws, but recognize them as complementary descriptions of a single underlying phenomenon—one-dimensional motion with *constant acceleration*. Having several equations provides convenient starting points for approaching problems. Don't memorize these equations, but grow familiar with them as you work problems. We now offer a strategy for solving problems about one-dimensional motion with *constant acceleration* using these equations.

PROBLEM-SOLVING STRATEGY 2.1 Motion with Constant Acceleration

INTERPRET Interpret the problem to be sure it asks about motion with *constant acceleration*. Next, identify the object(s) whose motion you're interested in.

DEVELOP Draw a diagram with appropriate labels, and choose a coordinate system. For instance, sketch the initial and final physical situations, or draw a position-versus-time graph. Then determine which equations of motion from Table 2.1 contain the quantities you're given and will be easiest to solve for the unknown(s).

EVALUATE Solve the equations in symbolic form and then evaluate numerical quantities.

ASSESS Does your answer make sense? Are the units correct? Do the numbers sound reasonable? What happens in special cases—for example, when a distance, velocity, acceleration, or time becomes very large or very small?

The next two examples are typical of problems involving constant acceleration. Example 2.3 is a straightforward application of the equations we've just derived to a single object. Example 2.4 involves two objects, in which case we need to write equations describing the motions of both objects.

EXAMPLE 2.3 Motion with Constant Acceleration: Landing a Jetliner

A jetliner touches down at 270 km/h. The plane then decelerates (i.e., undergoes acceleration directed opposite its velocity) at 4.5 m/s^2. What's the minimum runway length on which this aircraft can land?

INTERPRET We interpret this as a problem involving one-dimensional motion with constant acceleration and identify the airplane as the object of interest.

DEVELOP We determine that Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, relates distance, velocity, and acceleration; so our plan is to solve that equation for the minimum runway length. We want the airplane to come to a stop, so the final velocity v is 0, and v_0 is the initial touchdown velocity. If x_0 is the touchdown point, then the quantity $x - x_0$ is the distance we're interested in; we'll call this Δx.

EVALUATE Setting $v = 0$ and solving Equation 2.11 then give

$$\Delta x = \frac{-v_0^2}{2a} = \frac{-[(270 \text{ km/h})(1000 \text{ m/km})(1/3600 \text{ h/s})]^2}{(2)(-4.5 \text{ m/s}^2)} = 625 \text{ m}$$

Note that we used a negative value for the acceleration because the plane's acceleration is directed opposite its velocity—which we chose as the positive x-direction. We also converted the speed to m/s for compatibility with the SI units given for acceleration.

ASSESS Make sense? That 625 m is just over one-third of a mile, which seems a bit short. However, this is an absolute minimum with no margin of safety. For full-size jetliners, the standard for minimum landing runway length is about 5000 feet or 1.5 km.

✓TIP Be Careful with Mixed Units

Frequently problems are stated in units other than SI. Although it's possible to work consistently in other units, when in doubt, convert to SI. In this problem, the acceleration is originally in SI units but the velocity isn't—a sure indication of the need for conversion.

■

EXAMPLE 2.4 Motion with Two Objects: Speed Trap!

A speeding motorist zooms through a 50 km/h zone at 75 km/h (that's 21 m/s) without noticing a stationary police car. The police officer immediately heads after the speeder, accelerating at 2.5 m/s^2. When the officer catches up to the speeder, how far down the road are they, and how fast is the police car going?

INTERPRET We interpret this as *two* problems involving one-dimensional motion with constant acceleration. We identify the objects in question as the speeding car and the police car. Their motions are related because we're interested in the point where the two coincide.

DEVELOP It's helpful to draw a sketch showing qualitatively the position-versus-time graphs for the two cars. Since the speeding car moves with constant speed, its graph is a straight line. The police car is accelerating from rest, so its graph starts flat and gets increasingly steeper. Our sketch in Fig. 2.10 shows clearly the point we're interested in, when the two cars coincide for the second time. Equation 2.10, $x = x_0 + v_0 t + \frac{1}{2}at^2$, gives position versus time with constant acceleration. Our plan is (1) to write versions of this equation specialized to each car, (2) to equate the resulting position expressions to find the time when the cars coincide, and (3) to find the corresponding position and the police car's velocity. For the latter we'll use Equation 2.7, $v = v_0 + at$.

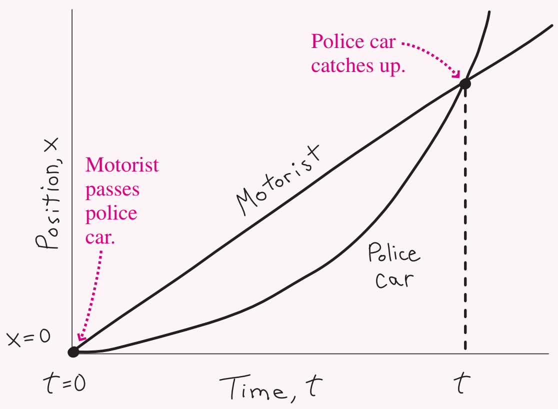


FIGURE 2.10 Our sketch of position versus time for the cars in Example 2.4.

