CHAPTER 3

Motion in a Plane

This image shows an example of one of the giant pumpkin-shooting cannons used in the world championship "Pumpkin Chuckin" games. These compressed air cannons are capable of firing a pumpkin projectile more than 1200 m downrange. The trajectory of the pumpkin is well described by the projectile motion equations presented in this chapter.

By the end of this chapter, you will be able to:

1. Calculate the displacement of an object and relate the displacement to the object's average velocity for motion in a plane.
2. Calculate the average acceleration of an object moving in two dimensions.
3. Use knowledge of how the speed and direction of an object are changing to determine the relative direction of the velocity and acceleration vectors.
4. Use the equations of motion for constant acceleration to solve for unknown quantities for an object moving under constant acceleration in two dimensions.
5. Determine the relative velocity of an object for observers in different frames of reference in two dimensions.

In straight-line problems (such as those in Chapter 2), we developed the formalism we need to describe the position and velocity of a particle as a function of time. We placed particular emphasis on motion with constant acceleration, primarily because an object in free fall has a constant downward acceleration of $9.8\ \text{m/s}^2$. In this chapter, we will extend our description of motion to more than one dimension. We'll focus on motion in two dimensions—that is, in a plane—but the principles that we develop will also apply to three-dimensional motion. So, instead of simply considering the motion of, let's say, a baseball that has been thrown straight up into the air, we will expand our analysis to handle the more complex motion of a baseball that has been thrown from home plate to second base. For this two-dimensional motion, the vector quantities *displacement*, *velocity*, and *acceleration* have two components, one for each axis of our two-dimensional coordinate system. We'll also generalize the concept of relative velocity to motion in a plane, such as an airplane flying in a crosswind.

This chapter represents a merging of the vector language we have learned (in Chapter 1) with kinematic language (which we learned in Chapter 2). As before, we're concerned with *describing* motion, not with analyzing its causes. But the language you learn here will be an essential tool in later chapters when you use Newton's laws of motion to study the relationship between force and motion.

3.1 Velocity in a Plane

To describe the *motion* of an object in a plane, we first need to be able to describe the object's *position.* (In this chapter, as in the preceding one, we assume that the objects we describe can be modeled as particles.) Often, it's useful to use a familiar x-y axis system (Figure 3.1a). For example, when a football player kicks a field goal, the ball (represented by point P) moves in a vertical plane. The ball's horizontal distance from the origin O at any time is x, and its vertical distance above the ground at any time is y. The numbers x and y are called the *coordinates* of point P. The vector $\vec{r}$ from the origin O to point P is called the **position vector** of point P, and the Cartesian coordinates x and y of point P are the x and y components, respectively, of vector $\vec{r}$. (You may want to review Section 1.8, "Components of Vectors.") The *distance* of point P from the origin is the magnitude of vector $\vec{r}$:

$$r = |\vec{r}| = \sqrt{x^2 + y^2}.$$

Figure 3.1b shows the ball at two points in its curved path. At time t_1, it is at point P_1 with position vector $\vec{r}_1$; at the later time t_2, it is at point P_2 with position vector $\vec{r}_2$. The ball moves from P_1 to P_2 during the time interval $\Delta t = t_2 - t_1$. The change in position (the displacement) during this interval is the vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$. We define the **average velocity** $\vec{v}_{av}$ during the interval in the same way we did in Chapter 2 for straight-line motion:

Definition of average velocity

The average velocity of a particle is the displacement $\Delta\vec{r}$ divided by the time interval Δt:

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}. \tag{3.1}$$

Units: m/s

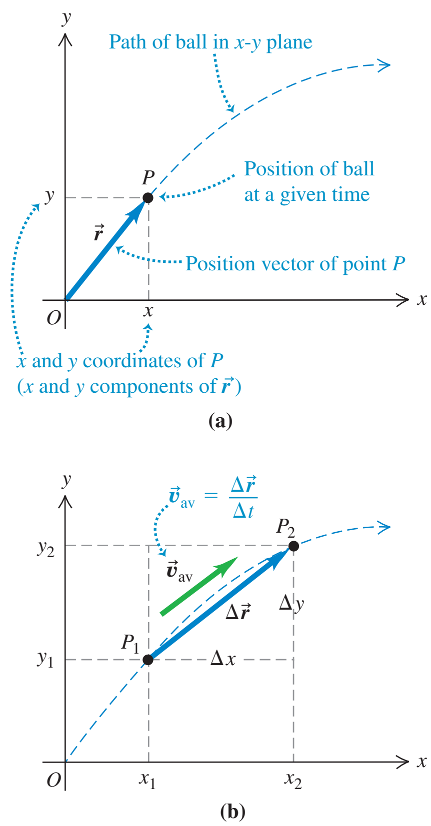
Notes:

- If the object is moving in the x-y plane, $\vec{v}_{av}$ has both an x and a y component.
- If the object is moving only along the x axis, then the y component of $\vec{v}_{av}$ is zero.
- If the object is moving only along the y axis, then the x component of $\vec{v}_{av}$ is zero.

That is, the average velocity is a vector quantity that has the same direction as $\Delta\vec{r}$ and a magnitude equal to the magnitude of $\Delta\vec{r}$ divided by the time interval Δt. The magnitude of $\Delta\vec{r}$ is always the straight-line distance from P_1 to P_2, regardless of the actual path taken by the object. Thus, the average velocity is the same for *any* path that takes the particle from P_1 to P_2 in the same time interval Δt.

Figure 3.1b also shows that, during any displacement $\Delta\vec{r}$, Δx and Δy are the components of the vector $\Delta\vec{r}$; it follows that the components $v_{av,x}$ and $v_{av,y}$ of the average velocity vector are

$$v_{av,x} = \frac{\Delta x}{\Delta t} \quad \text{and} \quad v_{av,y} = \frac{\Delta y}{\Delta t}. \tag{3.2}$$



▶ **FIGURE 3.1** Position vectors can specify the location and displacement of a point in an x-y coordinate system. A position vector points from the origin to the point.

We define the **instantaneous velocity** $\vec{v}$ as follows:

Definition of instantaneous velocity $\vec{v}$ in a plane

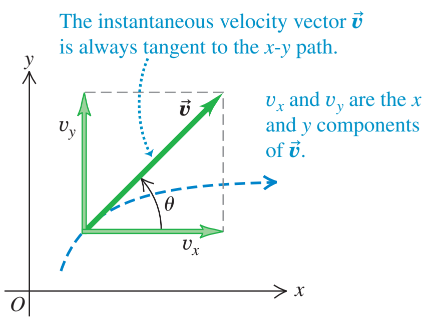
The instantaneous velocity is the limit of the average velocity as the time interval Δt approaches zero:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}. \tag{3.3}$$

Units: m/s

Notes:

- At every point along the path, the instantaneous velocity is tangent to the path.
- Speed is the magnitude of the instantaneous velocity.



▲ **FIGURE 3.2** The two velocity components for motion in the x-y plane.

As $\Delta t \longrightarrow 0$, points P_1 and P_2 move closer and closer together. In the limit, the vector $\Delta \vec{r}$ becomes *tangent* to the curve, as shown in Figure 3.2. The direction of $\Delta \vec{r}$ in the limit is also the direction of the instantaneous velocity $\vec{v}$.

During any displacement $\Delta \vec{r}$, the changes Δx and Δy in the coordinates x and y are the *components* of $\Delta \vec{r}$. It follows that the components v_x and v_y of the instantaneous velocity $\vec{v}$ are

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \quad \text{and} \quad v_y = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}.$$

The instantaneous *speed* of the object is the *magnitude* v of the instantaneous velocity vector $\vec{v}$. This is given by the Pythagorean theorem:

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2} \quad \text{(speed of a particle in a plane).}$$

The direction of $\vec{v}$ is given by the angle θ in the figure. We see that when we measure θ in the usual way (counterclockwise from the $+x$ axis, as in Section 1.8),

$$\theta = \tan^{-1}\frac{v_y}{v_x}.$$

NOTE In this text, we always use Greek letters for angles; we'll use θ (theta) for the direction of vectors measured counterclockwise from the $+x$ axis and ϕ (phi) for most other angles.

Because velocity is a vector quantity, we may represent it either in terms of its components or in terms of its magnitude and direction, as described in Chapter 1. The *direction* of an object's instantaneous velocity at any point is *always* tangent to the path at that point. But, in general, the position vector $\vec{r}$ does *not* have the same direction as the instantaneous velocity $\vec{v}$. (The direction of the position vector depends on where you place the origin, while the direction of $\vec{v}$ is determined by the shape of the path.)

EXAMPLE 3.1 A model car

For our first example of motion in a plane, suppose you are operating a radio-controlled model car on a vacant tennis court. The surface of the court represents the x-y plane, and you place the origin at your own location. At time $t_1 = 2.0$ s the car has x and y coordinates (4.0 m, 2.0 m), and at time $t_2 = 2.5$ s it has coordinates (7.0 m, 6.0 m). For the time interval from t_1 to t_2, find **(a)** the components of the average velocity of the car and **(b)** the magnitude and direction of the average velocity.

Video Tutor Solution

CONTINUED

SOLUTION

SET UP Figure 3.3 shows our sketch for this problem. We see that $\Delta x = 7.0\text{ m} - 4.0\text{ m} = 3.0\text{ m}$, $\Delta y = 6.0\text{ m} - 2.0\text{ m} = 4.0\text{ m}$, and $\Delta t = 0.50\text{ s}$.

SOLVE **Part (a):** To find the components of $\vec{v}_{av}$, we use their definitions (Equations 3.2):

$$v_{av,x} = \frac{\Delta x}{\Delta t} = \frac{3.0\text{ m}}{0.50\text{ s}} = 6.0\text{ m/s},$$

$$v_{av,y} = \frac{\Delta y}{\Delta t} = \frac{4.0\text{ m}}{0.50\text{ s}} = 8.0\text{ m/s}.$$

Part (b): The magnitude of $\vec{v}_{av}$ is obtained from the Pythagorean theorem:

$$|\vec{v}_{av}| = \sqrt{v_{av,x}{}^2 + v_{av,y}{}^2}$$
$$= \sqrt{(6.0\text{ m/s})^2 + (8.0\text{ m/s})^2} = 10.0\text{ m/s}.$$

The direction of $\vec{v}_{av}$ is most easily described by its angle, measured counterclockwise from the positive x axis. Calling this angle θ, we have

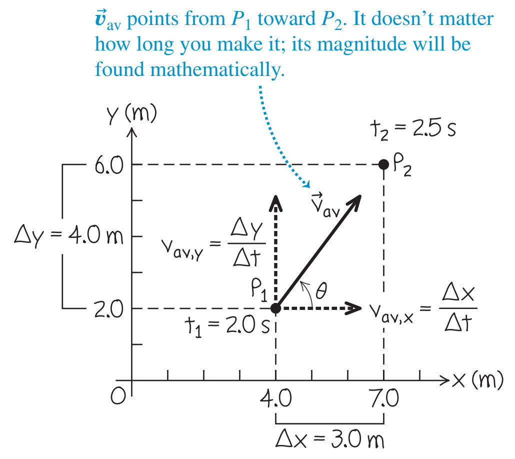
$$\theta = \tan^{-1}\frac{v_{av,y}}{v_{av,x}} = \tan^{-1}\frac{8.0\text{ m/s}}{6.0\text{ m/s}} = 53°.$$

Alternative Solution: Alternatively, the magnitude of $\vec{v}_{av}$ is the distance between the points (4.0 m, 2.0 m) and (7.0 m, 6.0 m)—that is, 5.0 m (found using Pythagoras's theorem)—divided by the time interval (0.5 s):

$$|\vec{v}_{av}| = \frac{5.0\text{ m}}{0.50\text{ s}} = 10.0\text{ m/s}.$$

Since the direction of $\vec{v}_{av}$ is the same as the direction of the displacement $\Delta\vec{r}$ between the two points, we can calculate it from Δx and Δy rather than from $v_{av,x}$ and $v_{av,y}$:

$$\theta = \tan^{-1}\frac{\Delta y}{\Delta x} = \tan^{-1}\frac{4.0\text{ m}}{3.0\text{ m}} = 53°.$$



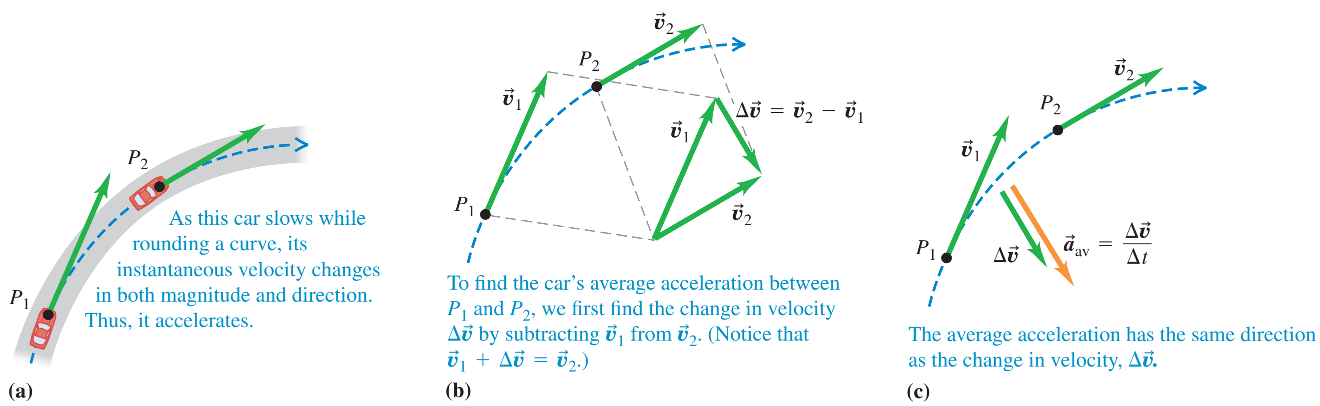
▲ **FIGURE 3.3** Our sketch for this problem.

REFLECT Be sure you understand the relationship between the two solutions to part (b). In the first, we calculated the magnitude and direction of $\vec{v}_{av}$ from the components of this vector quantity. In the alternative solution, we used the fact that the average velocity $\vec{v}_{av}$ and the displacement $\Delta\vec{r}$ have the same direction.

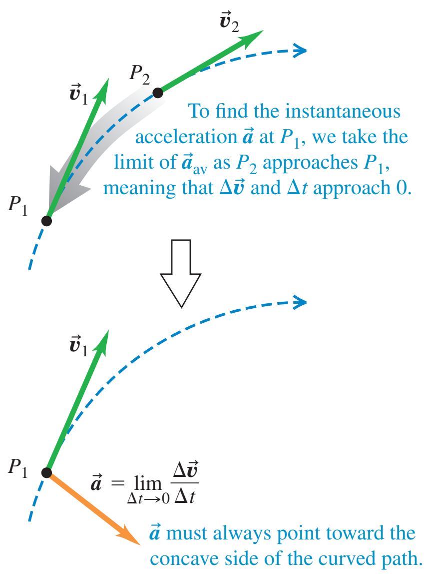
Practice Problem: Suppose you reverse the car's motion, so that it retraces its path in the opposite direction in the same time. Find the components of the average velocity of the car and the magnitude and direction of the average velocity. *Answers:* −6.0 m/s, −8.0 m/s, −10.0 m/s, 233°.

3.2 Acceleration in a Plane

Now let's consider the acceleration of an object moving on a curved path in a plane. In Figure 3.4, the vector $\vec{v}_1$ represents the particle's instantaneous velocity at point P_1 at time t_1, and the vector $\vec{v}_2$ represents the particle's instantaneous velocity at point P_2 at time t_2. In general, the two velocities differ in both magnitude and direction.



▲ **FIGURE 3.4** Finding the average acceleration between two points in the x-y plane.



▲ **FIGURE 3.5** The instantaneous acceleration of a point in the x-y plane.

▲ **Application**
Can you spot the three accelerators?

Almost everyone knows that the gas pedal is an accelerator—pressing it makes the car speed up. As a physics student, you probably also recognize that the brake pedal is an accelerator—it slows the car down. The third accelerator is the steering wheel, which changes the *direction* of the car's velocity.

We define the **average acceleration** $\vec{a}_{av}$ of the particle as follows:

Definition of average acceleration $\vec{a}_{av}$

As an object undergoes a displacement during a time interval Δt, its average acceleration is its change in velocity $\Delta\vec{v}$ divided by Δt:

$$\text{Average acceleration} = \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}. \qquad (3.4)$$

Units: m/s^2
Note:

- Even if a particle is moving at a constant speed, it can have a nonzero average acceleration if the particle changes the direction of its motion.

