Restoration of the continuous phase transition induced by frozen disorder in systems with two interacting order parameters

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(Received 31 December 2003; revised manuscript received 28 April 2004; published 3 August 2004)

A model which in the thermodynamic limit calculates the partition function exactly is used to study the phase transitions of systems with two interacting order parameters coupled to their respective random fields. An ordered phase is always unstable for the spatial range $d \leqslant 4$ when both random fields are present. A continuous phase transition is possible when only one of the random fields is nonzero. For this case, a diagram of the equilibrium order parameter as a function of temperature for three different strengths of the random field is constructed. The critical temperature decreases with increasing randomness. The slope of the order parameter becomes steeper as the random field decreases and diverges as the randomness vanishes. These results can be contrasted with pure systems of coupled parameters where a fluctuation-induced first-order transition occurs.

DOI: 10.1103/PhysRevB.70.054401 PACS number(s): 64.60.Cn, 64.60.Fr, 64.60.Kw

I. INTRODUCTION

The influence of random fields is frequently used as a reason to account for qualitative differences between experimental results and theoretical predictions concerning a phase transition. Renormalization group (RG) theory has investigated such systems and in many cases found interesting results.[1–12] In some cases exactly solvable models were used to recover, add to, and/or clarify RG resu1ts.[13–17] This paper considers the critical behavior of systems described by two coupled scalar order parameters, each one of which is influenced by its respective random field. There are many examples of physical systems with competing order parameters.[18] In the present work the study of such systems is done within the context of a model which in the thermodynamic limit calculates the partition function exactly[15] by taking into account fluctuation interactions of equal and opposite momentum.[19] This model belongs to the same universality class as the spherical model[20] and has demonstrated major qualitative results obtained by RG theory. For example, physical systems encompassed by this model are, orthorombic high T_c superconductors with d-pairing[21] and oxygen ordering near a structural phase transition in Y–Ba–Cu–O.[22] Both RG and the model exhibit similar critical behavior at the phase transition in such systems. The model was also applied successfully in pure systems with coupled order parameters[23] where it derived results such as a fluctuation-induced first-order phase transition, as well as random Ising models[15] where it derived a dimensional reduction. Pure systems with coupled order parameters are complex and when RG techniques were applied the results were not always proven rigorously.[24] But since the study of these systems from another point of view, such as the method which is used in this paper, yielded results analogous to those of RG theory, it could be a serious argument in support of the correctness of these results. Thus, we are confident that this model will yield correct results when applied in the random-field type functional of two coupled order parameters. For this case it will be shown that, regardless of the presence of fluctuation interactions, which is the reason for the existence of the first-order phase transition in the pure system of two coupled order parameters,[23,24] the influence of only one random field coupled to one of the order parameters restores the second-order phase transition. Two random fields coupled to their corresponding order parameters forbid an ordered phase for $d \leqslant 4$.

The Ginzburg-Landau free energy functional of a system with two coupled order parameters under the influence of two frozen-in fields is described by

$$F[\varphi_1,\varphi_2] = \frac{1}{2}\int d^dx \left\{ \sum_{i=1}^{2} \left[\tau_i \varphi_i^2(x) + c_i[\nabla \varphi_i(x)]^2 + \frac{1}{4} g_i \varphi_i^4(x) \right] + \frac{1}{2} w \varphi_1^2(x)\varphi_2^2(x) - \sum_{i=1}^{2} [h_i \varphi_i(x) + h_i(x)\varphi_i(x)] \right\}, \tag{1}$$

where $\tau_i = (T - T_{ci})/T_{ci}$ with T_{ci} a trial critical temperature for the continuous scalar order parameter $\varphi_i(x)$, h_i is a uniform external conjugate field, $h_i(x)$ is a quenched random field, w is the coupling strength between the order parameters, and g_i and c_i are the usual constants of interactions. Here is how the rest of the paper is organized: In Sec. II the steps for the solution of functional Eq. (1) are set up, by first applying the replica method and second by applying the defining approximation of the model. In Sec. III the equations are solved and an expression of the equilibrium order parameter of the relevant phase is obtained. The results are used to construct a diagram of the equilibrium order parameter as a function of temperature for different strengths of the random field. Section IV closes with a summary.