EVALUATE Let's take the origin to be the point where the speeder passes the police car and $t = 0$ to be the corresponding time, as marked in Fig. 2.10. Then $x_0 = 0$ in Equation 2.10 for both cars, while the speeder has no acceleration and the police car has no initial velocity. Thus our two versions of Equation 2.10 are

$$x_s = v_{s0}t \quad \text{(speeder)} \quad \text{and} \quad x_p = \tfrac{1}{2}a_p t^2 \text{ (police car)}$$

Equating x_s and x_p tells when the speeder and the police car are at the same place, so we write $v_{s0}t = \frac{1}{2}a_p t^2$. This equation is satisfied when $t = 0$ or $t = 2v_{s0}/a_p$. Why two answers? We asked for *any* times when the two cars are in the same place. That includes the initial encounter at $t = 0$ as well as the later time $t = 2v_{s0}/a_p$ when the police car catches the speeder; both points are shown on our sketch. *Where* does this occur? We can evaluate using $t = 2v_{s0}/a_p$ in the speeder's equation:

$$x_s = v_{s0}t = v_{s0}\frac{2v_{s0}}{a_p} = \frac{2v_{s0}^2}{a_p} = \frac{(2)(21 \text{ m/s})^2}{2.5 \text{ m/s}^2} = 350 \text{ m}$$

Equation 2.7 then gives the police car's speed at this time:

$$v_p = a_p t = a_p\frac{2v_{s0}}{a_p} = 2v_{s0} = 150 \text{ km/h}$$

ASSESS Make sense? As Fig. 2.10 shows, the police car starts from rest and undergoes constant acceleration, so it has to be going faster at the point where the two cars meet. In fact, it's going twice as fast—again, as in Example 2.2, that's because the police car's position depends quadratically on time. That quadratic dependence also tells us that the police car's position-versus-time graph in Fig. 2.10 is a parabola. ■

FIGURE 2.11 Strobe photo of a falling ball. Successive images are farther apart, showing that the ball is accelerating.

GOT IT? 2.4 The police car in Example 2.4 starts with zero velocity and is going at twice the car's velocity when it catches up to the car. So at some intermediate instant it must be going at the same velocity as the car. Is that instant (a) halfway between the times when the two cars coincide, (b) closer to the time when the speeder passes the stationary police car, or (c) closer to the time when the police car catches the speeder?

2.5 The Acceleration of Gravity

Drop an object, and it falls at an increasing rate, accelerating because of gravity (Fig. 2.11). The acceleration is constant for objects falling near Earth's surface, and furthermore it has the same value for all objects. This value, the **acceleration of gravity**, is designated g and is approximately 9.8 m/s^2 near Earth's surface.

The acceleration of gravity applies strictly only in **free fall**—motion under the influence of gravity alone. Air resistance, in particular, may dramatically alter the motion, giving the false impression that gravity acts differently on lighter and heavier objects. As early as the year 1600, Galileo is reputed to have shown that all objects have the same acceleration by dropping objects off the Leaning Tower of Pisa. Astronauts have verified that a feather and a hammer fall with the same acceleration on the airless Moon—although that acceleration is less than on Earth.

Although g is approximately constant near Earth's surface, it varies slightly with latitude and even local geology. The variation with altitude becomes substantial over distances of tens to hundreds of kilometers. But nearer Earth's surface it's a good approximation to take g as strictly constant. Then an object in free fall undergoes constant acceleration, and the equations of Table 2.1 apply. In working gravitational problems, we usually replace x with y to designate the vertical direction. If we make the arbitrary but common choice that the upward direction is positive, then acceleration a becomes $-g$ because the acceleration is downward.

EXAMPLE 2.5 Constant Acceleration due to Gravity: Cliff Diving

A diver drops from a 10-m-high cliff. At what speed does he enter the water, and how long is he in the air?

INTERPRET This is a case of constant acceleration due to gravity, and the diver is the object of interest. The diver drops a known distance starting from rest, and we want to know the speed and time when he hits the water.

DEVELOP Figure 2.12 is a sketch showing what the diver's position versus time should look like. We've incorporated what we know: the initial position 10 m above the water, the start from rest, and the downward acceleration that results in a parabolic position-versus-time curve. Given the dive height, Equation 2.11 determines the speed v. Following our newly adopted convention that y designates the vertical direction, we write Equation 2.11 as $v^2 = v_0^2 + 2a(y - y_0)$. Since the diver starts from rest, $v_0 = 0$ and the equation becomes $v^2 = -2g(y - y_0)$. So our plan is first to solve for the speed at the water; then use Equation 2.7, $v = v_0 + at$, to get the time.

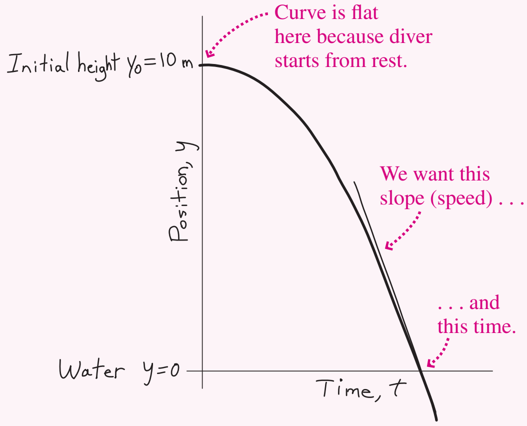


FIGURE 2.12 Our sketch for Example 2.5.

EVALUATE Our sketch shows that we've chosen $y = 0$ at the water, so $y_0 = 10$ m and Equation 2.11 gives

$$|v| = \sqrt{-2g(y - y_0)} = \sqrt{(-2)(9.8\ \text{m/s}^2)(0\ \text{m} - 10\ \text{m})} = 14\ \text{m/s}$$

This is the magnitude of the velocity, hence the absolute value sign; the actual value is $v = -14$ m/s, with the minus sign indicating downward motion. Knowing the initial and final velocities, we use Equation 2.7 to find how long the dive takes. Solving that equation for t gives

$$t = \frac{v_0 - v}{g} = \frac{0\ \text{m/s} - (-14\ \text{m/s})}{9.8\ \text{m/s}^2} = 1.4\ \text{s}$$

Note the careful attention to signs here; we wrote v with its negative sign and used $a = -g$ in Equation 2.7 because we defined downward to be the negative direction in our coordinate system.