Average acceleration is a vector quantity in the same direction as the vector $\Delta\vec{v}$. In Chapter 2, we stressed that acceleration is a quantitative description of the way an object's motion is *changing* with time. Figure 3.4b shows that **the final velocity $\vec{v}_2$ (at time t_2) is the vector sum of the original velocity $\vec{v}_1$ (at time t_1) and the change in velocity $\Delta\vec{v}$ during the interval Δt.**

We define the **instantaneous acceleration** $\vec{a}$ at point P_1 as follows (Figure 3.5):

Definition of instantaneous acceleration $\vec{a}$

When the velocity of a particle changes by an amount $\Delta\vec{v}$ as the particle undergoes a displacement $\Delta\vec{r}$ during a time interval Δt, the instantaneous acceleration is the limit of the average acceleration as Δt approaches zero:

$$\text{Instantaneous acceleration} = \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}. \qquad (3.5)$$

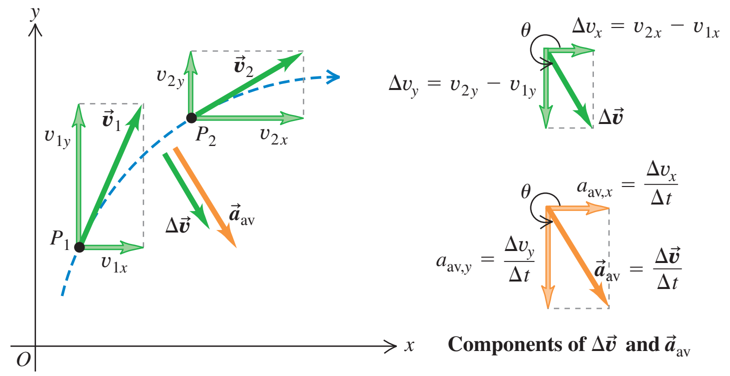
Units: m/s^2
Notes:

- The acceleration vector does not necessarily point in the direction of the motion.
- When a particle moves on a curved path, it *always* has a nonzero acceleration, even if it is moving with a constant speed.

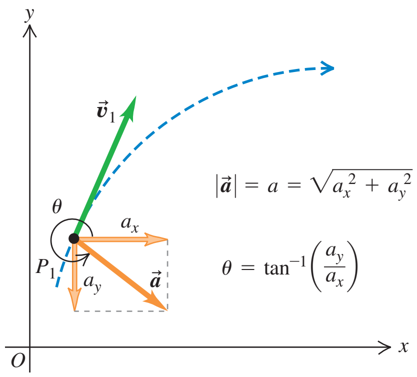
The instantaneous acceleration vector at point P_1 in Figure 3.5 does *not* have the same direction as the instantaneous velocity vector $\vec{v}$ at that point; in general, there is no reason it should. (Recall from Chapter 2 that the velocity and acceleration components of a particle moving along a line could have opposite signs.) The construction in Figure 3.5 shows that the acceleration vector must always point toward the *concave* side of the curved path. **When a particle moves in a curved path, it *always* has nonzero acceleration, even when it moves with constant speed.** More generally, acceleration is associated with change of speed, change of direction of velocity, or both.

We often represent the acceleration of a particle in terms of the components of this vector quantity. Like Figure 3.2, Figure 3.6 shows the motion of a particle as described in a rectangular coordinate system. During a time interval Δt, the velocity of the particle changes by an amount $\Delta\vec{v}$, with components Δv_x and Δv_y. So we can represent the average acceleration $\vec{a}_{av}$ in terms of its x and y components:

$$a_{av,x} = \frac{\Delta v_x}{\Delta t}, \quad a_{av,y} = \frac{\Delta v_y}{\Delta t}. \qquad (3.6)$$



(a) Components of average acceleration for the interval from P_1 to P_2.



(b) Components of instantaneous acceleration at P_1.

▲ **FIGURE 3.6** Components of average and instantaneous acceleration.

Similarly, the x and y components of instantaneous acceleration, a_x and a_y, are

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}, \quad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}.$$

If we know the components a_x and a_y, we can find the magnitude and direction of the acceleration vector $\vec{a}$, just as we did with velocity:

$$|\vec{a}| = a = \sqrt{a_x^2 + a_y^2}, \quad \theta = \tan^{-1}\frac{a_y}{a_x}. \tag{3.7}$$

The angle θ gives the direction of $\vec{a}$, measured counterclockwise from the $+x$ axis.

PhET: Ladybug Motion 2D

▲ **BIO Application**
Knowing up from down.

Plant roots have exquisitely developed mechanisms for sensing gravity and growing downward. If the direction of a root is changed—say, by running into a rock in the soil—the root is forced to grow horizontally; however, as soon as it can, it again turns downward. This ability involves a number of cellular signals, but the primary sensor detects acceleration. The sensing cells contain statoliths, specialized starch-containing granules that are denser than the fluid in the cells and, in response to changes in the direction of gravity, fall (are accelerated) to a different location within the cells. This triggers an active localized response of the cells and causes the growth direction of the root to reorient so that the root once again grows downward.

EXAMPLE 3.2 The model car again

Video Tutor Solution

Let's look again at the radio-controlled model car in Example 3.1. Suppose that at time $t_1 = 2.0$ s the car has components of velocity $v_x = 1.0$ m/s and $v_y = 3.0$ m/s and that at time $t_2 = 2.5$ s the components are $v_x = 4.0$ m/s and $v_y = 3.0$ m/s. Find **(a)** the components of average acceleration and **(b)** the magnitude and direction of the average acceleration during this interval.

CONTINUED

SOLUTION

SET UP Figure 3.7 shows our sketch.

SOLVE Figure 3.6a outlines the relationships we'll use.

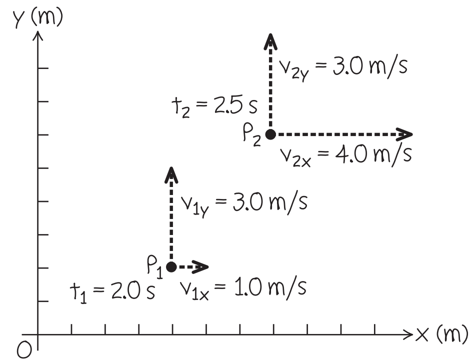
Part (a): To find the components of average acceleration, we need the components of the change in velocity, Δv_x and Δv_y, and the time interval, $\Delta t = 0.5$ s. The change in v_x is $\Delta v_x = v_{2x} - v_{1x} = (4.0\text{ m/s} - 1.0\text{ m/s}) = 3.0$ m/s, so the x component of average acceleration in the interval $\Delta t = 0.5$ s is

$$a_{\text{av},x} = \frac{\Delta v_x}{\Delta t} = \frac{3.0\text{ m/s}}{0.5\text{ s}} = 6.0\text{ m/s}^2.$$

The change in v_y is zero, so $a_{\text{av},y}$ in this interval is also zero.

Part (b): The vector $\vec{a}_{\text{av}}$ has only an x component. The vector points in the $+x$ direction and has magnitude 6.0 m/s^2.

REFLECT We can always represent a vector quantity (such as displacement, velocity, or acceleration) either in terms of its components or in terms of its magnitude and direction.



▲ **FIGURE 3.7** Our sketch for this problem.

Practice Problem: Suppose that as the car continues to move, it has velocity components $v_x = 3.5$ m/s and $v_y = -1.0$ m/s at time $t_3 = 3.0$ s. What are the magnitude and direction of the average acceleration between $t_2 = 2.0$ s and $t_3 = 3.0$ s? *Answers:* 4.7 m/s, $-58°$ from $+x$.

CONCEPTUAL ANALYSIS 3.1

A visitor at a picnic

An ant crawls at a constant speed on top of a square blanket that you have laid out for a picnic. You have picked a flat, horizontal lawn for your picnic and oriented the blanket so that one side of it runs north–south and the other side runs east–west. The ant crawls onto the blanket at its northeast corner, traveling due west. Sixty seconds later, the ant crawls off the blanket at its southwest corner, traveling due south. In which direction does the average velocity of the ant point for the 60 s it is on the blanket?

A. Due west
B. Due south
C. Southwest

SOLUTION The key idea is that the average velocity of the ant will point in the same direction as its displacement. Regardless of how the ant moves during the 60 s it is on the blanket, the ant's displacement during that interval points from the northeast corner of the blanket (its initial position) to the southwest corner (its final position). Therefore, the ant's displacement points in the southwest direction, so the average velocity must point to the southwest as well, and the correct answer is C.

In which direction does the average acceleration of the ant point for the 60 s it is on the blanket?

A. Due south
B. Southeast
C. Southwest
D. The average acceleration is zero because the ant moves at a constant speed.

SOLUTION Even though the ant travels at a constant speed across the blanket, the direction of its velocity changes as it crawls, so there must be an average acceleration over the 60 s interval. The average acceleration of the ant will point in the same direction as the change in the ant's velocity vector, which is $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. (Remember that subtracting $\vec{v}_1$ is the same as adding $-\vec{v}_1$.) Therefore, $\Delta\vec{v}$ is the vector sum of a vector that points south ($\vec{v}_2$) and a vector that points east ($-\vec{v}_1$). So the correct answer is B.

3.3 Projectile Motion

A **projectile** is any object that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance. A batted baseball, a thrown football, and a package dropped from an airplane are all examples of projectiles. The path followed by a projectile is called its *trajectory*. In Chapter 2 we looked at a special case of projectile motion: free fall.

To analyze projectile motion, we'll start with an idealized model. We represent the projectile as a single particle with an acceleration (due to the earth's gravitational pull) that is constant in both magnitude and direction. We'll ignore the effects of air resistance and the curvature and rotation of the earth. Like all models, this one has limitations. The curvature of the earth has to be considered in the flight of long-range ballistic missiles, and air resistance is of crucial importance to a skydiver. Nevertheless, we can learn a lot by analyzing this simple model.

Video Tutor Demo

We first notice that projectile motion is always confined to a vertical plane determined by the direction of the initial velocity. We'll call this plane the *x-y coordinate plane,* with the x axis horizontal and the y axis directed vertically upward. Figure 3.8 shows a view of this plane from the side, along with a typical trajectory.

The key to analyzing projectile motion is the fact that *we can treat the x and y coordinates separately.* Why is this so? Anticipating a relationship that we'll study in detail later (in Chapter 4), we note that the instantaneous acceleration of an object is proportional to (and in the same direction as) the net *force* acting on the object. Because of the assumptions made in our model, the only force acting on the projectile is the earth's gravitational attraction; we assume that this is constant in magnitude and always vertically downward in direction. Thus, the vertical component of acceleration is the same as if the projectile moved in only the y direction (as it did in Section 2.6). Figure 3.9 shows two trajectories; the vertical displacements of the two objects at any time are the same, even though their horizontal displacements are different.

We conclude that the x component of acceleration a_x is zero and the y component a_y is constant and equal in magnitude to the acceleration of free fall:

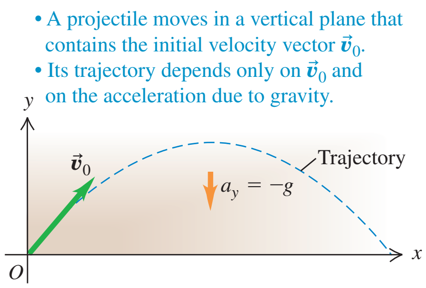
Acceleration vector in projectile motion

$$a_x = 0; \quad a_y = -g.$$

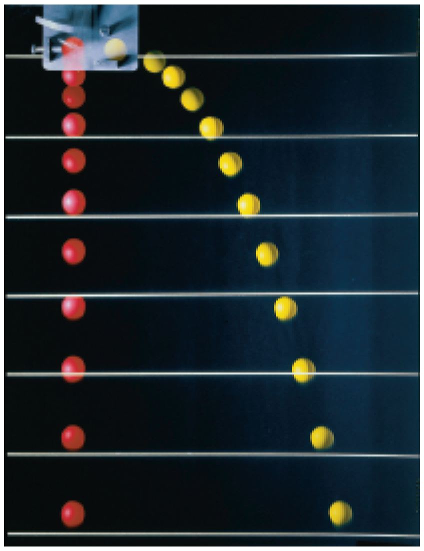
Notes:

- Here g is defined as the magnitude of the acceleration due to gravity: $g = +9.8\ \text{m/s}^2$.
- The minus sign assumes that our coordinate system is one in which the positive y direction is straight upward.

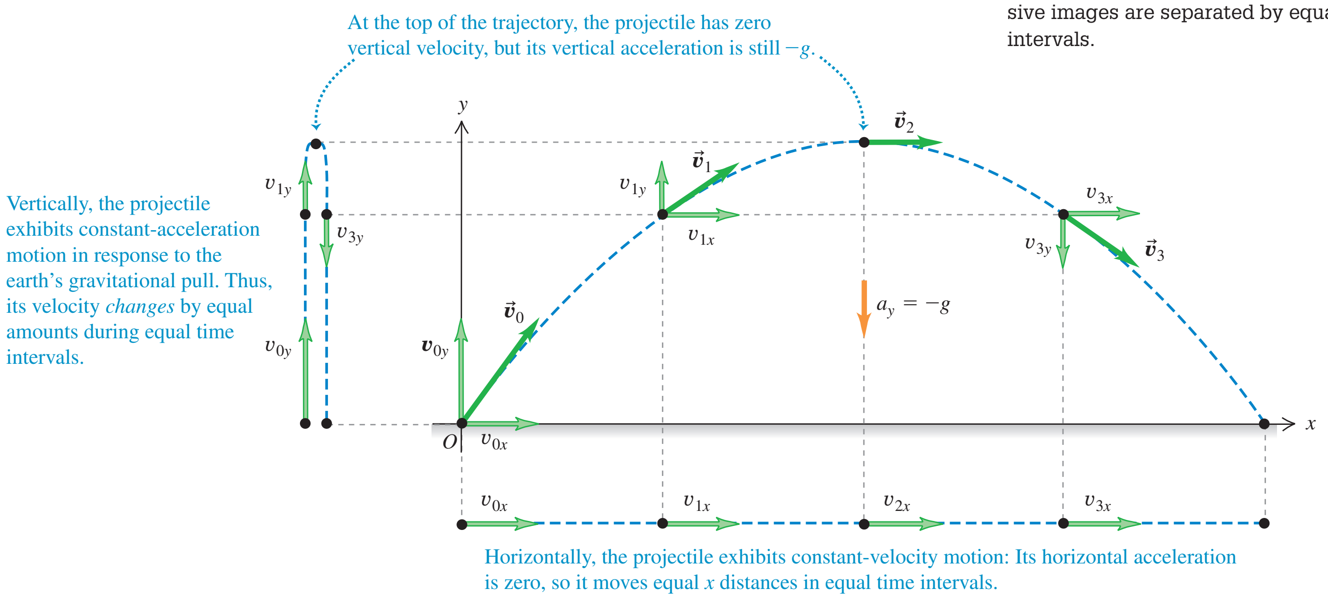
So we can think of projectile motion as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration. We can then express all the vector relationships in terms of separate equations for the horizontal and vertical components. The actual motion is a combination of these separate motions. Figure 3.10 shows the horizontal and vertical components of motion for a projectile that starts at (or passes through) the origin of coordinates at time $t = 0$. As in Figure 3.9, the projectile is shown at equal time intervals.



▲ **FIGURE 3.8** The trajectory of a projectile.



▲ **FIGURE 3.9** Independence of horizontal and vertical motion: At any given time, both balls have the same y position, velocity, and acceleration, despite having different x positions and velocities. Successive images are separated by equal time intervals.



▲ **FIGURE 3.10** Independence of horizontal and vertical motion. In the vertical direction, a projectile behaves like a freely falling object; in the horizontal direction, it moves with constant velocity.

CONCEPTUAL ANALYSIS 3.2

Horizontal stone throw

If you throw a stone horizontally out over the surface of a lake, the time it is in the air (i.e., before it hits the water) is determined by *only*

A. the height from which you throw it.
B. the initial speed of the stone.
C. the height from which you throw it and its initial speed.

SOLUTION As we've just learned, the horizontal and vertical components of a projectile's motion are independent. Therefore, the stone reaches the water at the same time as a rock that is dropped vertically from the same starting height. The time required for the stone to reach the water depends only on the height from which it is thrown, so the correct answer is A. The *distance* the stone travels before splashing down, however, depends on both the height from which it is thrown and the speed with which it is thrown.