II. METHOD OF SOLUTION

To find the partition function corresponding to functional Eq. (1) we first apply the replica method.[25] To calculate the average with respect to $h_i(x)$ free energy of the quenched system, one must average over all the free energies corresponding to all possible random configurations of the random field $h_i(x)$

$$-F \equiv \left\langle \ln\left[\int \prod_{i=1}^{2} [D\varphi_i(x)]\exp\{-F[\varphi_i(x)]\}\right]\right\rangle = \frac{\partial}{\partial n}\left(\int \prod_{i=1}^{2} [D^n\varphi_i(x)]\ \exp(-F_{eff}[\varphi_i(x)])\right)\Bigg|_{n=0} \quad (2)$$

where $\varphi_i(x)$ is an n-component vector, $\varphi_i(x) \equiv [\varphi_{i1}(x), \ldots, \varphi_{in}(x)]$, and $F_{eff}[\varphi_i]$ is defined as

$$F_{eff}[\varphi_i(x)] \equiv \frac{1}{2}\int_{-\infty}^{\infty} d^dx\Bigg\{\sum_{i=1}^{2}\Bigg[\tau_i|\varphi_i(x)|^2 + c_i(\nabla\varphi_i(x))^2 + \sum_{j=1}^{n}\left(\frac{g_i}{4}\varphi_{ij}^4(x) - h_i\varphi_{ij}(x)\right)\Bigg] + \frac{w}{2}\sum_{j=1}^{n}\varphi_{1j}^2(x)\varphi_{2j}^2(x)\Bigg\} - Q[\varphi_i] \quad (3)$$

with

$$Q[\varphi_i] = \int d^dx\ \ln\Bigg\{\prod_{i=1}^{2}\left(\int dh_i(x)\rho_i[h_i(x)]\right) \times\exp\frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{n}h_i(x)\varphi_{ij}(x)\Bigg\}, \quad (4)$$

and where the averaging in Eq. (4) is done with respect to the probability distribution function $P_i\{h(x)\}=\Pi_x\rho_i\{h(x)\}$. Note that for short-range (e.g., Gaussian) random correlations $P_i\{h(x)\}$ can be decomposed into a product of independent probabilities at the various locations in the system, $\Pi_x\rho_i[h(x)]$. Thus choosing

$$\rho_i[h_i(x)] = \frac{e^{-h_i^2(x)/2B_i}}{\sqrt{2\pi B_i}}, \quad (5)$$

with B_i measuring the strength of the respective random fields, $F_{\mathrm{eff}}[\varphi_i]$ takes the form

$$F_{\mathrm{eff}}[\varphi_i(x)] \equiv \frac{1}{2}\int_{-\infty}^{\infty} d^dx\Bigg\{\sum_{i=1}^{2}\Bigg[\tau_i|\varphi_i(x)|^2 + c_i[\nabla\varphi_i(x)]^2 + \sum_{j=1}^{n}\left(\frac{g_i}{4}\varphi_{ij}^4(x) - h_i\varphi_{ij}(x)\right)\Bigg] + \frac{w}{2}\sum_{j=1}^{n}\varphi_{1j}^2(x)\varphi_{2j}^2(x) - B_1\left(\sum_{j=1}^{n}\varphi_{1j}(x)\right)^2 - B_2\left(\sum_{j=1}^{n}\varphi_{2j}(x)\right)^2\Bigg\}. \quad (6)$$