ASSESS Make sense? Our expression for v gives a higher speed with a greater acceleration or a greater distance $y - y_0$—both as expected. Our approach here isn't the only one possible; we could also have found the time by solving Equation 2.10 and then evaluating the speed using Equation 2.7. ■

In Example 2.5 the diver was moving downward, and the downward gravitational acceleration steadily increased his speed. But, as Conceptual Example 2.1 suggested, the acceleration of gravity is downward regardless of an object's motion. Throw a ball straight up, and it's accelerating *downward* even while moving *upward*. Since velocity and acceleration are in opposite directions, the ball slows until it reaches its peak, then pauses instantaneously, and then gains speed as it falls. All the while its acceleration is 9.8 m/s^2 downward.

EXAMPLE 2.6 Constant Acceleration due to Gravity: Tossing a Ball

You toss a ball straight up at 7.3 m/s; it leaves your hand at 1.5 m above the floor. Find when it hits the floor, the maximum height it reaches, and its speed when it passes your hand on the way down.

INTERPRET We have constant acceleration due to gravity, and here the object of interest is the ball. We want to find time, height, and speed.

DEVELOP The ball starts by going up, eventually comes to a stop, and then heads downward. Figure 2.13 is a sketch of the height versus time that we expect, showing what we know and the three quantities we're after. Equation 2.10, $y = y_0 + v_0 t + \frac{1}{2}at^2$, determines position as a function of time, so our plan is to use that equation to find the time the ball hits the floor (again, we've replaced horizontal position x with height y in Equation 2.10). Then we can use Equation 2.11, $v^2 = v_0^2 + 2a(y - y_0)$, to find the height at which $v = 0$—that is, the peak height. Finally, Equation 2.11 will also give us the speed at any height, letting us answer the question about the speed when the ball passes the height of 1.5 m on its way down.

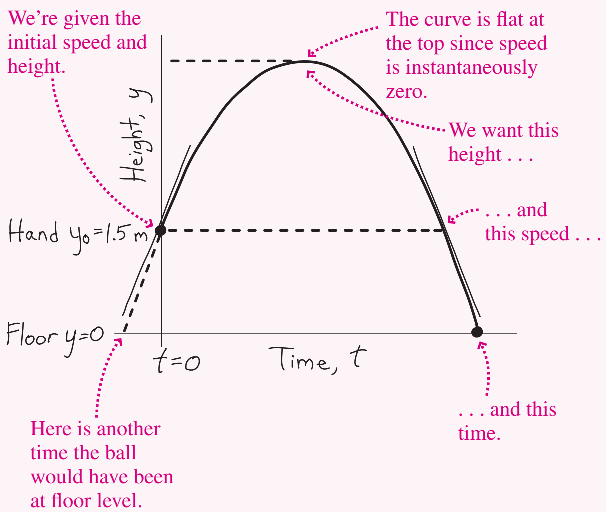


FIGURE 2.13 Our sketch for Example 2.6.

EVALUATE Our sketch shows that we've taken $y = 0$ at the floor; so when the ball is at the floor, Equation 2.10 becomes $0 = y_0 + v_0 t - \frac{1}{2}gt^2$, which we can solve for t using the quadratic formula [Appendix A; $t = (v_0 \pm \sqrt{v_0^2 + 2y_0 g})/g$]. Here v_0 is the initial velocity, 7.3 m/s; it's positive because the motion is initially upward. The initial position is the hand height, so $y_0 = 1.5$ m, and g of course is 9.8 m/s^2 (we accounted for the downward acceleration by putting $a = -g$ in Equation 2.10). Putting in these numbers gives $t = 1.7$ s or -0.18 s; the answer we want is 1.7 s. At the peak of its flight, the ball's velocity is instantaneously zero because it's moving neither up nor down. So we set $v^2 = 0$ in Equation 2.11 to get $0 = v_0^2 - 2g(y - y_0)$. Solving for y then gives the peak height:

$$y = y_0 + \frac{v_0^2}{2g} = 1.5 \text{ m} + \frac{(7.3 \text{ m/s})^2}{(2)(9.8 \text{ m/s}^2)} = 4.2 \text{ m}$$

To find the speed when the ball reaches 1.5 m on the way down, we set $y = y_0$ in Equation 2.11. The result is $v^2 = v_0^2$, so $v = \pm v_0$ or ± 7.3 m/s. Once again, there are two answers. The equation has given us *all* the velocities the ball has at 1.5 m—including the initial upward velocity and the later downward velocity. We've shown here that an upward-thrown object returns to its initial height with the same speed it had initially.

ASSESS Make sense? With no air resistance to sap the ball of its energy, it seems reasonable that the ball comes back down with the same speed—a fact we'll explore further when we introduce energy conservation in Chapter 7. But why are there two answers for time and velocity? Equation 2.10 doesn't "know" about your hand or the floor; it "assumes" the ball has always been undergoing downward acceleration g. We asked of Equation 2.10 when the ball would be at $y = 0$. The second answer, 1.7 s, was the one we wanted. But if the ball had always been in free fall, it would also have been on the floor 0.18 s earlier, heading upward. That's the meaning of the other answer, -0.18 s, as we've indicated on our sketch. Similarly, Equation 2.11 gave us all the velocities the ball had at a height of 1.5 m, including both the initial upward velocity and the later downward velocity. ■

✓TIP Multiple Answers

Frequently the mathematics of a problem gives more than one answer. Think about what each answer means before discarding it! Sometimes an answer isn't consistent with the physical assumptions of the problem, but other times all answers are meaningful even if they aren't all what you're looking for.

GOT IT? 2.5 Standing on a roof, you simultaneously throw one ball straight up and drop another from rest. Which hits the ground first? Which hits the ground moving faster?