In projectile motion, the vertical and horizontal coordinates both vary with constant-acceleration motion. (The horizontal component of acceleration is constant at zero.) Therefore, we can use the same equations we derived in Section 2.4 for constant-acceleration motion. Here's a reminder of these relationships:

$$v_x = v_{0x} + a_x t \quad \text{(velocity as a function of time)}, \tag{2.6}$$

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \quad \text{(position as a function of time)}, \tag{2.10}$$

$$v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0) \quad \text{(velocity as a function of position)}. \tag{2.11}$$

Our procedure will be to use this set of equations separately for each coordinate. That is, we use this set of equations for the x coordinate, and then we use a second set with all the x's replaced by y's for the y coordinate. Next, we have to choose the appropriate components of the constant acceleration $\vec{a}$ and the initial velocity $\vec{v}_0$. At time $t = 0$, the particle is at the point (x_0, y_0), and its velocity components have the initial values v_{0x} and v_{0y}. The components of acceleration are constant: $a_x = 0$, $a_y = -g$. When we put all the pieces together, here's what we get:

Equations for projectile motion (assuming that $a_x = 0$, $a_y = -g$)

Considering the x motion, we substitute $a_x = 0$ in Equations 2.6 and 2.10, obtaining

$$v_x = v_{0x}, \tag{3.8}$$

$$x = x_0 + v_{0x}t. \tag{3.9}$$

For the y motion, we substitute y for x, v_y for v_x, v_{0y} for v_{0x}, and $-g$ for a to get

$$v_y = v_{0y} - gt, \tag{3.10}$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2. \tag{3.11}$$

Notes:

- v_{0x} and v_{0y} represent the x and y components of the initial velocity vector, respectively.
- v_x and v_y represent the x and y components of the velocity at a later time t.
- x_0 and y_0 represent the x and y components of the initial position of the object.
- x and y represent the x and y components of the position at a later time t.

Usually it is simplest to take the initial position (at time $t = 0$) as the origin; in this case, $x_0 = y_0 = 0$. The initial position might be, for example, the position of a ball at the instant it leaves the thrower's hand or the position of a bullet at the instant it leaves the barrel of the gun.

As shown in Figure 3.10, the x component of acceleration for a projectile is zero, so v_x is constant, but v_y changes by equal amounts in equal times, corresponding to a constant

y component of acceleration. At the highest point in the trajectory, $v_y = 0$. But a_y is still equal to $-g$ at this point. Make sure you understand why!

In many problems the initial velocity $\vec{v}_0$ will be specified by its magnitude v_0 (the initial speed) and its angle θ_0 with the positive x axis, as shown in Figure 3.11. It is easier to visualize the initial velocity when it is presented in terms of a magnitude and direction, but we have to find its x and y components before we can use Equations 3.8 through 3.11. In terms of v_0 and θ, the components v_{0x} and v_{0y} of initial velocity are

$$v_{0x} = v_0 \cos\theta_0, \quad v_{0y} = v_0 \sin\theta_0.$$

We can get a lot of information from the equations for projectile motion. For example, the distance r of the projectile from the origin at any time (the magnitude of the position vector $\vec{r}$) is given by

$$r = \sqrt{x^2 + y^2}.$$

The projectile's speed v (the magnitude of its velocity) at any time is

$$v = \sqrt{v_x{}^2 + v_y{}^2}.$$

The *direction* of the velocity at any time, in terms of the angle θ it makes with the positive x axis, is given by

$$\theta = \tan^{-1}\frac{v_y}{v_x}.$$

The velocity vector $\vec{v}$ is tangent to the trajectory at each point. Note that v_x is constant but that the *direction* of the velocity changes because v_y changes continuously.

The formulation just described gives us the coordinates and velocity components of a projectile as functions of time. The actual *shape* of the trajectory is a graph of y as a function of x. We can derive an equation for this relationship by assuming, for simplicity, that $x_0 = y_0 = 0$. First we solve Equation 3.9 for t, and then we substitute the resulting expression for t into Equation 3.11 and simplify the result. We find that $t = x/(v_{0x})$ and

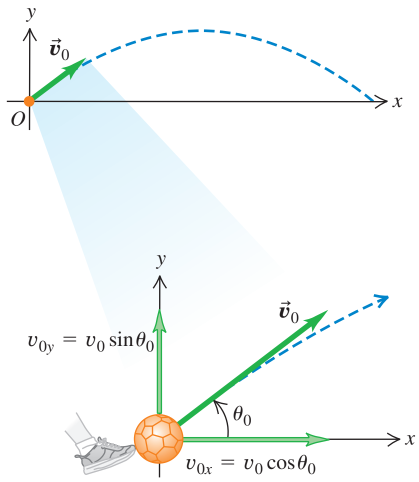
$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \tfrac{1}{2}g\left(\frac{x}{v_{0x}}\right)^2,$$

$$y = \left(\frac{v_{0y}}{v_{0x}}\right)x - \left(\frac{g}{2v_{0x}{}^2}\right)x^2.$$

Note that in the last equation the quantities in parentheses are simply constants. So the equation is a relationship between the variables x and y, with the general form

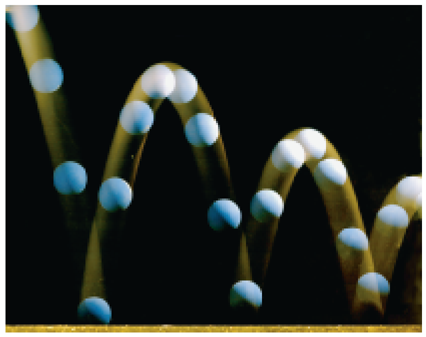
$$y = bx - cx^2,$$

where b and c are positive constants. This is the equation of an upside-down *parabola* (Figure 3.12).



▲ **FIGURE 3.11** The initial velocity of a projectile, showing the components and the launch angle θ_0.

MP® **PhET:** Projectile Motion



▲ **FIGURE 3.12** Strobe photo of a bouncing ball; the images are separated by equal time intervals. The ball follows a parabolic trajectory after each bounce. It rises a little less after each bounce because it loses energy during each collision with the floor.

CONCEPTUAL ANALYSIS 3.3

Horizontal paintball shot

A paintball is shot horizontally in the positive x direction. At time Δt after the ball is shot, it is 3 cm to the right and 3 cm below its starting point. Over the next interval Δt, the *changes* in the horizontal and vertical positions are

A. $\Delta x = 3$ cm, $\Delta y = -3$ cm.
B. $\Delta x = 3$ cm, $\Delta y = -9$ cm.
C. $\Delta x = 6$ cm, $\Delta y = -6$ cm.

SOLUTION The ball is a projectile, so we assume that its horizontal component of velocity is constant. Therefore, the changes Δx in the horizontal position during equal time intervals are equal. This result rules out answer C. Because the ball speeds up as it falls, the second Δy must be greater than the first, which eliminates answer A. However, is answer B correct? With constant acceleration, the distance of the fall from rest is proportional to t^2. This means that, as time increases from 1 to 2 to 3 in units of t, the distance fallen will be y, $4y$, and $9y$, or -3 cm, -12 cm, and -27 cm, in vertical height. The change in y from one time interval to the next is -3 cm, -9 cm $[=-12\text{ cm} - (-3\text{ cm})]$, and -15 cm $[=-27\text{ cm} - (-12\text{ cm})]$. The ratio of these numbers is 1:3:5, the sequence of odd integers. Answer B is correct.

EXAMPLE 3.3 Paintball gun

Video Tutor Solution

Now we are going to look at a classic example of projectile motion: a bullet fired from a gun. As shown in Figure 3.13, a paintball is fired horizontally at a speed of 75.0 m/s from a point 1.50 m above the ground. The ball misses its target and hits the ground some distance away. **(a)** For how many seconds is the ball in the air? **(b)** Find the maximum horizontal displacement (which we'll call the *range* of the ball). Ignore air resistance.

SOLUTION

SET UP We choose to place the origin of the coordinate system at ground level, directly below the end of the gun barrel, as shown in Figure 3.14. (This choice of position for the origin avoids having to deal with negative values of y, a modest convenience.) Then $x_0 = 0$ and $y_0 = 1.50$ m. The gun is fired horizontally, so $v_{0x} = 75.0$ m/s and $v_{0y} = 0$. The final position of the ball, at ground level, is $y = 0$.

SOLVE **Part (a):** We're asked to find the total time the paintball is in the air. This is equal to the time it would take the paintball to fall vertically from its initial height to the ground. In each case, the vertical position is given as a function of time by Equation 3.11, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$. In this problem, $y_0 = 1.50$ m, $y = 0$, and $v_{0y} = 0$, so that equation becomes simply

$$0 = y_0 - \tfrac{1}{2}gt^2.$$

We need to find the time t. Solving for t and substituting numerical values, we obtain

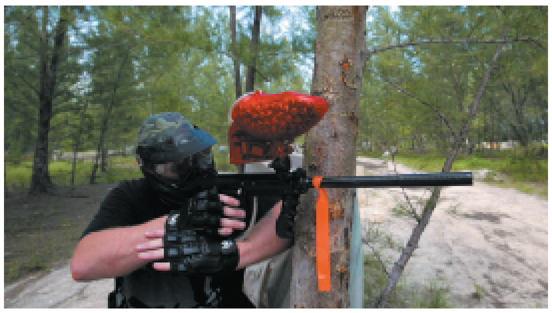
$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(1.50\ \text{m})}{9.80\ \text{m/s}^2}} = 0.533\ \text{s}.$$

Part (b): Now that we know the time t of the ball's flight through the air, we can find the range—that is, the horizontal distance x it travels during time t. We use Equation 3.9: $x = x_0 + v_{0x}t$. Setting $x_0 = 0$, we find

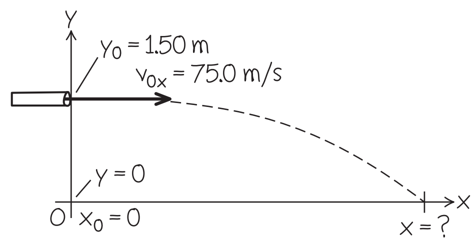
$$x = x_0 + v_{0x}t = 0 + (75.0\ \text{m/s})(0.553\ \text{s}) = 41.5\ \text{m}.$$

REFLECT Actual ranges of paintballs are less than this, typically about 30 m. The difference is due primarily to air resistance, which decreases the horizontal component of velocity.

Practice Problem: If air resistance is ignored, what initial speed is required for a range of 20 m? *Answer:* 36.1 m/s.



▲ **FIGURE 3.13**



▲ **FIGURE 3.14** Our sketch for this problem.

Video Tutor Demo

PROBLEM-SOLVING STRATEGY 3.1 Projectile motion

The strategies that we used in Sections 2.4 and 2.6 for solving straight-line, constant-acceleration motion problems are equally useful for projectile motion.

SET UP

1. Define your coordinate system and make a sketch showing your axes. Usually it is easiest to place the origin at the initial ($t = 0$) position of the projectile, with the x axis horizontal and the y axis pointing upward. Then $x_0 = 0$, $y_0 = 0$, $a_x = 0$, and $a_y = -g$.
2. List the known and unknown quantities. In some problems, the components (or magnitude and direction) of the initial velocity are given, and you can use Equations 3.12 through 3.15 to find the coordinates and velocity components at some later time. In other problems, you might be given two points on the trajectory and be asked to find the initial velocity. Be sure you know which quantities are given and which are to be found.
3. It often helps to state the problem in prose and then translate into symbols. For example, *when* does the particle arrive at a certain point (i.e, at what value of t)? *Where*

CONTINUED

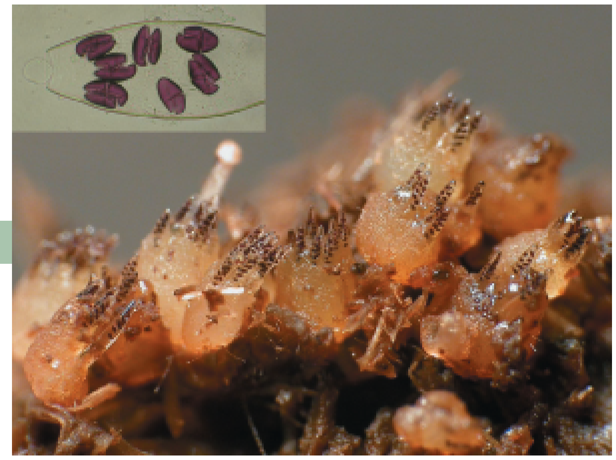
is the particle when a velocity component has a certain value? (That is, what are the values of x and y when v_x or v_y has the specified value?)

SOLVE

4. At the highest point in a trajectory, $v_y = 0$. So the question "When does the projectile reach its highest point?" translates into "What is the value of t when $v_y = 0$?" Similarly, if $y_0 = 0$, then "When does the projectile return to its initial elevation?" translates into "What is the value of t when $y = 0$?" and so on.
5. Resist the temptation to divide the trajectory into segments and analyze each one separately. You don't have to start all over, with a new axis system and a new time scale, when the projectile reaches its highest point. It is usually easier to set up Equations 3.8 through 3.11 at the start and use the same axes and time scale throughout the problem.

REFLECT

6. Try to make rough estimates as to what your answers should be, and then ask whether your calculations confirm your estimates. Ask, "Does this result make sense?"



▶ BIO Application Ballistic spores.

Animals generally depend on muscles to move about, while other organisms such as plants and fungi are relatively immobile and require specialized mechanisms for seed and spore dispersal. Among the fungi, some of those living on animal dung have compartments called asci (shown) that contain spores (inset) and generate substantial hydrostatic pressure. The asci rupture and behave as "squirt guns," propelling the spores into the air. The initial acceleration of the spores has only recently been measured with specialized video cameras capturing up to 250,000 frames per second. These data reveal that the spores are ejected with accelerations of up to 1,800,000 m/s^2, or about 180,000g. These are the largest natural accelerations ever measured.

EXAMPLE 3.4 A home-run hit

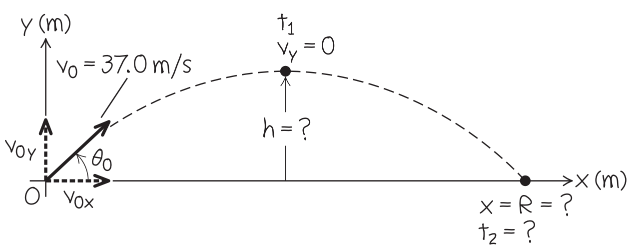
Now let's consider a projectile problem in which the initial velocity is specified in terms of a magnitude and an angle. Suppose a home-run baseball is hit with an initial speed $v_0 = 37.0$ m/s at an initial angle $\theta_0 = 53.1°$. **(a)** Find the ball's position, and the magnitude and direction of its velocity, when $t = 2.00$ s. **(b)** Find the time the ball reaches the highest point of its flight, and find its height h at that point. **(c)** Find the horizontal range R (the horizontal distance from the starting point to the point where the ball hits the ground).

Video Tutor Solution

SOLUTION

SET UP The ball is probably struck a meter or so above ground level, but we'll ignore this small distance and assume that it starts at ground level ($y_0 = 0$). Figure 3.15 shows our sketch. We place the origin of coordinates at the starting point, so $x_0 = 0$. The components of the initial velocity are

$$v_{0x} = v_0 \cos\theta_0 = (37.0\text{ m/s})(0.600) = 22.2\text{ m/s},$$
$$v_{0y} = v_0 \sin\theta_0 = (37.0\text{ m/s})(0.800) = 29.6\text{ m/s}.$$



▲ **FIGURE 3.15** Our sketch for this problem.

SOLVE Part (a): We ignore the effects of the air. We want to find x, y, v_x, and v_y at time $t = 2.00$ s. We substitute these values into Equations 3.8 through 3.11, along with the value $t = 2.00$ s. The coordinates at $t = 2.00$ s are

$$x = v_{0x}t = (22.2\text{ m/s})(2.00\text{ s}) = 44.4\text{ m},$$
$$y = v_{0y}t - \tfrac{1}{2}gt^2$$
$$= (29.6\text{ m/s})(2.00\text{ s}) - \tfrac{1}{2}(9.80\text{ m/s}^2)(2.00\text{ s})^2 = 39.6\text{ m}.$$

The components v_x and v_y of the velocity vector $\vec{v}$ at time $t = 2.00$ s are

$$v_x = v_{0x} = 22.2\text{ m/s},$$
$$v_y = v_{0y} - gt = 29.6\text{ m/s} + (-9.80\text{ m/s}^2)(2.00\text{ s})$$
$$= 10.0\text{ m/s}.$$

The magnitude and direction of the velocity vector $\vec{v}$ at time $t = 2.00$ s are, respectively,

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{(22.2\text{ m/s})^2 + (10.0\text{ m/s})^2} = 24.3\text{ m/s},$$
$$\theta = \tan^{-1}\frac{10.0\text{ m/s}}{22.2\text{ m/s}} = \tan^{-1}0.450 = 24.2°.$$

CONTINUED

Part (b): At the highest point in the ball's path, the vertical velocity component v_y is zero. When does this happen? Call that time t_1; then, using $v_y = v_{0y} - gt$, we find that

$$v_y = 0 = 29.6\ \text{m/s} - (9.80\ \text{m/s}^2)t_1,$$
$$t_1 = 3.02\ \text{s}.$$

The height h at this time is the value of y when $t = 3.02$ s. We use Equation 3.11,

$$y = y_0 + v_{0y}t + \tfrac{1}{2}(-g)t^2,$$

to obtain

$$h = 0 + (29.6\ \text{m/s})(3.02\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(3.02\ \text{s})^2$$
$$= 44.7\ \text{m}.$$

Alternatively, we can use the constant-acceleration formula,

$$v_y{}^2 = v_{0y}{}^2 + 2a(y - y_0) = v_{0y}{}^2 + 2(-g)(y - y_0).$$

At the highest point, $v_y = 0$ and $y = h$. Substituting these values in, along with $y_0 = 0$, we find that

$$0 = (29.6\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)h,$$
$$h = 44.7\ \text{m}.$$

This is roughly half the height of the Statue of Liberty above the playing field.