The above effective free energy functional is now treated within the context of the model which reduces the quartic terms as follows:[19]

$$\int d^dx\varphi_{ij}^4(x) \to \frac{a_{ij}^2[\varphi_i]}{V}; \quad a_{ij}[\varphi_{ij}] \equiv \int d^dx\varphi_{ij}^2$$

$$\int d^dx\varphi_{1j}^2(x)\varphi_{2j}^2(x) \to \frac{a_{1j}[\varphi_{1j}]a_{2j}[\varphi_{2j}]}{V}, \quad (7)$$

with V the volume of the system. When above reductions are expressed in momentum space their physical meaning is clear: only fluctuation interactions of equal and antiparallel momentum are considered. When this model was applied to functional Eq. (1) with zero random fields, it demonstrated a rich picture of phase transitions[23] and in particular, it proved the existence of fluctuation-induced first-order phase transition which was in accordance with RG theory predictions.[24] Below it is seen how the random fields replace such a first-order phase transition by a continuous one.

After the reductions of Eq. (7) and the use of a transformation analogous to Hubbard-Stratonovich the partition function becomes

$$Z = \int \prod_{i=1,j=1}^{2,n}(D\varphi_{iq}dx_{ij}dy_{ij})\exp\Bigg\{-\frac{V}{2}\sum_{i=1,j=1}^{2,n}\Bigg[\tau_i x_{ij} + \frac{g_i}{4}x_{ij}^2 + \frac{w}{4}x_{1j}x_{2j} - x_{ij}y_{ij} - \frac{h_i}{V}\varphi_{ij0}\Bigg] - \frac{1}{2}\sum_{i,j,q}^{2,n,\infty}|\varphi_{ijq}|^2(y_{ij} + c_iq^2 - B_i) + \sum_{i=1,q}^{2,\infty}\sum_{j\neq j'}^{n}\frac{B_i}{2}\varphi_{ijq}\varphi_{ij'-q}\Bigg\}. \quad (8)$$

On the one hand the transformation introduced extra variables the x_{ij} and y_{ij}, but on the other hand, it eliminated of the quartic terms of Eq. (6), thus making the calculation of the partition function possible. Therefore, functional integrals in Eq. (8) may be calculated after the diagonalization with respect to the n components of the vector φ_i. Note that the nondiagonal terms in the free energy are due to the random fields B_i. After diagonalization, it becomes required that y_{ij} are independent of j since only this choice reproduces the pure system of coupled order parameters upon suppression of the random fields. This is so because as the random fields vanish, the degeneracy of the eigenvalues of every other choice of y_{ij} does not reduce to n-fold, as expected from the study of the pure system of coupled parameters treated within the context of the replica method. This simplifies the integration since we can define $y_{ij} \equiv y_i$. Hence integrals with respect to y_i and x_{ij} may be performed using the steepest descend method, and the partition function maybe obtained. In the thermodynamic limit $V \to \infty$ the calculation is exact. Consequently,

$$Z = \int \prod_{i=1}^{2}(Dx_{ij}dy_i)\exp\left[-\frac{V}{2}F(x_{ij},y_i;h_1,h_2)\right] \quad (9)$$

with

$$F(x_{ij},y_i;h_1h_2) = \sum_{i=1}^{2}\left[\sum_{j=1}^{n}\left(t_i x_{ij} + \frac{g_i}{4}x_{ij}^2 - y_i x_{ij} + \frac{w}{4}x_{1j}x_{2j}\right) + (n-1)f_d(y_i;c_i) + f_d(y_i - nB_i;c_i) - \frac{nh_i^2}{y_i - nB_i}\right]. \tag{10}$$