APPLICATION **Keeping Time**

The NIST-F1 atomic clock, shown here with its developers, sets the U.S. standard of time. The clock is so accurate that it won't gain or lose more than a second in 100 million years! It gets its remarkable accuracy by monitoring a super-cold clump of freely falling cesium atoms for what is, in this context, a long time period of about 1 s. The atom clump is put in free fall by a more sophisticated version of the ball toss in Example 2.6. In the NIST-F1 clock, laser beams gently "toss" the ball of atoms upward with a speed that gives it an up-and-down travel time of about 1 s (see Problem 66). For this reason NIST-F1 is called an atomic fountain clock. In the photo you can see the clock's towerlike structure that accommodates this atomic fountain.

2.6 When Acceleration Isn't Constant

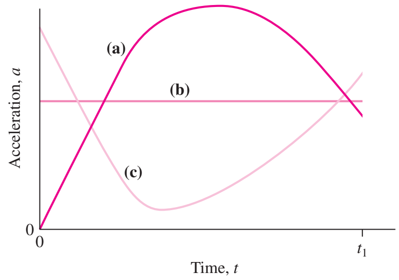
Sections 2.4 and 2.5 both dealt with *constant acceleration*. Fortunately, there are many important applications, such as situations involving gravity near Earth's surface, where acceleration *is* constant. But when it isn't, then the equations listed in Table 2.1 don't apply. In Chapter 3 you'll see that acceleration can vary in magnitude, direction, or both. In the one-dimensional situations of the current chapter, a nonconstant acceleration a would be specified by giving a as a function of time t: $a(t)$. If you've already studied integral calculus, then you know that integration is the opposite of differentiation. Since acceleration is the derivative of velocity, you get from acceleration to velocity by integration; from there you can get to position by integrating again. Mathematically, we express these relations as

$$v(t) = \int a(t)\, dt \qquad (2.12)$$

$$x(t) = \int v(t)\, dt \qquad (2.13)$$

These results don't fully determine v and x; you also need to know the *initial conditions* (usually, the values at time $t = 0$); these provide what are called in calculus the constants of integration. In Problem 87, you can evaluate the integrals in Equations 2.12 and 2.13 for the case of constant acceleration, giving an alternate derivation of Equations 2.7 and 2.10. Problems 82, 88, and 89 challenge you to use integral calculus to find an object's position in the case of nonconstant accelerations, while Problem 90 explores the case of an exponentially decreasing acceleration.

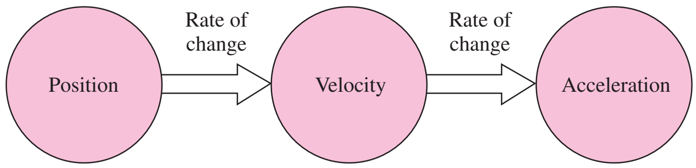
GOT IT? 2.6 The graph shows acceleration versus time for three different objects, all of which start at rest from the same position. Only object (b) undergoes constant acceleration. Which object is going fastest at the time t_1?



CHAPTER 2 SUMMARY

Big Idea

The big ideas here are those of **kinematics**—the study of motion without regard to its cause. **Position**, **velocity**, and **acceleration** are the quantities that characterize motion:

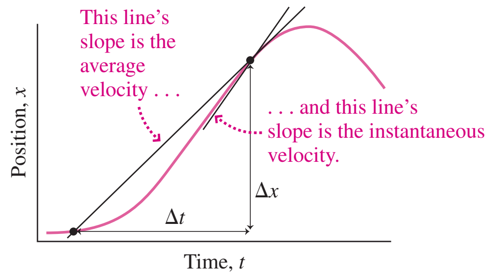


Key Concepts and Equations

Average velocity and acceleration involve changes in position and velocity, respectively, occurring over a time interval Δt:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

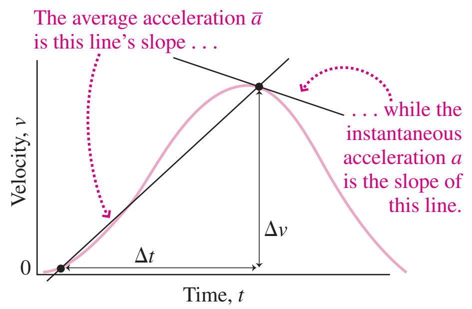


Here Δx is the **displacement**, or change in position, and Δv is the change in velocity.

Instantaneous values are the limits of infinitesimally small time intervals and are given by calculus as the time derivatives of position and velocity:

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$



Applications

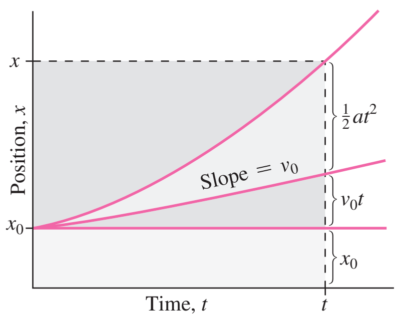
Constant acceleration is a special case that yields simple equations describing one-dimensional motion:

$$v = v_0 + at$$

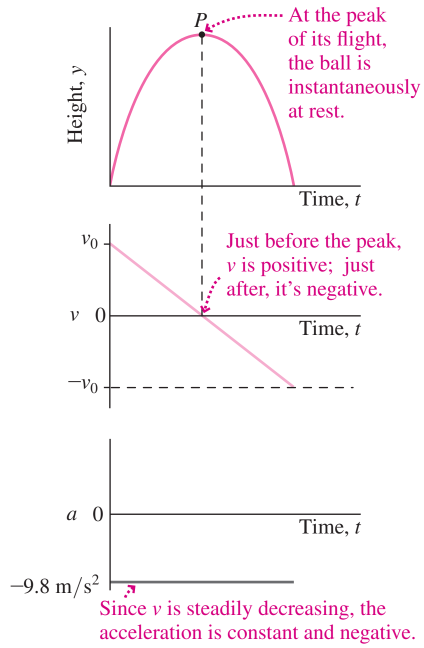
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

These equations apply only in the case of constant acceleration.



An important example is the acceleration of gravity, essentially constant near Earth's surface, with magnitude approximately 9.8 m/s^2.