Part (c): To find the range R, we start by asking *when* the ball hits the ground. This occurs when $y = 0$. Call that time t_2; then, from Equation 3.11,

$$y = 0 = (29.6\ \text{m/s})t_2 + \tfrac{1}{2}(-9.80\ \text{m/s}^2)t_2{}^2.$$

This is a quadratic equation for t_2; it has two roots:

$$t_2 = 0 \quad \text{and} \quad t_2 = 6.04\ \text{s}.$$

There are two times at which $y = 0$: $t_2 = 0$ is the time the ball *leaves* the ground, and $t_2 = 6.04$ s is the time of its return. The latter is exactly twice the time the ball takes to reach the highest point, so the time of descent equals the time of ascent. (This is *always* true if the starting and ending points are at the same elevation and air resistance is ignored. We'll prove it in Example 3.5.)

The range R is the value of x when the ball returns to the ground—that is, when $t = 6.04$ s:

$$R = v_{0x}t_2 = (22.2\ \text{m/s})(6.04\ \text{s}) = 134\ \text{m}.$$

For comparison, the distance from home plate to center field at Pittsburgh's PNC Park is 399 ft (about 122 m). If we could ignore air resistance, the ball really would be a home run.

At the instant the ball returns to $y = 0$, its vertical component of velocity is

$$v_y = 29.6\ \text{m/s} + (-9.80\ \text{m/s}^2)(6.04\ \text{s}) = -29.6\ \text{m/s}.$$

That is, v_y has the same magnitude as the initial vertical velocity v_{0y} but the opposite sign. Since v_x is constant, the angle $\theta = -53.1°$ (below the horizontal) at this point is the negative of the initial angle $\theta_0 = 53.1°$.

REFLECT The actual values of the maximum height h and the range R are substantially lower than the values we've found because air resistance is *not* negligible. In fact, the range of a batted ball is substantially greater (on the order of 10 m for a home-run ball) in Denver than in Pittsburgh because the density of air is almost 20% lower in Denver.

Practice Problem: If the ball could continue to travel *below* its original level (through an appropriately shaped hole in the ground), then negative values of y corresponding to times greater than 6.04 s would be possible. Compute the ball's position and velocity 8.00 s after the start of its flight. *Answers:* $x = 178$ m, $y = -76.8$ m, $v_x = 22.2$ m/s, $v_y = -48.8$ m/s.

EXAMPLE 3.5 Range and maximum height of a home-run ball

For the situation of Example 3.4, we will derive general expressions for the maximum height h and the range R of a ball hit with an initial speed v_0 at an angle θ_0 above the horizontal (between 0 and 90°). To simplify our task, we will assume that the ball is hit from ground level and that it returns to ground level. In addition, for a given v_0, we want to calculate the value of θ_0 that gives the maximum height and the maximum horizontal range.

Video Tutor Solution

SOLUTION

SET UP We can use the same coordinate system and diagram as in Example 3.4, so $y_0 = 0$.

SOLVE The solution will follow the same pattern as in Example 3.4, but now no numerical values are given. Thus the results won't be numbers, but rather symbolic expressions from which we can extract general relationships and proportionalities.

First, for a given θ_0, *when* does the projectile reach its highest point? First we note that the initial velocity in the y direction is given by $v_{0y} = v_0 \sin\theta_0$. At the maximum height, $v_y = 0$, so, from Equation 3.10, the time t_1 at the highest point ($y = h$) is given by

$$v_y = v_0 \sin\theta_0 - gt_1 = 0, \quad t_1 = \frac{v_0 \sin\theta_0}{g}.$$

Next, in terms of v_0 and θ_0, what is the value of y at this time? From Equations 3.10 and 3.11,

$$h = v_{0y}t - \tfrac{1}{2}gt^2 = v_0 \sin\theta_0\left(\frac{v_0 \sin\theta_0}{g}\right) - \tfrac{1}{2}g\left(\frac{v_0 \sin\theta_0}{g}\right)^2,$$
$$h = \frac{v_0{}^2 \sin^2\theta_0}{2g}.$$

To derive a general expression for the range R, we again follow the procedure we used in Example 3.4. First we find an expression for the time t_2 that the projectile returns to its initial elevation. At that time, $y = 0$; from Equation 3.11,

$$0 = (v_0 \sin\theta_0)t_2 - \tfrac{1}{2}gt_2{}^2$$

CONTINUED

The two roots of this quadratic equation are $t_2 = 0$ (the launch time) and $t_2 = 2v_0 \sin\theta_0/g$. The range R is the value of x at the *second* time. Now we can use the fact that the initial velocity in the x direction is $v_{0x} = v_0 \cos\theta_0$ and substitute the above expression for t_2 into Equation 3.9,

$$R = (v_0 \cos\theta_0)\frac{2v_0 \sin\theta_0}{g}.$$

Finally, this expression can be simplified by using the trigonometric identity $2 \sin\theta_0 \cos\theta_0 = \sin 2\theta_0$,

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

REFLECT *Remember that this expression for the range is valid only for the case where the projectile returns to the height from which it was launched*. It does not work for the case where the projectile is launched from the top of a building, for instance. In the above height equation, note that if we vary θ_0, the *maximum* value of h occurs when $\sin\theta_0 = 1$ and $\theta_0 = 90°$—in other words, when the ball is hit straight upward. That's what we should expect. In that case, $h = v_0^2/2g$. If the ball is launched horizontally, then $\theta_0 = 0$ and the maximum height is zero! The maximum range R occurs when $\sin 2\theta_0$ has its greatest value—namely, unity. That occurs when $2\theta_0 = 90°$, or $\theta_0 = 45°$. This angle gives the maximum range for a given initial speed.

Note that both h and R are proportional to the *square* of the initial speed v_0; this relationship may be a little surprising. If we double the initial speed, the range and maximum height increase by a factor of 4! We can also use the proportional-reasoning methods of Section 2.5, along with the equations for h and R just derived, to confirm that both of these quantities are proportional to v_0^2 and inversely proportional to g.

Practice Problem: Show that the range is the same when the launch angle is 30° as when it is 60° and that the range for any launch angle θ_0 is the same as for the complementary angle $(90° - \theta_0)$. *Answer:* $\sin\phi = \sin(180° - \phi)$, so $\sin 2\theta_0 = \sin 2(90° - \theta_0)$.

Symmetry in projectile motion

Comparing the expressions for t_1 and t_2 in Example 3.5, we see that $t_2 = 2t_1$; that is, the total flight time is twice the time required to reach the highest point. It follows that the time required to reach the highest point equals the time required to fall from there back to the initial elevation, as we asserted in Example 3.4. More generally, Figure 3.10 shows that the path of the particle is symmetric about the highest point.

▶ Application Faster than a speeding bullet?

In a human cannonball act, the cannon must be angled so that the human projectile lands in the net. If the available space is short, this angle is greater than 45°. Also, allowance must be made for the level of the net—it may be higher or lower than the level of the muzzle. Finally, a human cannonball is *not* an ideal projectile—air resistance has to be taken into account.

CONCEPTUAL ANALYSIS 3.4

Throwing stones

Two stones are launched from the top of a tall building. One stone is thrown in a direction 20° above the horizontal with a speed of 10 m/s; the other is thrown in a direction 20° below the horizontal with the same speed. How do their speeds compare just before they hit the ground below? (Ignore air friction.)

A. The one thrown upward is traveling faster.
B. The one thrown downward is traveling faster.
C. Both are traveling at the same speed.

SOLUTION We've just learned that the portion of a projectile's trajectory that lies above the initial height is symmetric about its highest point. Thus, when the stone launched upward comes back to the level of the building's top, it is moving at 10 m/s in a direction 20° below the horizontal (symmetric to its launch velocity). At this point, it has the same speed and angle as the stone launched downward. The velocities of the two stones therefore match exactly at any position below the top of the building. So answer C is correct.

EXAMPLE 3.6 Kicking a field goal

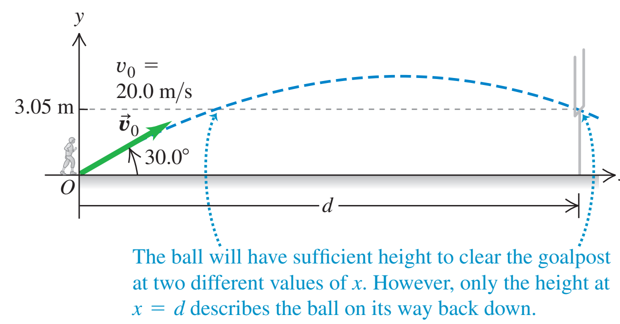
In this example we will look at the more complex question of whether or not a projectile will clear a certain height at a specific point in its trajectory. Consider a field goal attempt, where a football is kicked from a point on the ground that is a horizontal distance d from the goalpost. For the attempt to be successful, the ball must clear the crossbar, 10 ft (about 3.05 m) above the ground, as shown in Figure 3.16. The ball leaves the kicker's foot with an initial speed of 20.0 m/s at an angle of 30.0° above the horizontal. What is the distance d between kicker and goalpost if the ball barely clears the crossbar on its way back down?

Video Tutor Solution

SOLUTION

SET UP We use our idealized model of projectile motion, in which we assume level ground, ignore the effects of air resistance, and treat the football as a point particle. We place the origin of coordinates at the point where the ball is kicked. Then $x_0 = y_0 = 0$,

$$v_{0x} = v_0 \cos 30.0° = (20.0 \text{ m/s})(0.866) = 17.3 \text{ m/s, and}$$
$$v_{0y} = v_0 \sin 30.0° = (20.0 \text{ m/s})(0.500) = 10.0 \text{ m/s}.$$



▲ **FIGURE 3.16** Kicking a field goal.

SOLVE We first ask *when* (i.e., at what value of t) the ball is at a height of 3.05 m above the ground; then we find the value of x at that time. When that value of x is equal to the distance d, the ball is just barely passing over the crossbar.

To find the time t when $y = 3.05$ m, we use the equation $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$. Substituting numerical values, we obtain

$$3.05 \text{ m} = (10.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2.$$

This is a quadratic equation; to solve it, we first write it in standard form: $(4.90 \text{ m/s}^2)t^2 - (10.0 \text{ m/s})t + 3.05 \text{ m} = 0$. Then we use the quadratic formula. (See Chapter 0 if you need to review it.) We get

$$t = \frac{1}{2(4.9 \text{ m/s}^2)} \times \left(10.0 \text{ m/s} \pm \sqrt{(10.0 \text{ m/s})^2 - 4(4.90 \text{ m/s}^2)(3.05 \text{ m})}\right) = 0.373 \text{ s}, 1.67 \text{ s}.$$

There are two roots: $t = 0.373$ s and $t = 1.67$ s. The ball passes the height 3.05 m twice, once on the way up and once on the way down. We need to find the value of x at each of these times, using the equation for x as a function of time. Because $x_0 = 0$, we have simply $x = v_{0x}t$. For $t = 0.373$ s, we get $x = (17.3 \text{ m/s})(0.373 \text{ s}) = 6.45$ m, and for $t = 1.67$ s, we get $x = (17.3 \text{ m/s})(1.67 \text{ s}) = 28.9$ m. So, if the goalpost is located between 6.45 m and 28.9 m from the initial point, the ball will pass over the crossbar; otherwise, it will pass under it.

REFLECT The distance of 28.9 m is about 32 yd; field goal attempts are often successful at that distance. The ball passes the height $y = 10$ ft twice, once on the way up (when $t = 0.373$ s) and once on the way down (when $t = 1.67$ s). To verify this, we could calculate v_y at both times; when we do, we find that it is positive (upward) at $t = 0.373$ s and negative (downward) with the same magnitude at $t = 1.67$ s.

Practice Problem: If the kicker gives the ball the same initial speed and angle but the ball is kicked from a point 25 m from the goalpost, what is the height of the ball above the crossbar as it crosses over the goalpost? *Answer:* 1.2 m.

EXAMPLE 3.7 Shooting a falling pear

Here we will look at a classic projectile motion problem involving two objects that are moving simultaneously. In an archery competition, a contestant is challenged to hit a falling pear with an arrow. At the sound of a horn, he is to shoot his arrow, and at the same instant the pear will be dropped from the top of a tall tower. As shown in Figure 3.17, the archer aims *directly* at the initial position of the pear, seemingly making no allowance for the fact that the pear is dropping as the arrow moves toward it. His rivals assume that this is a mistake, but to their shock, the arrow hits the pear. Show that if the arrow is aimed directly at the initial position of the pear, it will *always* hit the pear, regardless of the pear's initial location or initial speed (assuming that neither pear nor arrow hits the ground first).

Video Tutor Solution

CONTINUED

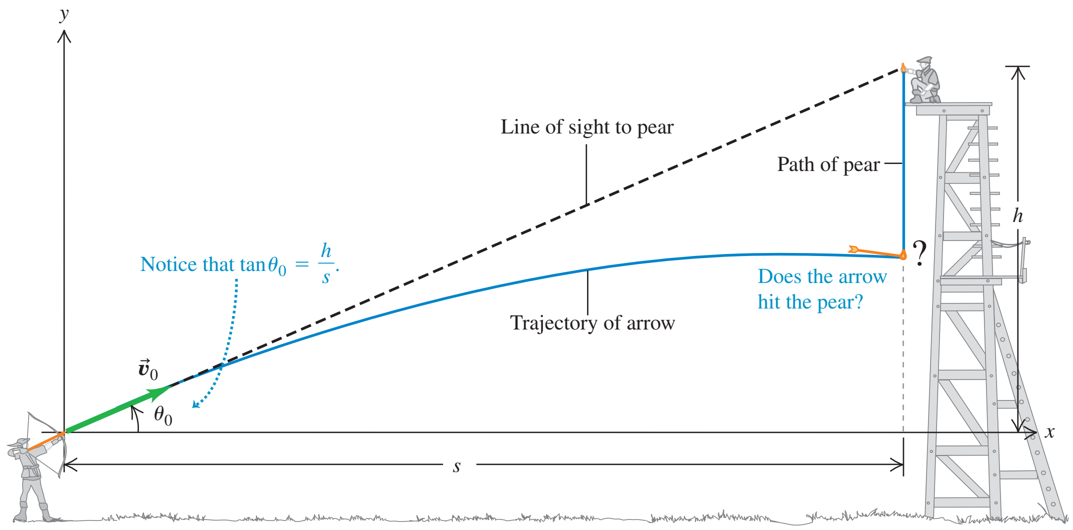


FIGURE 3.17 The archer's shot.

SOLUTION

SET UP To show that the arrow hits the pear, we have to prove that there is some time when the arrow and the pear have the same x and y coordinates. We place the origin at the point from which the archer shoots the arrow. Figure 3.17 shows that, initially, the pear is a horizontal distance s and a vertical distance $h = s\tan\theta_0$ from this point. First, we derive an expression for the time when the x coordinates are the same. Then we ask whether the y coordinates are *also* the same at this time; if they are, the arrow hits the pear.