In Eq. (10) we have made the following definitions: the normalized trial critical temperature t_i is $t_i = \tau_i + 1/2[g_i\theta(\Lambda;c_i) + w\theta(\Lambda;c_{i'\neq i})] \equiv (T-T_i)/T_i$ and with S_d the surface area of a d-dimensional unit-radius sphere we define $\theta(\Lambda;c_i)$ and f_d

$$\theta(\Lambda;c_i) = \frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{\Lambda^{d-2}}{c_i(d-2)} & d \neq 2\\[2ex] \dfrac{1+\ln(c_i\Lambda^2)}{2c_i} & d = 2\end{cases}$$

$$f_d(y_i;c_i) = \frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{\pi y_i^{d/2}}{dc_i^{d/2}\sin\left[\dfrac{\pi d}{2}\right]} \equiv \kappa(c_i)y_i^{d/2} & d \neq \text{even}\\[2ex] \dfrac{1}{d}\left(-\dfrac{y_i}{c_i}\right)^{d/2}\ln|y_i| \equiv \mu(c_i)y_i^{d/2}\ln|y_i| & d = \text{even}\end{cases}, \tag{11}$$

where Λ is a momentum cutoff.

III. RESULTS

Expressions of the disorder, average free energy and order parameter φ_i, are given by

$$F_{av} = \lim_{n\to 0}\frac{1}{n}F(x_{ij},y_i;h_1,h_2), \tag{12}$$

and

$$\varphi_i = -\frac{\partial F_{av}}{\partial h_i} = \lim_{n\to 0}\frac{h_i}{y_i(h_i)}, \tag{13}$$

with x_{ij} and y_i obtained from equilibrium equations $\partial F/\partial x_{ij} = 0$ and $\partial F/\partial y_i = 0$ using Eq. (10). From $\partial F/\partial x_{ij} = 0$ it is derived that for a fixed i all n of the variables x_{ij} are equal to one another hence $x_{ij} \equiv x_i$. After eliminating x_i we obtain two equations for y_1 and y_2:

$$\frac{-2ng_{i'}}{\Delta}(y_i - t_i) - \frac{-2nw}{\Delta}(t_{i'} - y_{i'}) + (n-1)f_d'(y_i;c_i) + f_d'(y_i - nB_i;c_i) + \frac{nh_i^2}{(y_i - nB_i)^2} = 0, \tag{14}$$

with $\Delta = g_1g_2 - w^2$ and $i \neq i'$. When Eq. (14) is expanded in powers of n, for any d and up to order n (which is sufficient since we consider the $n \to 0$ limit), the solution for $y_i(h_i)$ is independent of n. Using Eq. (13), then, respectively, for a noneven d (including nonintegers) and d even the resulting equations for the averaged order parameter φ_i, are

$$\varphi_i^2 - \frac{2g_{i'}}{\Delta}\frac{h_i}{\varphi_i} + \frac{2g_{i'}t_i}{\Delta} - \frac{2wt_{i'}}{\Delta} + \frac{2w}{\Delta}\frac{h_{i'}}{\varphi_{t'}} + \begin{cases}\dfrac{d}{2}\kappa(c_i)\left(\dfrac{h_i}{\varphi_i}\right)^{(d-2)/2}\\[2ex] \mu(c_i)\left(\dfrac{h_i}{\varphi_i}\right)^{(d-2)/2}\left[1 + \dfrac{d}{2}\ln\left(\dfrac{h_i}{\varphi_i}\right)\right]\end{cases} + \begin{cases}\dfrac{-d(d-2)\kappa(c_i)}{4}B_i\left(\dfrac{h_i}{\varphi_i}\right)^{(d-4)/2}\\[2ex] -\mu(c_i)B_i\left(\dfrac{h_i}{\varphi_i}\right)^{(d-4)/2}\left[d - 1 + \dfrac{d(d-2)}{4}\ln\left(\dfrac{h_i}{\varphi_i}\right)\right]\end{cases} = 0 \quad \text{for} \quad \begin{cases}d = \text{noneven}\\ d = \text{even}\end{cases}. \tag{15}$$