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

BIO *Biology and/or medicine-related problems* **DATA** *Data problems* **ENV** *Environmental problems* **CH** *Challenge problems* **COMP** *Computer problems*

For Thought and Discussion

1. Under what conditions are average and instantaneous velocity equal?
2. Does a speedometer measure speed or velocity?
3. You check your odometer at the beginning of a day's driving and again at the end. Under what conditions would the difference between the two readings represent your displacement?
4. Consider two possible definitions of average speed: (a) the average of the values of the instantaneous speed over a time interval and (b) the magnitude of the average velocity. Are these definitions equivalent? Give two examples to demonstrate your conclusion.
5. Is it possible to be at position $x = 0$ and still be moving?
6. Is it possible to have zero velocity and still be accelerating?
7. If you know the initial velocity v_0 and the initial and final heights y_0 and y, you can use Equation 2.10 to solve for the time t when the object will be at height y. But the equation is quadratic in t, so you'll get two answers. Physically, why is this?
8. Starting from rest, an object undergoes acceleration given by $a = bt$, where t is time and b is a constant. Can you use bt for a in Equation 2.10 to predict the object's position as a function of time? Why or why not?
9. In which of the velocity-versus-time graphs shown in Fig. 2.14 would the average velocity over the interval shown equal the average of the velocities at the ends of the interval?

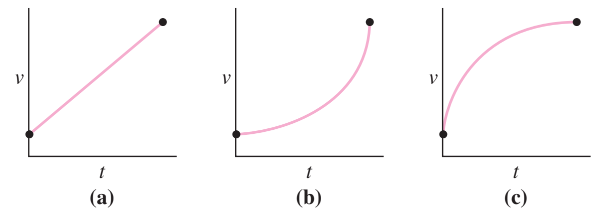


FIGURE 2.14 For Thought and Discussion 9

10. If you travel in a straight line at 50 km/h for 1 h and at 100 km/h for another hour, is your average velocity 75 km/h? If not, is it more or less?
11. If you travel in a straight line at 50 km/h for 50 km and then at 100 km/h for another 50 km, is your average velocity 75 km/h? If not, is it more or less?

Exercises and Problems

Exercises

Section 2.1 Average Motion

12. In 2009, Usain Bolt of Jamaica set a world record in the 100-m dash with a time of 9.58 s. What was his average speed?
13. The standard 26-mile, 385-yard marathon dates to 1908, when the Olympic marathon started at Windsor Castle and finished before the Royal Box at London's Olympic Stadium. Today's top marathoners achieve times around 2 hours, 3 minutes for the standard marathon. (a) What's the average speed of a marathon run in this time? (b) Marathons before 1908 were typically about 25 miles. How much longer does the race last today as a result of the extra mile and 385 yards, assuming it's run at part (a)'s average speed?
14. Starting from home, you bicycle 24 km north in 2.5 h and then turn around and pedal straight home in 1.5 h. What are your (a) displacement at the end of the first 2.5 h, (b) average velocity over the first 2.5 h, (c) average velocity for the homeward leg of the trip, (d) displacement for the entire trip, and (e) average velocity for the entire trip?
15. The Voyager 1 spacecraft is expected to continue broadcasting data until at least 2020, when it will be some 14 billion miles from Earth. How long will it take Voyager's radio signals, traveling at the speed of light, to reach Earth from this distance?
16. In 2008, Australian Emma Snowsill set an unofficial record in the women's Olympic triathlon, completing the 1.5-km swim, 40-km bicycle ride, and 10-km run in 1 h, 58 min, 27.66 s. What was her average speed?
17. Taking Earth's orbit to be a circle of radius 1.5×10^8 km, determine Earth's orbital speed in (a) meters per second and (b) miles per second.
18. What's the conversion factor from meters per second to miles per hour?

Section 2.2 Instantaneous Velocity

19. On a single graph, plot distance versus time for the first two trips from Houston to Des Moines described on page 16. For each trip, identify graphically the average velocity and, for each segment of the trip, the instantaneous velocity.
20. For the motion plotted in Fig. 2.15, estimate (a) the greatest velocity in the positive x-direction, (b) the greatest velocity in the negative x-direction, (c) any times when the object is instantaneously at rest, and (d) the average velocity over the interval shown.

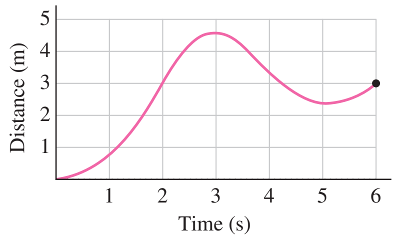


FIGURE 2.15 Exercise 20

21. A model rocket is launched straight upward. Its altitude y as a function of time is given by $y = bt - ct^2$, where $b = 82$ m/s, $c = 4.9$ m/s^2, t is the time in seconds, and y is in meters. (a) Use differentiation to find a general expression for the rocket's velocity as a function of time. (b) When is the velocity zero?

Section 2.3 Acceleration

22. A giant eruption on the Sun propels solar material from rest to 450 km/s over a period of 1 h. Find the average acceleration.
23. Starting from rest, a subway train first accelerates to 25 m/s, then brakes. Forty-eight seconds after starting, it's moving at 17 m/s. What's its average acceleration in this 48-s interval?

24. A space shuttle's main engines cut off 8.5 min after launch, at which time its speed is 7.6 km/s. What's the shuttle's average acceleration during this interval?
25. An egg drops from a second-story window, taking 1.12 s to fall and reaching 11.0 m/s just before hitting the ground. On contact, the egg stops completely in 0.131 s. Calculate the magnitudes of its average acceleration (a) while falling and (b) while stopping.
26. An airplane's takeoff speed is 320 km/h. If its average acceleration is 2.9 m/s^2, how much time is it accelerating down the runway before it lifts off?
27. ThrustSSC, the world's first supersonic car, accelerates from rest to 1000 km/h in 16 s. What's its acceleration?