SOLVE The pear drops straight down, so $x_{\text{pear}} = s$ at *all* times. For the arrow, x_{arrow} is given by Equation 3.9, with $v_{0x} = v_0\cos\theta_0$, $x_{\text{arrow}} = (v_0\cos\theta_0)t$. The time when the x coordinates of the arrow and pear are equal (i.e., $x_{\text{pear}} = x_{\text{arrow}}$) is given by $s = (v_0\cos\theta_0)t$, or

$$t = \frac{s}{v_0\cos\theta_0}.$$

Now we ask whether y_{arrow} and y_{pear} are also equal at this time; if they are, we have a hit. The pear is in one-dimensional free fall, and its position at *any* time t is given by Equation 3.11; that is, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$. The initial height is $y_0 = s\tan\theta_0$, the initial y component of velocity v_y is zero, and we find that

$$y_{\text{pear}} = s\tan\theta_0 - \tfrac{1}{2}gt^2.$$

For the arrow, we use Equation 3.11,

$$y_{\text{arrow}} = (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2.$$

If the y coordinates are equal ($y_{\text{pear}} = y_{\text{arrow}}$) at the same time that $t = s/v_0\cos\theta_0$, the time when their x coordinates are equal, then we have a hit. We see that this happens if $s\tan\theta_0 = (v_0\sin\theta_0)t$ at time $t = s/v_0\cos\theta_0$ (the time when the x coordinates are equal). When we substitute this expression for t into the preceding equations for y_{pear} and y_{arrow}, we have

$$y_{\text{pear}} = s\tan\theta_0 - \tfrac{1}{2}g\left(\frac{s}{v_0\cos\theta_0}\right)^2;$$

$$y_{\text{arrow}} = (v_0\sin\theta_0)\left(\frac{s}{v_0\cos\theta_0}\right) - \tfrac{1}{2}g\left(\frac{s}{v_0\cos\theta_0}\right)^2$$
$$= s\tan\theta_0 - \tfrac{1}{2}g\left(\frac{s}{v_0\cos\theta_0}\right)^2.$$

We see that the expressions for y_{pear} and y_{arrow} are identical! Therefore, the y coordinates of the pear and arrow are the same at the time when the x coordinates of the pear and arrow are also equal.

REFLECT We have established the fact that, at the time the x coordinates are equal, the y coordinates are also equal. Thus, an arrow aimed at the initial position of the pear *always* hits it, no matter what v_0 is. With no gravity ($g = 0$), the pear would remain motionless, and the arrow would travel in a straight line to hit it. With gravity, both objects "fall" the same additional distance $(-\frac{1}{2}gt^2)$ below their $g = 0$ positions, and the arrow still hits the pear. We can see this explicitly in the expressions for the y positions of the two objects, where the effects of gravity are highlighted:

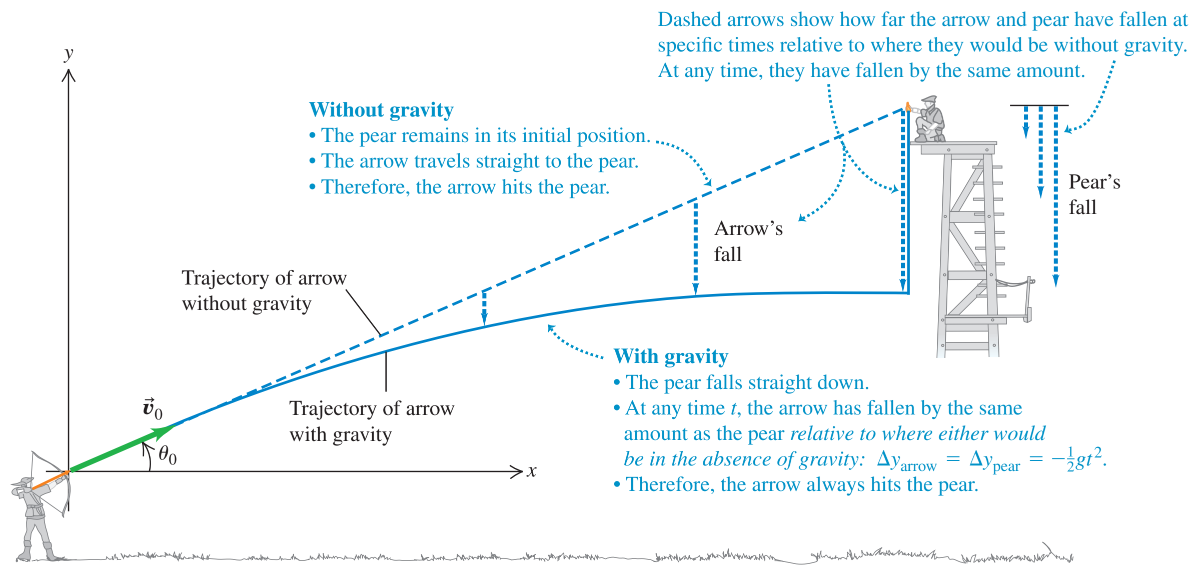
Arrow: $y_{\text{arrow}} = v_{0y}t \underbrace{-\tfrac{1}{2}gt^2}$ and Pear: $y_{\text{pear}} = y_{0,\text{pear}} \underbrace{-\tfrac{1}{2}gt^2}$

Effect of gravity is the same for both.

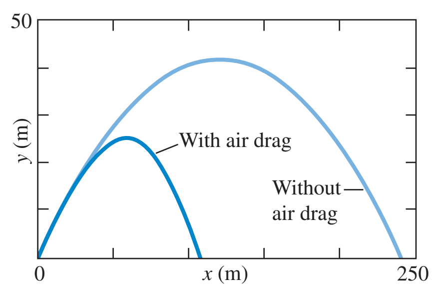
Figure 3.18 shows a comparison of the situations with and without gravity.

Practice Problem: Suppose the pear is released from a height of 6.00 m above the archer's arrow, the arrow is shot at a speed of 30.0 m/s, and the distance between the archer and the base of the tower is 15.0 m. Find the time at which the arrow hits the pear, the distance the pear has fallen, and the height of the arrow above its release point. *Answers:* 0.54 s, 1.4 m, 4.6 m.

CONTINUED



▲ **FIGURE 3.18** An explanation of why an arrow that is aimed directly at a falling target always hits it (provided neither the arrow nor the target hits the ground first).



▲ **FIGURE 3.19** Computer-generated trajectories of a baseball with and without air resistance. Air resistance has a large cumulative effect on the flight of a baseball.

We mentioned at the beginning of this section that air resistance isn't always negligible. When it has to be included, the calculations become a lot more complicated because the forces of air resistance depend on velocity and the acceleration is no longer constant. Figure 3.19 shows a computer simulation of the trajectory of a baseball for 10 s of flight, with an air-resistance force proportional to the square of the particle's speed. We see that air resistance decreases the maximum height and range substantially, and the trajectory becomes asymmetric. In this case, the initial angle θ_0 that gives the maximum range (for a given value of v_0) is less than 45°.

3.4 Uniform Circular Motion

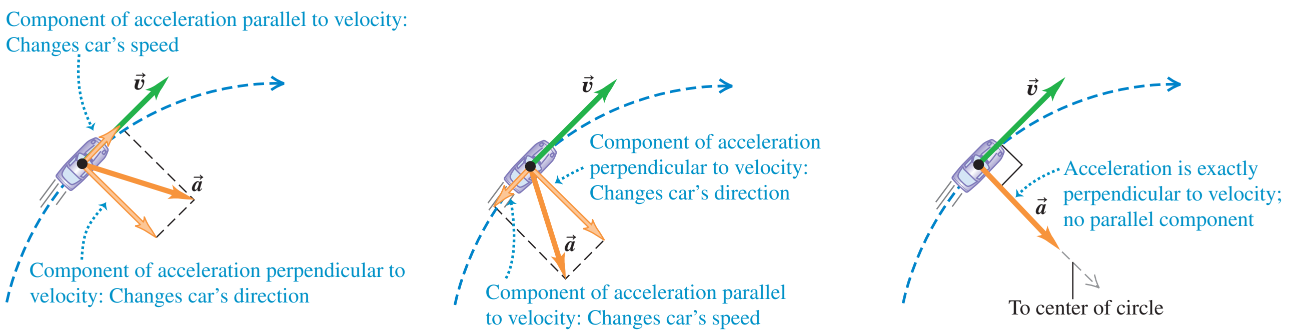
We discussed the components of acceleration in Section 3.2. When a particle moves along a curved path, the direction of its velocity changes. Because of this, it *must* have a component of acceleration perpendicular to the path, even if its speed is constant.

Here we will consider motion on the simplest type of curved path: a circle. When a particle moves in a circle with constant speed, the motion is called **uniform circular motion.** A car rounding a curve with a constant radius at constant speed, a satellite moving in a circular orbit, and an ice skater skating in a circle with constant speed are all examples of uniform circular motion. There is no component of acceleration parallel (tangent) to the path; otherwise, the speed would change (Figure 3.20). The component of acceleration perpendicular to the path, which causes the direction of the velocity to change, is related in a simple way to the speed v of the particle and the radius R of the circle. Our next project is to derive that relationship.

First we note that this is a different problem from the projectile-motion situation in Section 3.3, in which the acceleration was always straight downward and was constant in both magnitude and direction. Here the acceleration is perpendicular to the velocity at each instant; as the direction of the velocity changes, the direction of the acceleration also changes. As we will see, the acceleration vector at each point in the circular path is directed toward the *center* of the circle.

MP

PhET: Ladybug Revolution



(a) Car speeding up along a circular path

(b) Car slowing down along a circular path

(c) Uniform circular motion: Constant speed along a circular path

▲ **FIGURE 3.20** In uniform circular motion, an object moves at constant speed in a circular path, and its acceleration is perpendicular to its velocity.

Figure 3.21a shows an object (represented by a dot) moving with constant speed in a circular path with radius R and center at O. The object moves from A to B, a distance Δs, in a time Δt. The vector change in velocity $\Delta\vec{v}$ during this time is shown in Figure 3.21b.

The two triangles, one in Figure 3.21a and the other in Figure 3.21b, are *similar* because both are isosceles triangles and the angles $\Delta\phi$ are the same. From geometry, the ratios of corresponding sides of similar triangles are equal, so

$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R}, \quad \text{or} \quad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s.$$

The magnitude a_{av} of the average acceleration $\vec{a}_{\text{av}}$ during Δt is therefore

$$a_{\text{av}} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}.$$

The magnitude a of the instantaneous acceleration $\vec{a}$ at point A is the limit of this expression as Δt approaches zero and point B gets closer and closer to point A:

$$a = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}.$$

But the limit of $\Delta s/\Delta t$ is just the *speed* v_1 at point A. Also, A can be any point on the path, and the speed is the same at every point on the path. So we can drop the subscript and let v represent the speed at *any* point. Then we obtain the following relationship:

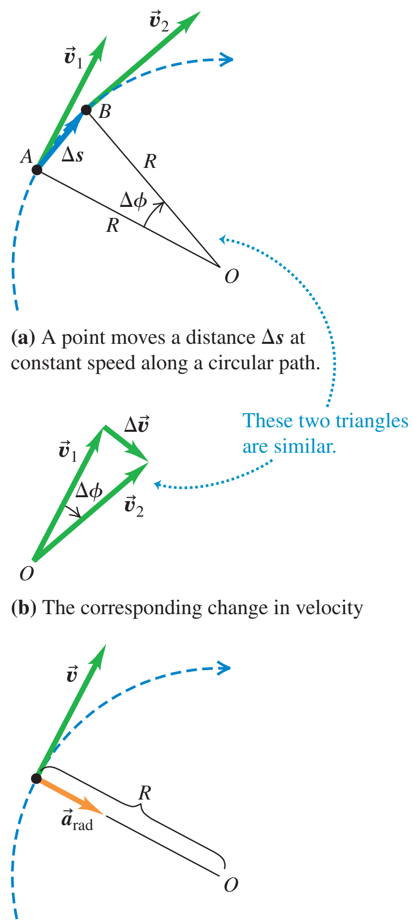
Acceleration in uniform circular motion

The acceleration of an object in uniform circular motion is *radial,* meaning that it always points toward the center of the circle and is perpendicular to the object's velocity $\vec{v}$. We denote it as $\vec{a}_{\text{rad}}$; its magnitude a_{rad} is given by

$$a_{\text{rad}} = \frac{v^2}{R}. \tag{3.12}$$

Units: m/s^2
Notes:

- Not only does the direction of the velocity constantly change as the object moves around the circle, but the direction of the vector $\vec{a}_{\text{rad}}$ also changes.
- The radial acceleration increases as the square of the speed.



(a) A point moves a distance Δs at constant speed along a circular path.

(b) The corresponding change in velocity

(c) The acceleration in uniform circular motion always points toward the center of the circle.

▲ **FIGURE 3.21** Finding the change in velocity $\vec{v}$ of an object moving in a circle at constant speed.

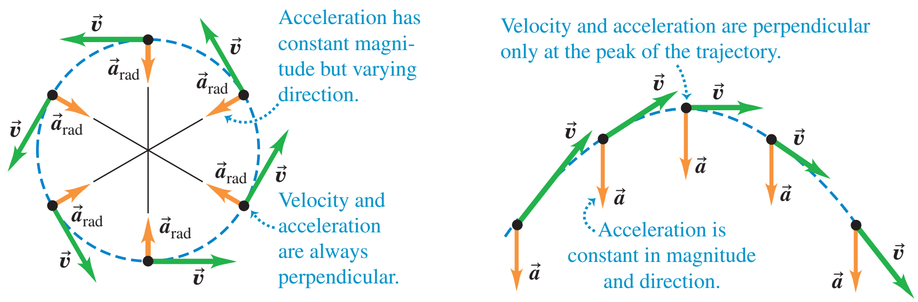
Because the acceleration of an object in uniform circular motion is always directed toward the center of the circle, it is sometimes called **centripetal acceleration.** The word *centripetal* is derived from two Latin words meaning "seeking the center."

Figure 3.22a shows the directions of the velocity and acceleration vectors at several points for an object moving with uniform circular motion. Compare the motion shown in this figure with the projectile motion in Figure 3.22b, in which the acceleration is always directed straight downward and is *not* perpendicular to the path, except at the highest point in the trajectory.

It may seem odd that the centripetal acceleration is proportional to the *square* of the object's speed, rather than simply proportional to the speed. Here's a way to make that relationship more plausible. Suppose that in Figure 3.21 we double the object's speed. This doubles the magnitude of the velocity vector, which would seem to double its rate of change when its direction changes. But note that the direction is also changing twice as rapidly, an effect that, by itself, would double the magnitude of the acceleration. So each of the two effects separately would double a_{rad}, and the combined effect is to increase a_{rad} by a factor of 4 when v is doubled. Hence, we say that a_{rad} is proportional to v^2.

▲ Application
Where am I?

If you've ever used a global positioning system (GPS) unit for navigating or geocaching, you've used an application of uniform circular motion. This system uses a group of 24 satellites to pinpoint locations anywhere on the earth's surface or in the air, often to within as little as 3 m. Although these satellites are moving at speeds greater than 11,000 km/h, their orbits are precisely known, and their exact positions at any instant can be determined. In the field, distance readings taken simultaneously from several of the satellites to a GPS unit provide position vectors that are used to determine precise positions anywhere on the earth.



(a) Uniform circular motion

(b) Projectile motion

▲ **FIGURE 3.22** Acceleration and velocity (a) for a particle in uniform circular motion and (b) for a projectile with no air resistance.

EXAMPLE 3.8 Fast car, flat curve

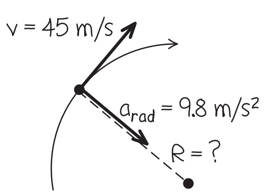
Now let's look at a specific case of radial acceleration. The 2012 Audi R8 GT grips the pavement well enough to enter a curve and sustain a maximum radial acceleration of $1.0\,g = 9.8\ \text{m/s}^2$. If this represents the maximum centripetal acceleration that can be attained without skidding out of the circular path, and if the car is traveling at a constant 45 m/s (about 101 mi/h), what is the minimum radius of curvature the car can negotiate? (Assume that the curve is unbanked.)

Video Tutor Solution

SOLUTION

SET UP AND SOLVE Figure 3.23 shows our diagram. The solution is a straightforward application of Equation 3.12. We find that

$$R = \frac{v^2}{a_{\text{rad}}} = \frac{(45\ \text{m/s})^2}{9.8\ \text{m/s}^2} = 207\ \text{m}.$$



▲ **FIGURE 3.23** Our diagram for this problem.

REFLECT Don't try this at home! The acceleration given represents an absolute maximum with smooth, dry pavement and very grippy tires. If the curve is banked, the radius can be smaller (as we'll see in Chapter 5.)

Practice Problem: A more reasonable maximum acceleration for varying pavement conditions is $5.0\ \text{m/s}^2$. Under these conditions, what is the maximum speed at which a car can negotiate a flat curve with radius 230 m? *Answer:* 34 m/s = 76 mi/h.

EXAMPLE 3.9 A high-speed carnival ride

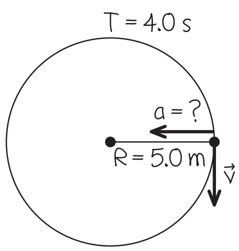
Video Tutor Solution

In this example the motion is in a vertical circle. Passengers in a carnival ride travel in a circle with radius 5.0 m (Figure 3.24). The ride moves at a constant speed and makes one complete circle in a time $T = 4.0$ s. What is the acceleration of the passengers?

◀ **FIGURE 3.24** Circular motion in a carnival ride.

SOLUTION

SET UP Figure 3.25 shows our diagram.



▲ **FIGURE 3.25** Our diagram for this problem.