Before we proceed with the random problem we will review some of the results of the pure case ($B_i \to 0$).[23,24] There are three low-symmetry solutions for the system of Eqs. (15). The first one is a mixed phase with both φ_1 and φ_2 nonzero. The second one (and analogously the third one) is when one of the order parameters is nonzero and the other one is: that is phase 1 is when $\varphi_1 \neq 0$ and $\varphi_2 = 0$ and phase 2 when $\varphi_1 = 0$ and $\varphi_2 \neq 0$. The system of $w > 0$, $\Delta < 0$ demonstrates a fluctuation-induced first-order phase transition from disorder into either one of the ordered phases 1 or 2. A phase transition between ordered phases 1 and 2 is of first order as well. Thus there exists a triple point. For this system the mixed phase is unstable. On the other hand, all three low symmetry phases can occur for systems described by either the set of inequalities $w > 0$, $\Delta > 0$, or $w < 0$, $\Delta > 0$ and the transitions from the disorder to the ordered phases or between the ordered phases are of the second order. Thus, they create a tetracritical point. The phase diagrams have been constructed elsewhere using both the model[23] and RG theory[24] and are qualitatively similar.

The critical behavior of the random case is, however, completely different. For example, when both B_1 and B_2 are nonzero it is derived from Eq. (15) that when $h_i \to 0$ no solution exists for φ_i for the spatial range $d \leq 4$. However, a solution exists and a phase transition is possible when only one of the random fields is nonzero. Additionally, unlike the pure case where the mixed phase can realize (at least when inequalities $w > 0$, $\Delta > 0$, or $w < 0$, $\Delta > 0$ are true), in the random case, the mixed phase is unstable. Moreover, it will be shown that when $B_{i'} \neq 0$ and $B_i = 0$ (when $i' = 2$ then $i = 1$) only phase i occurs and the transition is of the second order. Specifically, for $\Delta < 0$, the fluctuation-induced first-order transition present in the pure case which created a triple point, or the tetracritical point corresponding to $\Delta > 0$, are now replaced in the random case by a continuous phase transition into the unaccompanied phase i (i.e., phase i' cannot occur). We will first begin with systems obeying $\Delta < 0$ in 3 d, since in that case the pure version of functional Eq. (1)

demonstrates a discontinuous phase transition. Then we will also examine the case of $\Delta > 0$.

For examples, $\Delta < 0$, $d=3$, $B_1=0$, and $B_2 \neq 0$, then from Eq. (15), at zero constant external fields, $h_i \to 0$, the temperature dependence of the order parameter φ_1 of phase 1 is obtained. Notice that unlike the pure case for which both order parameters φ_1 and φ_2 had equilibrium values resulting into a triple point, in the random case a solution exists only for one of the order parameters, φ_1. Perhaps this is a hint that a second-order phase transition is to be expected. It is derived

$$\varphi_{1\pm}^2 = \frac{3w|\kappa(c_2)|}{2g_1}\left[-\frac{3|\kappa(c_2)|\Delta}{8g_1} \pm \sqrt{\left(\frac{3\kappa(c_2)\Delta}{8g_1}\right)^2 + \frac{(T-T_2)}{T_2} - \frac{w(T-T_1)}{T_1 g_1}}\right] - \frac{2(T-T_1)}{g_1 T_1} \mp \frac{3w|\kappa(c_2)|B_2}{4g_1\sqrt{\left(\frac{3\kappa(c_2)\Delta}{8g_1}\right)^2 + \frac{(T-T_2)}{T_2} - \frac{w(T-T_1)}{T_1 g_1}}}. \qquad (16)$$