Section 2.4 Constant Acceleration

28. You're driving at 70 km/h when you apply constant acceleration to pass another car. Six seconds later, you're doing 80 km/h. How far did you go in this time?
29. Differentiate both sides of Equation 2.10, and show that you get Equation 2.7.
30. An X-ray tube gives electrons constant acceleration over a distance of 15 cm. If their final speed is 1.2×10^7 m/s, what are (a) the electrons' acceleration and (b) the time they spend accelerating?
31. A rocket rises with constant acceleration to an altitude of 85 km, at which point its speed is 2.8 km/s. (a) What's its acceleration? (b) How long does the ascent take?
32. Starting from rest, a car accelerates at a constant rate, reaching 88 km/h in 12 s. Find (a) its acceleration and (b) how far it goes in this time.
33. A car moving initially at 50 mi/h begins slowing at a constant rate 100 ft short of a stoplight. If the car comes to a full stop just at the light, what is the magnitude of its acceleration?
34. **BIO** In a medical X-ray tube, electrons are accelerated to a velocity of 10^8 m/s and then slammed into a tungsten target. As they stop, the electrons' rapid acceleration produces X rays. If the time for an electron to stop is on the order of 10^{-9} s, approximately how far does it move while stopping?
35. California's Bay Area Rapid Transit System (BART) uses an automatic braking system triggered by earthquake warnings. The system is designed to prevent disastrous accidents involving trains traveling at a maximum of 112 km/h and carrying a total of some 45,000 passengers at rush hour. If it takes a train 24 s to brake to a stop, how much advance warning of an earthquake is needed to bring a 112-km/h train to a reasonably safe speed of 42 km/h when the earthquake strikes?
36. You're driving at speed v_0 when you spot a stationary moose on the road, a distance d ahead. Find an expression for the magnitude of the acceleration you need if you're to stop before hitting the moose.

Section 2.5 The Acceleration of Gravity

37. You drop a rock into a deep well and 4.4 s later hear a splash. How far down is the water? Neglect the travel time of sound.
38. Your friend is sitting 6.5 m above you on a tree branch. How fast should you throw an apple so it just reaches her?
39. A model rocket leaves the ground, heading straight up at 49 m/s. (a) What's its maximum altitude? Find its speed and altitude at (b) 1 s, (c) 4 s, and (d) 7 s.
40. A foul ball leaves the bat going straight up at 23 m/s. (a) How high does it rise? (b) How long is it in the air? Neglect the distance between bat and ground.
41. A Frisbee is lodged in a tree 6.5 m above the ground. A rock thrown from below must be going at least 3 m/s to dislodge the Frisbee. How fast must such a rock be thrown upward if it leaves the thrower's hand 1.3 m above the ground?
42. Space pirates kidnap an earthling and hold him on one of the solar system's planets. With nothing else to do, the prisoner amuses himself by dropping his watch from eye level (170 cm) to the floor. He observes that the watch takes 0.95 s to fall. On what planet is he being held? (*Hint*: Consult Appendix E.)

Problems

43. You allow 40 min to drive 25 mi to the airport, but you're caught in heavy traffic and average only 20 mi/h for the first 15 min. What must your average speed be on the rest of the trip if you're to make your flight?
44. A base runner can get from first to second base in 3.4 s. If he leaves first as the pitcher throws a 90 mi/h fastball the 61-ft distance to the catcher, and if the catcher takes 0.45 s to catch and rethrow the ball, how fast does the catcher have to throw the ball to second base to make an out? Home plate to second base is the diagonal of a square 90 ft on a side.
45. You can run 9.0 m/s, 20% faster than your brother. How much head start should you give him in order to have a tie race over 100 m?
46. A jetliner leaves San Francisco for New York, 4600 km away. With a strong tailwind, its speed is 1100 km/h. At the same time, a second jet leaves New York for San Francisco. Flying into the wind, it makes only 700 km/h. When and where do the two planes pass?
47. An object's position is given by $x = bt + ct^3$, where $b = 1.50$ m/s, $c = 0.640$ m/s^3, and t is time in seconds. To study the limiting process leading to the instantaneous velocity, calculate the object's average velocity over time intervals from (a) 1.00 s to 3.00 s, (b) 1.50 s to 2.50 s, and (c) 1.95 s to 2.05 s. (d) Find the instantaneous velocity as a function of time by differentiating, and compare its value at 2 s with your average velocities.
48. An object's position as a function of time t is given by $x = bt^4$, with b a constant. Find an expression for the instantaneous velocity, and show that the average velocity over the interval from $t = 0$ to any time t is one-fourth of the instantaneous velocity at t.
49. In a drag race, the position of a car as a function of time is given by $x = bt^2$, with $b = 2.000$ m/s^2. In an attempt to determine the car's velocity midway down a 400-m track, two observers stand at the 180-m and 220-m marks and note when the car passes. (a) What value do the two observers compute for the car's velocity over this 40-m stretch? Give your answer to four significant figures. (b) By what percentage does this observed value differ from the instantaneous value at $x = 200$ m?
50. Squaring Equation 2.7 gives an expression for v^2. Equation 2.11 also gives an expression for v^2. Equate the two expressions, and show that the resulting equation reduces to Equation 2.10.
51. During the complicated sequence that landed the rover *Curiosity* on Mars in 2012, the spacecraft reached an altitude of 142 m above the Martian surface, moving vertically downward at 32.0 m/s. It then entered a so-called constant deceleration (CD) phase, during which its velocity decreased steadily to 0.75 m/s while it dropped to an altitude of 23 m. What was the magnitude of the spacecraft's acceleration during this CD phase?