SOLVE We again use Equation 3.12: $a = v^2/R$. To find the speed v, we use the fact that a passenger travels a distance equal to the circumference of the circle $(2\pi R)$ in the time T for one revolution:

$$v = \frac{2\pi R}{T} = \frac{2\pi(5.0\text{ m})}{4.0\text{ s}} = 7.9\text{ m/s}.$$

The centripetal acceleration is

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{(7.9\text{ m/s})^2}{5.0\text{ m}} = 12\text{ m/s}^2.$$

REFLECT As in Example 3.8, the direction of $\vec{a}$ is toward the center of the circle. The magnitude of $\vec{a}$ is greater than g, the acceleration due to gravity, so this is not a ride for the faint-hearted. (But some roller coasters subject their passengers to accelerations as great as $4g$.)

Practice Problem: If the ride increases in speed so that $T = 2.0$ s, what is a_{rad}? (This question can be answered by using proportional reasoning, without much arithmetic.) *Answer:* 49 m/s^2.

3.5 Relative Velocity in a Plane

In Section 2.7, we introduced the concept of *relative velocity* for motion along a straight line. We can extend this concept to include motion in a plane by using vector addition to combine velocities. We suggest that you review Section 2.7 as a prelude to this discussion.

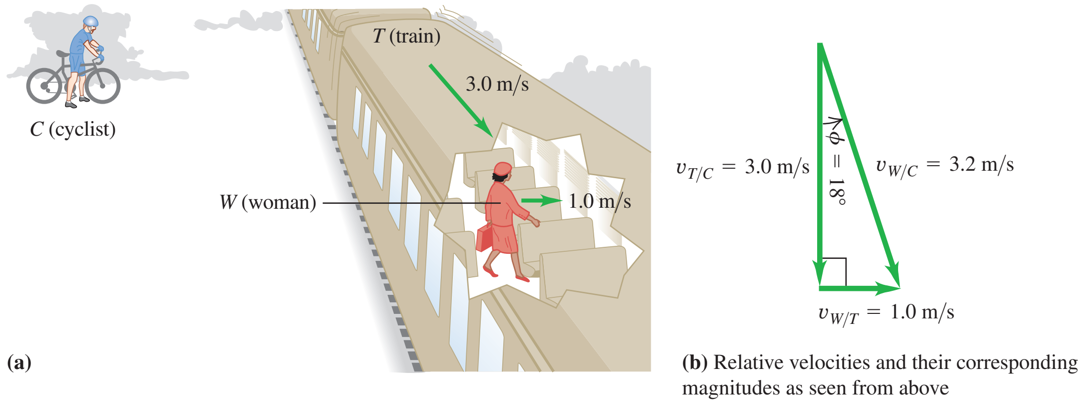
Suppose that the woman in Figure 2.29a is walking, not down the aisle of the railroad car, but from one side of the car to the other, with a speed of 1.0 m/s, as shown in Figure 3.26a. We can again describe the woman's position in two different frames of reference: that of the railroad car and that of the ground. Instead of coordinates x, we use position vectors $\vec{r}$. Let W represent the woman's position, C the frame of reference of the stationary ground observer (the cyclist), and T the frame of reference of the moving train. Then, as Figure 3.26b shows, the velocities are related by

$$\vec{v}_{W/C} = \vec{v}_{W/T} + \vec{v}_{T/C}.$$

Relative motion in a plane

When an object W is moving with velocity $\vec{v}_{W/T}$ relative to an object (or observer) T, and T is moving with velocity $\vec{v}_{T/C}$ with respect to an object (or observer) C, then the velocity $\vec{v}_{W/C}$ of W with respect to C is given by

$$\vec{v}_{W/C} = \vec{v}_{W/T} + \vec{v}_{T/C}. \qquad (3.13)$$



▲ **FIGURE 3.26** (a) A woman walking across a railroad car. (b) Vector diagram for the velocity of the woman relative to the cyclist. Recall that vector addition is commutative.

If the train's velocity relative to the cyclist has magnitude $v_{T/C} = 3.0\text{ m/s}$ and the woman's velocity relative to the train has magnitude $v_{W/T} = 1.0\text{ m/s}$, then her velocity $\vec{v}_{W/C}$ relative to the cyclist is as shown in the vector diagram of Figure 3.26b. The Pythagorean theorem then gives us

$$v_{W/C} = \sqrt{(3.0\text{ m/s})^2 + (1.0\text{ m/s})^2} = \sqrt{10\text{ m}^2/\text{s}^2} = 3.2\text{ m/s}.$$

We can also see from the diagram that the *direction* of the woman's velocity relative to the cyclist makes an angle ϕ with the train's velocity vector $\vec{v}_{T/C}$, where

$$\tan\phi = \frac{v_{W/T}}{v_{T/C}} = \frac{1.0\text{ m/s}}{3.0\text{ m/s}}, \quad \phi = 18°.$$

As in Section 2.7, we have the general rule that if A and B are any two points or frames of reference, then

$$\vec{v}_{A/B} = -\vec{v}_{B/A}. \tag{3.14}$$

The velocity of the woman relative to the train is the negative of the velocity of the train relative to the woman, and so on.

PROBLEM-SOLVING STRATEGY 3.2 **Relative velocity**

The strategy introduced in Section 2.7 is also useful here. The essential difference is that now the $\vec{v}$'s aren't all along the same line, so they have to be treated explicitly as vectors. For the double subscripts on the velocities, $\vec{v}_{A/B}$ always means "velocity of A relative to B." A useful rule for keeping the order of things straight is to regard each double subscript as a fraction. Then the fraction on the left side is the *product* of the fractions on the right side: $P/A = (P/B)(B/A)$. This is helpful when you apply Equation 3.14. If there are *three* different frames of reference, A, B, and C, you can write immediately

$$\vec{v}_{P/A} = \vec{v}_{P/C} + \vec{v}_{C/B} + \vec{v}_{B/A},$$

and so on. This is a *vector* equation, and you should always draw a vector diagram to show the addition of velocity vectors.

EXAMPLE 3.10 **Flying in a crosswind**

In this example we will look at relative velocity in the context of an aircraft flying in a crosswind. The compass of an airplane indicates that it is headed due north, and the airspeed indicator shows that the plane is moving through the air at 240 km/h. If there is a wind of 100 km/h from west to east, what is the velocity of the aircraft relative to the earth?

Video Tutor Solution

CONTINUED

SOLUTION

SET UP Figure 3.27 shows the appropriate vector diagram. We choose subscript *P* to refer to the plane and subscript *A* to the moving air (which now plays the role of the railroad car in Figure 3.26). Subscript *E* refers to the earth. The information given is

$$\vec{v}_{P/A} = 240\ \text{km/h} \quad \text{due north,}$$
$$\vec{v}_{A/E} = 100\ \text{km/h} \quad \text{due east,}$$

and we want to find the magnitude and direction of $\vec{v}_{P/E}$.

SOLVE We adapt Equation 3.13 to the notation of this example:

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}.$$

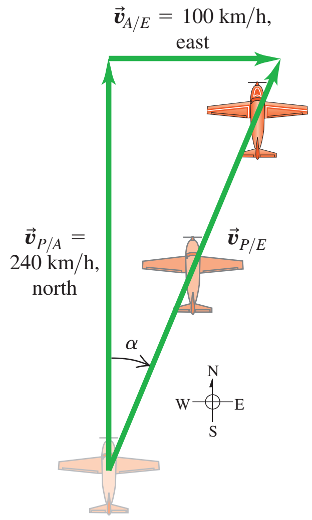
The three relative velocities and their relationship are shown in Figure 3.27. From this diagram, we find that

$$v_{P/E} = \sqrt{(240\ \text{km/h})^2 + (100\ \text{km/h})^2} = 260\ \text{km/h},$$

$$\alpha = \tan^{-1}\frac{100\ \text{km/h}}{240\ \text{km/h}} = 23°\ \text{E of N}.$$

REFLECT The plane's velocity with respect to the air is straight north, but the air's motion relative to earth gives the plane's velocity with respect to earth a component toward the east.

▶ **FIGURE 3.27** The plane is pointed north, but the wind blows east, giving the resultant velocity $\vec{v}_{P/E}$ relative to the earth.



Practice Problem: If the plane maintains its airspeed of 240 km/h, but the wind decreases, what is the wind speed if the plane's velocity with respect to earth is 15° east of north? *Answer:* 64 km/h.

EXAMPLE 3.11 Compensating for a crosswind

The lift and drag forces on an aircraft are functions of its velocity relative to the air, not to the ground. Consequently, pilots must compensate for the effects of wind. In this example we will determine in what direction the pilot in Example 3.10 should head in order for the plane to travel due north. Then we will determine the plane's velocity relative to the earth. (We will assume that the wind velocity and the magnitude of the airspeed are the same as in Example 3.10.)

Video Tutor Solution

SOLUTION

SET UP Now the information given is

$$\vec{v}_{P/A} = 240\ \text{km/h} \quad \text{direction unknown,}$$
$$\vec{v}_{A/E} = 100\ \text{km/h} \quad \text{due east.}$$

Figure 3.28 shows the appropriate vector diagram. Be sure you understand why this is not the same diagram as the one in Figure 3.27.

SOLVE We want to find $\vec{v}_{P/E}$; its magnitude is unknown, but we know that its direction is due north. Note that both this and the preceding example require us to determine two unknown quantities. In Example 3.10, these were the magnitude and direction of $\vec{v}_{P/E}$; in this example, the unknowns are the direction of $\vec{v}_{P/A}$ and the *magnitude* of $\vec{v}_{P/E}$.

The three relative velocities must still satisfy the vector equation

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}.$$

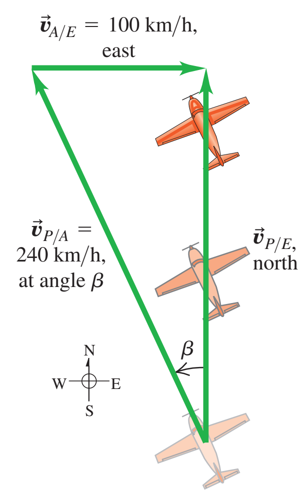
We find that

$$\beta = \sin^{-1}\frac{100\ \text{km/h}}{240\ \text{km/h}} = 25°,$$

$$v_{P/E} = \sqrt{(240\ \text{km/h})^2 - (100\ \text{km/h})^2} = 218\ \text{km/h}.$$

The pilot should head 25° west of north; her ground speed will then be 218 km/h.

▶ **FIGURE 3.28** The pilot must point the plane in the direction of the vector $\vec{v}_{P/A}$ in order to travel due north relative to the earth.



REFLECT When a plane flies in a crosswind that is at right angles to the plane's velocity relative to the ground, its speed relative to the ground is always slower than the airspeed; the trip takes longer than in calm air.

Practice Problem: If the pilot was forced to reduce the plane's speed to 150 km/h, how much would she need to increase the angle at which the plane is pointing (relative to north) in order for the plane to continue to travel due north? *Answer:* 8.7°.

FREQUENTLY ASKED QUESTIONS

Q: *How do I determine the specific direction that a projectile is traveling at some point on its trajectory?*

A: You determine the direction via the velocity vector. The angle that the velocity vector makes with the positive x axis is the direction of motion, and this angle is given by $\theta = \tan^{-1}(v_y/v_x)$. So you need to calculate the components of the velocity vector at the specific point in question.

Q: *Is the velocity of a projectile zero at its maximum height?*

A: No! Only the y component of the velocity vector is zero at the maximum height. The x component is the same as it was when the projectile was launched.

Q: *If I launch a projectile from a building, how do I determine when it will hit the ground?*

A: Assuming that you have chosen a coordinate system for which the ground level is $y = 0$, you simply set the left-hand side of Equation 3.11 to zero and solve for t. Of course, y_0 in that equation will be equal to the building height.

Q: *How do the equations for projectile motion distinguish between an object being thrown downward from a building and an object being thrown upward from a building?*

A: For the downward case, the y component of the initial velocity vector is negative. For the upward case, it is positive.

Q: *How can something that is moving at a constant speed still be accelerating?*

A: Acceleration is the time rate of change of the velocity *vector*. Speed is the magnitude of the velocity and, even if the magnitude is not changing, the direction of the velocity can be changing. If the direction changes, then there must be an associated acceleration.

Q: *How can a projectile still have a downward acceleration of 9.8 m/s^2 after it hits the ground?*

A: It can't! The equations for projectile motion are valid only from the moment after the projectile is fired until the moment before it hits something, like the ground, a glove, or a wall.

Q: *Do I need to keep track of which way the projectile is moving to use the equations for projectile motion? After all, first the projectile goes up and then it comes down.*

A: You need to specify only the initial direction of the velocity the moment after the projectile is launched. Of course, you do this by determining the x and y components of the initial velocity vector. After that, the equations take care of everything.

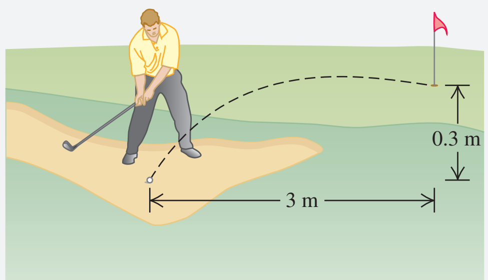
Bridging Problem

Video Tutor Solution

A golfer is attempting to hit a golf ball from a sand trap into a hole that is 3 m away (Figure 3.29). The hole is on a green that is at an elevation of 0.3 m above the sand trap. If the golfer hits the ball at an angle of $\theta = 60°$ above the horizontal, (a) what must be the speed v_0 of the ball for it to go directly into the hole without rolling along the green? (b) How long is the ball in the air? (c) What is the final speed of the ball just as it enters the hole? (d) At what angle is the ball traveling just before it enters the hole?

Set Up
- Choose an appropriate coordinate system for this problem.
- Write down the equations of motion for the golf ball.
- Write down the components of the initial velocity in terms of the angle θ.
- Identify the mathematical conditions for the ball to reach the hole.

Solve
- Use the x axis position equation to derive an expression for the time at which the ball reaches the hole in terms of v_0 and θ.
- Substitute this expression in the y axis equation.
- Determine v_0.
- Determine the final speed and the angle at which the ball is traveling.



▲ **FIGURE 3.29**

Reflect
- How would your equations change if the hole were at the same level as the sand trap?
- Are there any points along the ball's trajectory at which its acceleration is zero?
- How would you have approached this problem if, instead, you had been given the initial speed of the ball and had to find the angle θ?
- Explain why the golfer would miss his shot if he used the range equation from Example 3.5 to calculate the speed of the ball.

CHAPTER 3 SUMMARY

Position, Velocity, and Acceleration Vectors

(Sections 3.1 and 3.2) The position vector $\vec{r}$ of an object in a plane is the displacement vector from the origin to that object. Its components are the coordinates x and y. The average velocity $\vec{v}_{av}$ of the object during a time interval Δt is its displacement $\Delta\vec{r}$ (the change in the position vector $\vec{r}$) divided by Δt:

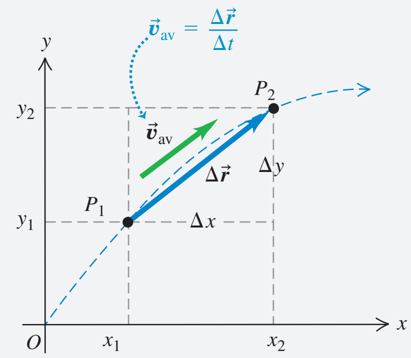
$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}. \quad (3.1)$$

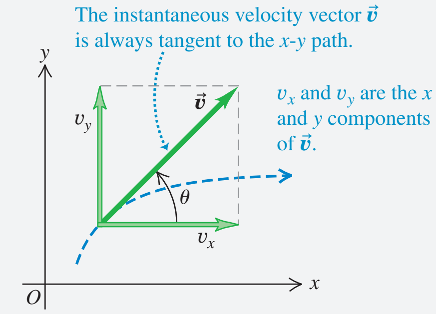
Because Δt is a scalar quantity, the direction of the average velocity vector $\vec{v}_{av}$ is determined entirely by the direction of the vector displacement $\Delta\vec{r}$.

The average acceleration $\vec{a}_{av}$ during the time interval Δt is the change in velocity $\Delta\vec{v}$ divided by the time interval Δt:

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}. \quad (3.4)$$

The instantaneous velocity and acceleration are given by $\vec{v} = \lim_{\Delta t \to 0}(\Delta\vec{r}/\Delta t)$ and $\vec{a} = \lim_{\Delta t \to 0}(\Delta\vec{v}/\Delta t)$, respectively. Graphically, the instantaneous velocity is the tangent to the position-versus-time curve.





Projectile Motion

(Section 3.3) **Projectile motion** occurs when an object is given an initial velocity and then follows a path determined entirely by the effect of a constant gravitational force. The path, called a *trajectory*, is a parabola in the x-y plane. The vertical motion of a projectile is independent of its horizontal motion.

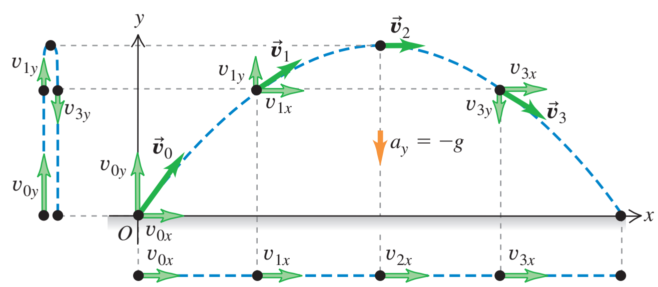
In projectile motion, $a_x = 0$ (there is no horizontal component of acceleration) and $a_y = -g$ (a constant vertical component of acceleration due to a constant gravitational force). The coordinates and velocity components, as functions of time, are

$$v_x = v_{0x}, \quad (3.8)$$

$$x = x_0 + v_{0x}t, \quad (3.9)$$

$$v_y = v_{0y} - gt, \quad (3.10)$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2. \quad (3.11)$$



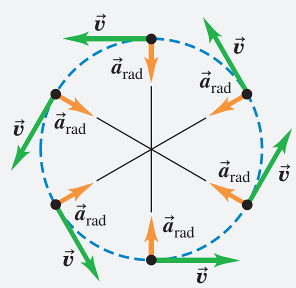
The vertical and horizontal components of a projectile's motion are independent.

Uniform Circular Motion

(Section 3.4) When a particle moves in a circular path with radius R and constant speed v, it has an acceleration with magnitude

$$a_{rad} = \frac{v^2}{R} \quad (3.12)$$

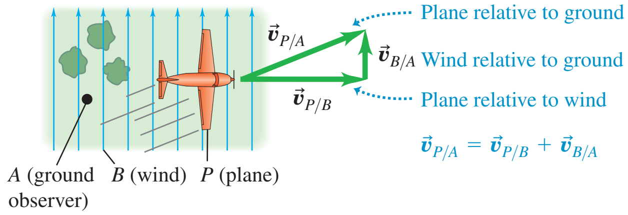
always directed toward the center of the circle and perpendicular to the instantaneous velocity v at each instant.



Relative Velocity

(Section 3.5) When an object P moves relative to another object (or reference frame) B, and B moves relative to a third object (or reference frame) A, we denote the velocity of P relative to B by $\vec{v}_{P/B}$, the velocity of P relative to A by $\vec{v}_{P/A}$, and the velocity of B relative to A by $\vec{v}_{B/A}$. These velocities are related by this variation of Equation 3.13:

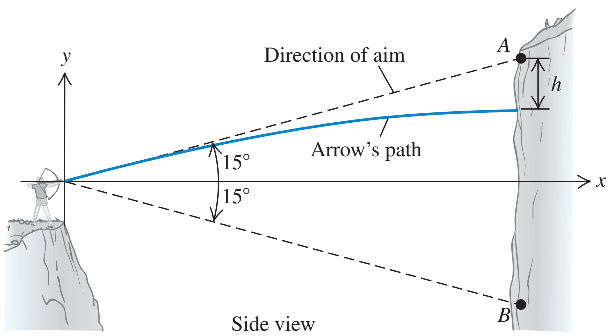
$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}.$$



MP For assigned homework and other learning materials, go to MasteringPhysics®.

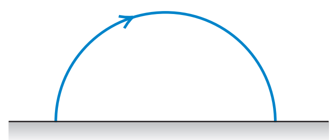
Conceptual Questions

1. A teacher stands in the middle of a parking lot and watches a small child peddle a tricycle around the lot. At time $t = 0$, the child is due north of the teacher, traveling west. Thirty seconds later, the child is due west of the teacher, traveling south. What are the directions of the child's (a) displacement, (b) average velocity, and (c) average acceleration? Express your answers in terms of these general directions: N of W, S of W, N of E, and S of E.
2. Suppose you shoot an arrow across a ravine at the vertical cliff opposite you, a distance d away, as shown in Figure 3.30. On your first try, you aim your arrow 15° above the horizontal, directly toward point A; it strikes the cliff at a distance h below point A. On your next try, you aim the arrow 15° below the horizontal, toward point B, shooting it with the same speed. Will the second arrow strike the cliff (a) the same distance h below point B, (b) less than the distance h below point B, or (c) more than the distance h below point B?



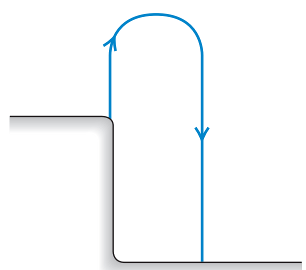
▲ **FIGURE 3.30** Question 2.

3. A football is thrown in a parabolic path. Is there any point at which the acceleration is parallel to the velocity? Perpendicular to the velocity?
4. If an athlete can give himself the same initial speed regardless of the direction in which he jumps, how is his maximum vertical jump (high jump) related to his maximum horizontal jump (long jump)?
5. The maximum range of a projectile occurs when it is aimed at a 45° angle if air resistance is ignored. At what angle should you launch it so that it will achieve the maximum time in the air? What would be its range in that case?
6. A projectile is fired at an angle above the horizontal from the edge of a vertical cliff. (a) Is its velocity ever only horizontal? If so, when? (b) Is its velocity ever only vertical? If so, why?
7. An archer shoots an arrow from the top of a vertical cliff at an angle θ above the horizontal. When the arrow reaches the level ground at the bottom of the cliff, will its speed depend on the angle θ at which it was shot?
8. An observer draws the path of a stone thrown into the air, as shown in Figure 3.31. What is wrong with the path shown? (There are *two* things wrong with it; can you spot both of them?)



▲ **FIGURE 3.31** Question 8.

9. In uniform circular motion, how does the acceleration change when the speed is increased by a factor of 3? When the radius is decreased by a factor of 2?
10. A hunter shoots a bullet from the top of a cliff. What is wrong with the drawing of the bullet's path in Figure 3.32?



▲ **FIGURE 3.32** Question 10.

11. You attach a weight to the end of a string of length L and twirl the weight in a horizontal circle. You find that when you twirl the weight at a speed v, the string breaks. What is the fastest speed at which you could twirl the string if you cut the string down to a length of $\frac{1}{4}L$? (Neglect the effects of gravity.)
12. According to what we have seen about circular motion, the earth is accelerating toward the sun, yet it is not getting any closer to the sun. To many people, this situation would seem like a contradiction. Explain why it really is *not* a contradiction.

Multiple-Choice Problems

1. A cannonball is fired toward a vertical building 400 m away with an initial velocity of 100 m/s at 36.9° above the horizontal. The ball will hit the building in
   A. 4.0 s. B. 5.0 s.
   C. less than 4.0 s. D. more than 5.0 s.
2. If the cannonball in the previous question is fired horizontally from a 150-m-high cliff, but still 400 m from the building, then the ball will hit the building in
   A. 4.0 s. B. 5.0 s.
   C. less than 4.0 s. D. more than 5.0 s.

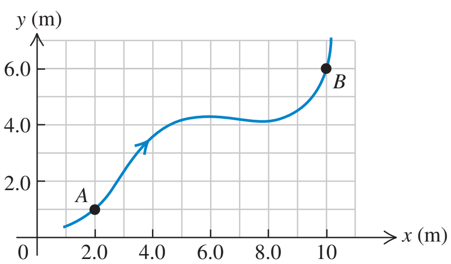
3. A ball thrown horizontally from the top of a building hits the ground in 0.50 s. If it had been thrown with twice the speed in the same direction, it would have hit the ground in
   A. 4.0 s.
   B. 1.0 s.
   C. 0.50 s.
   D. 0.25 s.
   E. 0.125 s.
4. Two balls are dropped from the top of the Leaning Tower of Pisa. The second ball is dropped a fraction of a second after the first ball. As they continue to accelerate to the ground, the distance between them will
   A. remain constant.
   B. decrease.
   C. increase.
5. Two balls are dropped at the same time from different heights. As they accelerate toward the ground, the distance between them will
   A. remain constant.
   B. decrease.
   C. increase.
6. An airplane flying at a constant horizontal velocity drops a package of supplies to a scientific mission in the Antarctic. If air resistance is negligibly small, the path of this package, *as observed by a person in the plane,* is
   A. a parabola.
   B. a straight line downward.
   C. a straight line pointing ahead of the plane.
   D. a straight line pointing behind the plane.
7. Your boat departs from the bank of a river that has a swift current parallel to its banks. If you want to cross this river in the shortest amount of time, you should direct your boat
   A. perpendicular to the current.
   B. upstream.
   C. downstream.
   D. so that it drifts with the current.
8. A ball is thrown horizontally from the top of a building and lands a distance d from the foot of the building after having been in the air for a time T and encountering no significant air resistance. If the building were twice as tall, the ball would have
   A. landed a distance $2d$ from the foot of the building.
   B. been in the air a time $2T$.
   C. been in the air a time $T\sqrt{2}$.
   D. reached the ground with twice the speed it did from the shorter building.
9. A child standing on a rotating carousel at a distance R from the center walks slowly inward until she's a distance $R/2$ from the center. Compared to her original centripetal acceleration, her acceleration is now
   A. half as great.
   B. twice as great.
   C. $\sqrt{2}$ times as great.
   D. unchanged.
10. An airplane whose airspeed is 600 mi/h is flying perpendicular to a jet stream whose speed relative to the earth's surface is 100 mi/h. The airplane's speed relative to the earth's surface is
    A. 600 mi/h.
    B. somewhat less than 600 mi/h.
    C. somewhat more than 600 mi/h.
    D. 100 mi/h.
11. At the same time that rock A is dropped from rest from the top of a building, rock B is thrown horizontally away from the building, also starting at the top. Air resistance is not large enough to worry about. Which of the following statements are correct? (More than one choice may be correct.)
    A. Rock B hits the ground before rock A does.
    B. Both rocks have the same speed just as they reach the ground.
    C. Rock B has more acceleration than rock A.
    D. Both rocks reach the ground at the same time.
12. A stone is thrown horizontally with a speed of 15 m/s from the top of a vertical cliff at the edge of a lake. If the stone hits the water 2.0 s later, the height of the cliff is closest to
    A. 10 m. B. 20 m. C. 30 m. D. 50 m.
13. An object traveling at constant speed V in a circle of radius R has an acceleration a. If *both* R and V are doubled, the new acceleration will be
    A. a. B. $2a$. C. $4a$. D. $8a$.

Problems

3.1 Velocity in a Plane

3.2 Acceleration in a Plane

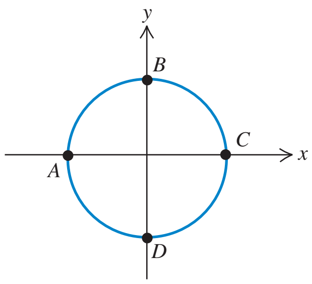
1. | A velocity vector has a magnitude of 25.0 m/s. If its y component is -13.0 m/s, what are the possible values of its x component?
2. | At an air show, a jet plane has velocity components $v_x = 625$ km/h and $v_y = 415$ km/h at time 3.85 s and $v_x = 838$ km/h and $v_y = 365$ km/h at time 6.52 s. For this time interval, find (a) the x and y components of the plane's average acceleration and (b) the magnitude and direction of its average acceleration.
3. || A dragonfly flies from point A to point B along the path shown in Figure 3.33 in 1.50 s. (a) Find the x and y components of its position vector at point A. (b) What are the magnitude and direction of its position vector at A? (c) Find the x and y components of the dragonfly's average velocity between A and B. (d) What are the magnitude and direction of its average velocity between these two points?



▲ **FIGURE 3.33** Problem 3.

4. | A coyote chasing a rabbit is moving 8.00 m/s due east at one moment and 8.80 m/s due south 4.00 s later. Find (a) the x and y components of the coyote's average acceleration during that time and (b) the magnitude and direction of the coyote's average acceleration during that time.
5. | A pool ball is rolling along a table with a constant velocity. The components of its velocity vector are $v_x = 0.5$ m/s and $v_y = 0.8$ m/s. Calculate the distance it travels in 0.4 s.

6. | An athlete starts at point A and runs at a constant speed of 6.0 m/s around a round track 100 m in diameter, as shown in Figure 3.34. Find the x and y components of this runner's average velocity and average acceleration between points (a) A and B, (b) A and C, (c) C and D, and (d) A and A (a full lap). (e) Calculate the magnitude of the runner's average velocity between A and B. Is his average speed equal to the magnitude of his average velocity? Why or why not? (f) How can his velocity be changing if he is running at constant speed?

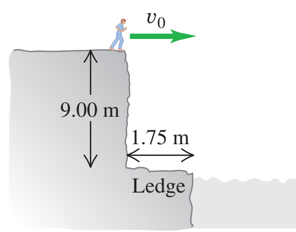


▲ **FIGURE 3.34** Problem 6.

7. | A particle starts from rest at the origin with an acceleration vector that has magnitude 4 m/s^2 and direction 30° above the positive x axis. (a) What are the components of its velocity vector 20 s later? (b) What is the particle's position at that time?

3.3 Projectile Motion

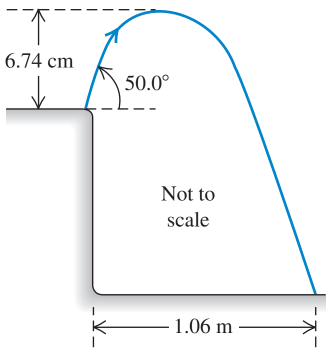
8. | A projectile is fired from ground level at an angle of 60° above the horizontal with an initial speed of 30 m/s. What are the magnitude and direction (relative to horizontal) of its instantaneous velocity at (a) the moment it is fired, (b) the moment it reaches its maximum height, and (c) the moment before it hits the ground?
9. | A stone is thrown horizontally at 30.0 m/s from the top of a very tall cliff. (a) Calculate its horizontal position and vertical position at 2 s intervals for the first 10.0 s. (b) Plot your positions from part (a) to scale. Then connect your points with a smooth curve to show the trajectory of the stone.
10. | A baseball pitcher throws a fastball horizontally at a speed of 42.0 m/s. Ignoring air resistance, how far does the ball drop between the pitcher's mound and home plate, 60 ft 6 in. away?
11. | A physics book slides off a horizontal tabletop with a speed of 1.10 m/s. It strikes the floor in 0.350 s. Ignore air resistance. Find (a) the height of the tabletop above the floor, (b) the horizontal distance from the edge of the table to the point where the book strikes the floor, and (c) the horizontal and vertical components of the book's velocity, and the magnitude and direction of its velocity, just before the book reaches the floor.
12. | A tennis ball rolls off the edge of a tabletop 0.750 m above the floor and strikes the floor at a point 1.40 m horizontally from the edge of the table. (a) Find the time of flight of the ball. (b) Find the magnitude of the initial velocity of the ball. (c) Find the magnitude and direction of the velocity of the ball just before it strikes the floor.
13. | A military helicopter on a training mission is flying horizontally at a speed of 60.0 m/s when it accidentally drops a bomb (fortunately, not armed) at an elevation of 300 m. You can ignore air resistance. (a) How much time is required for the bomb to reach the earth? (b) How far does it travel horizontally while falling? (c) Find the horizontal and vertical components of the bomb's velocity just before it strikes the earth. (d) Draw graphs of the horizontal distance vs. time and the vertical distance vs. time for the bomb's motion. (e) If the velocity of the helicopter remains constant, where is the helicopter when the bomb hits the ground?
14. | A daring swimmer dives off a cliff with a running horizontal leap, as shown in Figure 3.35. What must her minimum speed be just as she leaves the top of the cliff so that she will miss the ledge at the bottom, which is 1.75 m wide and 9.00 m below the top of the cliff?



▲ **FIGURE 3.35** Problem 14.