52. The position of a car in a drag race is measured each second, and
DATA the results are tabulated below.

Time t (s)	0	1	2	3	4	5
Position x (m)	0	1.7	6.2	17	24	40

Assuming the acceleration is approximately constant, plot position versus a quantity that should make the graph a straight line. Fit a line to the data, and from it determine the approximate acceleration.

53. A fireworks rocket explodes at a height of 82.0 m, producing fragments with velocities ranging from 7.68 m/s downward to 16.7 m/s upward. Over what time interval are fragments hitting the ground?
54. The muscles in a grasshopper's legs can propel the insect upward
BIO at 3.0 m/s. How high can the grasshopper jump?
55. On packed snow, computerized antilock brakes can reduce a car's stopping distance by 55%. By what percentage is the stopping time reduced?
56. A particle leaves its initial position x_0 at time $t = 0$, moving in the positive x-direction with speed v_0 but undergoing acceleration of magnitude a in the negative x-direction. Find expressions for (a) the time when it returns to x_0 and (b) its speed when it passes that point.
57. A hockey puck moving at 32 m/s slams through a wall of snow 35 cm thick. It emerges moving at 18 m/s. Assuming constant acceleration, find (a) the time the puck spends in the snow and (b) the thickness of a snow wall that would stop the puck entirely.
58. Amtrak's 20th-Century Limited is en route from Chicago to New York at 110 km/h when the engineer spots a cow on the track. The train brakes to a halt in 1.2 min, stopping just in front of the cow. (a) What is the magnitude of the train's acceleration? (b) What's the direction of the acceleration? (c) How far was the train from the cow when the engineer applied the brakes?
59. A jetliner touches down at 220 km/h and comes to a halt 29 s later. What's the shortest runway on which this aircraft can land?
60. A motorist suddenly notices a stalled car and slams on the brakes, negatively accelerating at 6.3 m/s^2. Unfortunately, this isn't enough, and a collision ensues. From the damage sustained, police estimate that the car was going 18 km/h at the time of the collision. They also measure skid marks 34 m long. (a) How fast was the motorist going when the brakes were first applied? (b) How much time elapsed from the initial braking to the collision?
61. A racing car undergoing constant acceleration covers 140 m in 3.6 s. (a) If it's moving at 53 m/s at the end of this interval, what was its speed at the beginning of the interval? (b) How far did it travel from rest to the end of the 140-m distance?
62. The maximum braking acceleration of a car on a dry road is about 8 m/s^2. If two cars move head-on toward each other at 88 km/h (55 mi/h), and their drivers brake when they're 85 m apart, will they collide? If so, at what relative speed? If not, how far apart will they be when they stop? Plot distance versus time for both cars on a single graph.
63. After 35 min of running, at the 9-km point in a 10-km race, you find yourself 100 m behind the leader and moving at the same speed. What should your acceleration be if you're to catch up by the finish line? Assume that the leader maintains constant speed.
64. You're speeding at 85 km/h when you notice that you're only 10 m behind the car in front of you, which is moving at the legal speed limit of 60 km/h. You slam on your brakes, and your car negatively accelerates at 4.2 m/s^2. Assuming the other car continues at constant speed, will you collide? If so, at what relative speed? If not, what will be the distance between the cars at their closest approach?
65. Airbags cushioned the Mars rover Spirit's landing, and the rover bounced some 15 m vertically after its first impact. Assuming no loss of speed at contact with the Martian surface, what was Spirit's impact speed?
66. Calculate the speed with which cesium atoms must be "tossed" in the NIST-F1 atomic clock so that their up-and-down travel time is 1.0 s. (See the Application on page 26.)
67. A falling object travels one-fourth of its total distance in the last
CH second of its fall. From what height was it dropped?
68. You're on a NASA team engineering a probe to land on Jupiter's moon Io, and your job is to specify the impact speed the probe can tolerate without damage. Rockets will bring the probe to a halt 100 m above the surface, after which it will fall freely. What speed do you specify? (Consult Appendix E.)
69. You're atop a building of height h, and a friend is poised to drop a ball from a window at $h/2$. Find an expression for the speed at which you should simultaneously throw a ball downward, so the two hit the ground at the same time.
70. A castle's defenders throw rocks down on their attackers from a 15-m-high wall, with initial speed 10 m/s. How much faster are the rocks moving when they hit the ground than if they were simply dropped?
71. Two divers jump from a 3.00-m platform. One jumps upward at
CH 1.80 m/s, and the second steps off the platform as the first passes it on the way down. (a) What are their speeds as they hit the water? (b) Which hits the water first and by how much?
72. A balloon is rising at 10 m/s when its passenger throws a ball straight up at 12 m/s relative to the balloon. How much later does the passenger catch the ball?
73. Landing on the Moon, a spacecraft fires its rockets and comes to a complete stop just 12 m above the lunar surface. It then drops freely to the surface. How long does it take to fall, and what's its impact speed? (*Hint*: Consult Appendix E.)
74. You're at mission control for a rocket launch, deciding whether to let the launch proceed. A band of clouds 5.3 km thick extends upward from 1.9 km altitude. The rocket will accelerate at 4.6 m/s^2, and it isn't allowed to be out of sight for more than 30 s. Should you allow the launch?
75. You're an investigator for the National Transportation Safety Board, examining a subway accident in which a train going at 80 km/h collided with a slower train traveling in the same direction at 25 km/h. Your job is to determine the relative speed of the collision, to help establish new crash standards. The faster train's "black box" shows that its brakes were applied and it began slowing at the rate of 2.1 m/s^2 when it was 50 m from the slower train, while the slower train continued at constant speed. What do you report?
76. You toss a book into your dorm room, just clearing a windowsill
CH 4.2 m above the ground. (a) If the book leaves your hand 1.5 m above the ground, how fast must it be going to clear the sill? (b) How long after it leaves your hand will it hit the floor, 0.87 m below the windowsill?
77. Consider an object traversing a distance L, part of the way at speed v_1 and the rest of the way at speed v_2. Find expressions for the object's average speed over the entire distance L when the object moves at each of the two speeds v_1 and v_2 for (a) half the total *time* and (b) half the total *distance*. (c) In which case is the average speed greater?
78. A particle's position as a function of time is given by $x = x_0 \sin\omega t$, where x_0 and ω are constants. (a) Find expressions for the velocity and acceleration. (b) What are the maximum values of velocity and acceleration? (*Hint*: Consult the table of derivatives in Appendix A.)