15. | A football is thrown with an initial upward velocity component of 15.0 m/s and a horizontal velocity component of 18.0 m/s. (a) How much time is required for the football to reach the highest point in its trajectory? (b) How high does it get above its release point? (c) How much time after it is thrown does it take to return to its original height? How does this time compare with what you calculated in part (a)? Is your answer reasonable? (d) How far has the football traveled horizontally from its original position?
16. | A tennis player hits a ball at ground level, giving it an initial velocity of 24 m/s at 57° above the horizontal. (a) What are the horizontal and vertical components of the ball's initial velocity? (b) How high above the ground does the ball go? (c) How long does it take the ball to reach its maximum height? (d) What are the ball's velocity and acceleration at its highest point? (e) For how long a time is the ball in the air? (f) When this ball lands on the court, how far is it from the place where it was hit?
17. | (a) A pistol that fires a signal flare gives it an initial velocity (muzzle velocity) of 125 m/s at an angle of 55.0° above the horizontal. You can ignore air resistance. Find the flare's maximum height and the distance from its firing point to its landing point if it is fired (a) on the level salt flats of Utah, and (b) over the flat Sea of Tranquility on the moon, where $g = 1.62$ m/s^2.
18. | A projectile is fired at an angle of 50° above the horizontal at a speed of 100 m/s. Calculate the magnitude and direction of its velocity (relative to the horizontal) at (a) $t = 5$ s, (b) $t = 10$ s, and (c) $t = 15$ s.
19. | A batted baseball leaves the bat at an angle of 30.0° above the horizontal and is caught by an outfielder 375 ft from home plate at the same height from which it left the bat. (a) What was the initial speed of the ball? (b) How high does the ball rise above the point where it struck the bat?
20. | A man stands on the roof of a 15.0-m-tall building and throws a rock with a velocity of magnitude 30.0 m/s at an angle of 33.0° above the horizontal. You can ignore air resistance. Calculate (a) the maximum height above the roof reached by the rock, (b) the magnitude of the velocity of the rock just before it strikes the ground, and (c) the horizontal distance from the base of the building to the point where the rock strikes the ground.
21. | **The champion jumper of the insect world.** The froghopper, BIO *Philaenus spumarius,* holds the world record for insect jumps. When leaping at an angle of 58.0° above the horizontal, some of the tiny critters have reached a maximum height of 58.7 cm above the level ground. (See *Nature,* Vol. 424, 31 July 2003, p. 509.) (a) What was the takeoff speed for such a leap? (b) What horizontal distance did the froghopper cover for this world-record leap?

22. || A grasshopper leaps into the air from the edge of a vertical cliff, as shown in Figure 3.36. Use information from the figure to find (a) the initial speed of the grasshopper and (b) the height of the cliff.



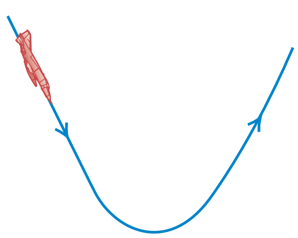
▲ **FIGURE 3.36** Problem 22.

23. || Show that a projectile achieves its maximum range when it is fired at 45° above the horizontal if $y = y_0$.
24. || A water balloon slingshot launches its projectiles essentially from ground level at a speed of 25.0 m/s. (a) At what angle should the slingshot be aimed to achieve its maximum range? (b) If shot at the angle you calculated in part (a), how far will a water balloon travel horizontally? (c) For how long will the balloon be in the air? (You can ignore air resistance.)
25. || Two archers shoot arrows in the same direction from the same place with the same initial speeds but at different angles. One shoots at 45° above the horizontal, while the other shoots at 60.0°. If the arrow launched at 45° lands 225 m from the archer, how far apart are the two arrows when they land? (You can assume that the arrows start at essentially ground level.)
26. | A bottle rocket can shoot its projectile vertically to a height of 25.0 m. At what angle should the bottle rocket be fired to reach its maximum horizontal range, and what is that range? (You can ignore air resistance.)
27. || An airplane is flying with a velocity of 90.0 m/s at an angle of 23.0° above the horizontal. When the plane is 114 m directly above a dog that is standing on level ground, a suitcase drops out of the luggage compartment. How far from the dog will the suitcase land? You can ignore air resistance.

3.4 Uniform Circular Motion

28. | You swing a 2.2 kg stone in a circle of radius 75 cm. At what speed should you swing it so its centripetal acceleration will be 9.8 m/s^2?
29. || Consult Appendix E. Calculate the radial acceleration (in m/s^2 and g's) of an object (a) on the ground at the earth's equator and (b) at the equator of Jupiter (which takes 0.41 day to spin once), turning with the planet.
30. | A model of a helicopter rotor has four blades, each 3.40 m in length from the central shaft to the tip of the blade. The model is rotated in a wind tunnel at 550 rev/min. (a) What is the linear speed, in m/s, of the blade tip? (b) What is the radial acceleration of the blade tip, expressed as a multiple of the acceleration g due to gravity?
31. || A wall clock has a second hand 15.0 cm long. What is the radial acceleration of the tip of this hand?
32. | A curving freeway exit has a radius of 50.0 m and a posted speed limit of 35 mi/h. What is your radial acceleration (in m/s^2) if you take this exit at the posted speed? What if you take the exit at a speed of 50 mi/h?
33. | **Pilot blackout in a power dive.** A jet plane comes in for a downward dive as shown in Figure 3.37. The bottom part of the path is a quarter circle having a radius of curvature of 350 m. According to medical tests, pilots lose consciousness at an acceleration of 5.5g. At what speed (in m/s and mph) will the pilot black out for this dive?

BIO


▲ **FIGURE 3.37** Problem 33.

34. || The rotation of the earth on its axis causes objects sitting on the equator to have a radial acceleration toward the center of the earth. How short would the earth's day (in hours) have to be in order for the radial acceleration at the equator to be equal to g?

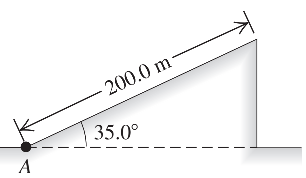
3.5 Relative Velocity in a Plane

35. || A canoe has a velocity of 0.40 m/s southeast relative to the earth. The canoe is on a river that is flowing 0.50 m/s east relative to the earth. Find the velocity (magnitude and direction) of the canoe relative to the river.
36. || **Crossing the river, I.** A river flows due south with a speed of 2.0 m/s. A man steers a motorboat across the river; his velocity relative to the water is 4.2 m/s due east. The river is 800 m wide. (a) What is his velocity (magnitude and direction) relative to the earth? (b) How much time is required for the man to cross the river? (c) How far south of his starting point will he reach the opposite bank?
37. || **Crossing the river, II.** (a) In which direction should the motorboat in the previous problem head in order to reach a point on the opposite bank directly east from the starting point? (The boat's speed relative to the water remains 4.2 m/s.) (b) What is the velocity of the boat relative to the earth? (c) How much time is required to cross the river?
38. | You're standing outside on a windless day when raindrops begin to fall straight down. You run for shelter at a speed of 5.0 m/s, and you notice while you're running that the raindrops appear to be falling at an angle of about 30° from the vertical. What's the vertical speed of the raindrops?
39. || **Bird migration.** Canada geese migrate essentially along a north–south direction for well over a thousand kilometers in some cases, traveling at speeds up to about 100 km/h. If one such bird is flying at 100 km/h relative to the air, but there is a 40 km/h wind blowing from west to east, (a) at what angle relative to the north–south direction should this bird head so that it will be traveling directly southward relative to the ground? (b) How long will it take the bird to cover a ground distance of 500 km from north to south? (*Note:* Even on cloudy nights, many birds can navigate using the earth's magnetic field to fix the north–south direction.)

BIO

General Problems

40. || A test rocket is launched by accelerating it along a 200.0-m incline at 1.25 m/s^2 starting from rest at point A (Figure 3.38). The incline rises at 35.0° above the horizontal, and at the instant the rocket leaves it, its engines turn off and it is subject only to gravity (air resistance can be ignored). Find (a) the maximum height above the ground that the rocket reaches, and (b) the greatest horizontal range of the rocket beyond point A.



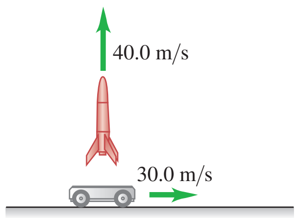
▲ **FIGURE 3.38** Problem 40.

41. || A player kicks a football at an angle of 40.0° from the horizontal, with an initial speed of 12.0 m/s. A second player standing at a distance of 30.0 m from the first (in the direction of the kick) starts running to meet the ball at the instant it is kicked. How fast must he run in order to catch the ball just before it hits the ground?

42. || **Fighting forest fires.** When fighting forest fires, airplanes work in support of ground crews by dropping water on the fires. A pilot is practicing by dropping a canister of red dye, hoping to hit a target on the ground below. If the plane is flying in a horizontal path 90.0 m above the ground and with a speed of 64.0 m/s (143 mi/h), at what horizontal distance from the target should the pilot release the canister? Ignore air resistance.

43. || Two projectiles are fired at 20 m/s from the top a 50-m-tall building. Projectile A is fired at an angle of 30° above the horizontal, while projectile B is fired at an angle of 30° below the horizontal. Calculate (a) the time for each projectile to hit the ground and (b) the speed at which each hits the ground. What can you conclude about the relationship between the launch angle and the speed at which a projectile hits the ground?

44. || A cart carrying a vertical missile launcher moves horizontally at a constant velocity of 30.0 m/s to the right (Figure 3.39). It launches a rocket vertically upward. The missile has an initial vertical velocity of 40.0 m/s relative to the cart. (a) How high does the rocket go? (b) How far does the cart travel while the rocket is in the air? (c) Where does the rocket land relative to the cart?



▲ **FIGURE 3.39** Problem 44.

45. || **The longest home run.** According to the *Guinness Book of World Records,* the longest home run ever measured was hit by Roy "Dizzy" Carlyle in a minor-league game. The ball traveled 188 m (618 ft) before landing on the ground outside the ballpark. (a) Assuming that the ball's initial velocity was 45° above the horizontal, and ignoring air resistance, what did the initial speed of the ball need to be to produce such a home run if the ball was hit at a point 0.9 m (3.0 ft) above ground level? Assume that the ground was perfectly flat. (b) How far would the ball be above a fence 3.0 m (10 ft) in height if the fence were 116 m (380 ft) from home plate?

46. || On the first play of a football game, the kicker kicks the ball with a speed of 25 m/s, and the ball travels a distance of 60 m (about 65 yd). (a) At what two possible angles, relative to the horizontal, could the ball have been kicked? (b) What two possible times could the ball have been in the air? The longer time is sometimes known as the *hang time*.

47. || DATA A projectile is fired horizontally at a speed of 15 m/s from the top of a 200-m-high cliff on Planet X. After it is fired, the height h of the projectile above the base of the cliff is measured as a function of time. The resulting data are shown here:

Time (sec)	h (m)
1.0	190
2.0	160
3.0	110
4.0	40

Make a linearized plot of these data with h on the y axis. Draw a best-fit line through the linearized data and use this line to estimate (a) the acceleration of gravity for Planet X, (b) the time when the projectile reaches the bottom of the cliff, and (c) the horizontal distance the projectile lands from the base of the cliff.

48. || A baseball thrown at an angle of 60.0° above the horizontal strikes a building 18.0 m away at a point 8.00 m above the point from which it is thrown. Ignore air resistance. (a) Find the magnitude of the initial velocity of the baseball (the velocity with which the baseball is thrown). (b) Find the magnitude and direction of the velocity of the baseball just before it strikes the building.

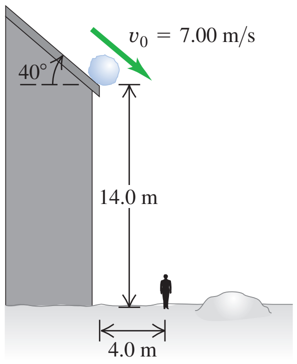
49. || A boy 12.0 m above the ground in a tree throws a ball for his dog, which is standing right below the tree and starts running the instant the ball is thrown. If the boy throws the ball horizontally at 8.50 m/s, (a) how fast must the dog run to catch the ball just as it reaches the ground, and (b) how far from the tree will the dog catch the ball?

50. | A football is kicked from ground level at a speed of 25 m/s. If it reaches a maximum height of 24 m, at what angle was it kicked (relative to horizontal)?

51. || A firefighting crew uses a water cannon that shoots water at 25.0 m/s at a fixed angle of 53.0° above the horizontal. The firefighters want to direct the water at a blaze that is 10.0 m above ground level. How far from the building should they position their cannon? There are *two* possibilities; can you get them both? (*Hint:* Start with a sketch showing the trajectory of the water.)

52. || An archer shoots an arrow at an apple that is sitting on a post. The arrow and the apple are initially at the same height. If the arrow has an initial speed of 75 m/s and the apple is 30 m away, at what launch angle should the archer aim the arrow?

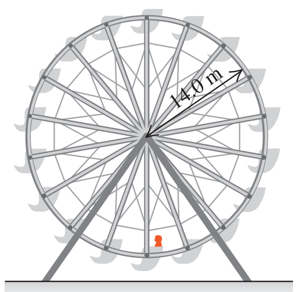
53. ||| **Look out!** A snowball rolls off a barn roof that slopes downward at an angle of 40.0°. (See Figure 3.40.) The edge of the roof is 14.0 m above the ground, and the snowball has a speed of 7.00 m/s as it rolls off the roof. Ignore air resistance. How far from the edge of the barn does the snowball strike the ground if it doesn't strike anything else while falling?



▲ **FIGURE 3.40** Problem 53.

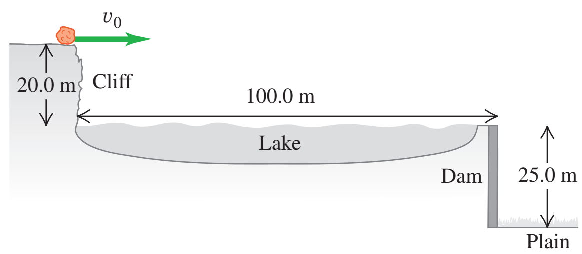
54. || BIO **Spiraling up.** It is common to see birds of prey rising upward on thermals. The paths they take may be spiral-like. You can model the spiral motion as uniform circular motion combined with a constant upward velocity. Assume a bird completes a circle of radius 8.00 m every 5.00 s and rises vertically at a rate of 3.00 m/s. Determine: (a) the speed of the bird relative to the ground; (b) the bird's acceleration (magnitude and direction); and (c) the angle between the bird's velocity vector and the horizontal.

55. || A Ferris wheel with radius 14.0 m is turning about a horizontal axis through its center, as shown in Figure 3.41. The linear speed of a passenger on the rim is constant and equal to 7.00 m/s. What are the magnitude and direction of the passenger's acceleration as she passes through (a) the lowest point in her circular motion and (b) the highest point in her circular motion? (c) How much time does it take the Ferris wheel to make one revolution?



▲ **FIGURE 3.41** Problem 55.

56. || A 76.0 kg boulder is rolling horizontally at the top of a vertical cliff that is 20.0 m above the surface of a lake, as shown in Figure 3.42. The top of the vertical face of a dam is located 100.0 m from the foot of the cliff, with the top of the dam level with the surface of the water in the lake. A level plain is 25.0 m below the top of the dam. (a) What must the minimum speed of the rock be just as it leaves the cliff so that it will travel to the plain without striking the dam? (b) How far from the foot of the dam does the rock hit the plain?



▲ **FIGURE 3.42** Problem 56.

57. || A batter hits a baseball at a speed of 35.0 m/s and an angle of 65.0° above the horizontal. At the same instant, an outfielder 70.0 m away begins running away from the batter in the line of the ball's flight, hoping to catch it. How fast must the outfielder run to catch the ball? (Ignore air resistance, and assume the fielder catches the ball at the same height at which it left the bat.)

Passage Problems

BIO Ballistic seed dispersal. Some plants disperse their seeds by having the fruit split and contract, propelling the seeds through the air. The trajectory of these seeds can be determined by using a high-speed camera. In one type of plant, seeds are projected at 20 cm above ground level with initial speeds between 2.3 m/s and 4.6 m/s. The launch angle is measured from the horizontal, with +90° corresponding to an initial velocity straight upward and −90° straight downward.

58. The experiment is designed so that the seeds will move no farther than 0.20 mm between photographic frames. What minimum frame rate for the high-speed camera is needed to achieve this?
A. 250 frames/s
B. 2500 frames/s
C. 25,000 frames/s
D. 250,000 frames/s

59. About how long does it take a seed launched at 90° at the highest possible initial speed to reach its maximum height? Ignore air resistance.
A. 0.23 s
B. 0.47 s
C. 1.0 s
D. 2.3 s

60. If a seed is launched at zero degrees with the maximum initial speed, how far from the plant will it land? Ignore air resistance, and assume that the ground is flat.
A. 20 cm
B. 93 cm
C. 2.2 m
D. 4.6 m

61. A large number of seeds are observed, and their initial launch angles are recorded. The range of projection angles is found to be −51° to 75°, with a mean of 31°. Approximately 65% of the seeds were launched between 6° and 56°. Which of the following hypotheses is best supported by these data? Seeds are preferentially launched
A. at angles that maximize the height they travel above the plant.
B. at angles below the horizontal in order to drive the seeds into the ground with more force.
C. at angles that maximize the horizontal distance traveled from the plant.
D. at angles that minimize the time the seeds spend exposed to the air.