79. CH Ice skaters, ballet dancers, and basketball players executing vertical leaps often give the illusion of "hanging" almost motionless near the top of the leap. To see why this is, consider a leap to maximum height h. Of the total time spent in the air, what fraction is spent in the upper half (i.e., at $y > \frac{1}{2}h$)?
80. You're staring idly out your dorm window when you see a water balloon fall past. If the balloon takes 0.22 s to cross the 1.3-m-high window, from what height above the window was it dropped?
81. A police radar's effective range is 1.0 km, and your radar detector's range is 1.9 km. You're going 110 km/h in a 70 km/h zone when the radar detector beeps. At what rate must you negatively accelerate to avoid a speeding ticket?
82. CH An object starts moving in a straight line from position x_0, at time $t = 0$, with velocity v_0. Its acceleration is given by $a = a_0 + bt$, where a_0 and b are constants. Use integration to find expressions for (a) the instantaneous velocity and (b) the position, as functions of time.
83. You're a consultant on a movie set, and the producer wants a car to drop so that it crosses the camera's field of view in time Δt. The field of view has height h. Derive an expression for the height above the top of the field of view from which the car should be released.
84. CH (a) For the ball in Example 2.6, find its velocity just before it hits the floor. (b) Suppose you had tossed a second ball straight down at 7.3 m/s (from the same place 1.5 m above the floor). What would its velocity be just before it hits the floor? (c) When would the second ball hit the floor? (Interpret any multiple answers.)
85. Your roommate is an aspiring novelist and asks your opinion on a matter of physics. The novel's central character is kept awake at night by a leaky faucet. The sink is 19.6 cm below the faucet. At the instant one drop leaves the faucet, another strikes the sink below and two more are in between on the way down. How many drops per second are keeping the protagonist awake?
86. You and your roommate plot to drop water balloons on students entering your dorm. Your window is 20 m above the sidewalk. You plan to place an X on the sidewalk to mark the spot a student must be when you drop the balloon. You note that most students approach the dorm at about 2 m/s. How far from the impact point do you place the X?
87. CH Derive Equation 2.10 by integrating Equation 2.7 over time. You'll have to interpret the constant of integration.
88. CH An object's acceleration increases quadratically with time: $a(t) = bt^2$, where $b = 0.041\ \text{m/s}^4$. If the object starts from rest, how far does it travel in 6.3 s?
89. CH An object's acceleration is given by the expression $a(t) = -a_0 \cos\omega t$, where a_0 and ω are positive constants. Find expressions for the object's (a) velocity and (b) position as functions of time. Assume that at time $t = 0$ it starts from rest at its greatest positive displacement from the origin. (c) Determine the magnitudes of the object's maximum velocity and maximum displacement from the origin.
90. CH An object's acceleration decreases exponentially with time: $a(t) = a_0 e^{-bt}$, where a_0 and b are constants. (a) Assuming the object starts from rest, determine its velocity as a function of time. (b) Will its speed increase indefinitely? (c) Will it travel indefinitely far from its starting point?
91. CH A ball is dropped from rest at a height h_0 above the ground. At the same instant, a second ball is launched with speed v_0 straight up from the ground, at a point directly below where the other ball is dropped. (a) Find a condition on v_0 such that the two balls will collide in mid-air. (b) Find an expression for the height at which they collide.

Passage Problems

A wildlife biologist is studying the hunting patterns of tigers. She anesthetizes a tiger and attaches a GPS collar to track its movements. The collar transmits data on the tiger's position and velocity. Figure 2.16 shows the tiger's velocity as a function of time as it moves on a one-dimensional path.

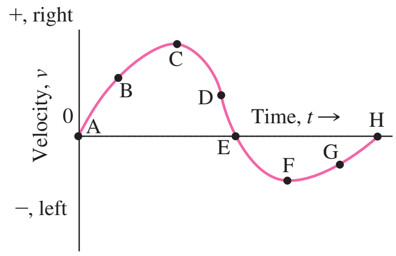


FIGURE 2.16 The tiger's velocity (Passage Problems 92–96)

92. At which marked point(s) is the tiger not moving?
    a. E only
    b. A, E, and H
    c. C and F
    d. none of the points (it's always moving)
93. At which marked point(s) is the tiger not accelerating?
    a. E only
    b. A, E, and H
    c. C and F
    d. all of the points (it's never accelerating)
94. At which point does the tiger have the greatest speed?
    a. B
    b. C
    c. D
    d. F
95. At which point does the tiger's acceleration have the greatest magnitude?
    a. B
    b. C
    c. D
    d. F
96. At which point is the tiger farthest from its starting position at $t = 0$?
    a. C
    b. E
    c. F
    d. H

Answers to Chapter Questions

Answer to Chapter Opening Question

Although the ball's velocity is zero at the top of its motion, its acceleration is $-9.8\ \text{m/s}^2$, as it is throughout the toss.

Answers to GOT IT? Questions

2.1 (a) and (b); average speed is greater for (c)
2.2 (b) moves with constant speed; (a) reverses; (d) speeds up
2.3 (b) downward
2.4 (a) halfway between the times; because its acceleration is constant, the police car's speed increases by equal amounts in equal times. So it gets from 0 to half its final velocity—which is twice the car's velocity—in half the total time.
2.5 The dropped ball hits first; the thrown ball hits moving faster.
2.6 (c)