The solution φ_{1+} corresponds to the lowest free energy. In the limit $B_2 \to 0$, Eq. (16) reduces to the expression corresponding to the pure case, $B_2=0$, where φ_{1+} (or φ_{2+}) has a jump-like behavior at the temperature given by $[3\kappa(c_2)\Delta/8g_1]^2 + t_2 - wt_1/g_1 = 0$.[23] On the other hand, in the random case where $B_2 \neq 0$, the requirement $\varphi_{1+}=0$ from Eq. (16) derives three solutions for a transition temperature. However, only one of them, which is symbolized by T_c is always real and positive and is the true critical temperature of the random case. T_c describes a second-order phase transition into the single phase φ_1 eliminating the triple point of the pure case. T_c is decreasing with increasing randomness. The expression of T_c is very long and cumbersome, thus it will not be provided. Instead we discuss the results in terms of the actual graph resulting from that cumbersome expression and a symbolic equation of the order parameter φ_{1+} which maintains the same mathematical form as Eq. (16). If we define $t \equiv T - T_c$, then for small t

$$\varphi_{1+}^2(t) = -\alpha_1 t + \frac{NAt}{\sqrt{\alpha_2 + NT_c}} + \frac{NA' B_2 t}{(\alpha_2 + NT_c)^{3/2}}. \qquad (17)$$

Using the solution of T_c from Eq. (16) one can find that α_1, α_2, A, A', and N (N is the only negative number), are explicit but unwieldy expressions of g_i, w, c_i, and T_i. For a small random field B_2 Eq. (17) becomes (note T_c depends on B_2):

$$\varphi_{1+}^2(t,B_2) = -t\left[\alpha_1 - \frac{N}{B_2^{1/2}}\left(\frac{A}{(NN')^{1/2}} + \frac{A'}{(NN')^{3/2}}\right)\right], \qquad (18)$$

where N also depends on g_i, w, c_i, and T_i. Based on Eq. (18) we construct a graph (Fig. 1) which shows the dependence of the φ_{1+} on the temperature T and on three different strengths of the random field B_2. We observe three characteristics: (1) the disorder–order phase transition is continuous; (2) the critical temperature is decreasing with increasing randomness B_2; (3) the slope of the curves become steeper with decreasing randomness B_2. In the latter, at a given temperature point t_0 it is derived from Eq. (18) that for $B_2 \neq 0$ the slope is equal to

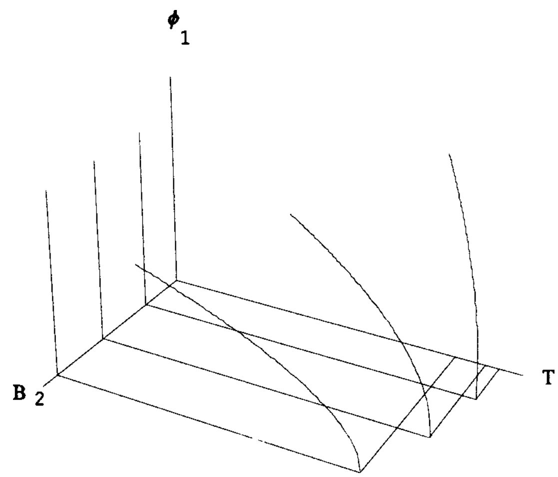

FIG. 1. Diagram of the equilibrium order parameter of phase 1 ($\varphi_1 \neq 0$ and $\varphi_2 \neq 0$) of systems having $\Delta < 0$, as a function of temperature for three different strengths of the random field B_2. The critical temperature decreases with increasing randomness. Also the curve becomes steeper with decreasing randomness, an indication of its divergence as $B_2 \to 0$.

$$\left.\frac{d\varphi_{1+}(t_0,B_2)}{dt}\right|_{t_0} = -\frac{1}{2}\sqrt{-\frac{1}{t_0}\left[\alpha_1 - \frac{N}{B_2^{1/2}}\left(\frac{A}{(NN')^{1/2}} + \frac{A'}{(NN')^{3/2}}\right)\right]}. \qquad (19)$$

It is clear from the above equation that as the random field B_2 vanishes the slope becomes infinite. These results can be contrasted with pure systems of coupled parameters where a fluctuation-induced first-order transition occurs.[23,24]

Let us now consider systems described by $\Delta > 0$. As in the pure problem we have a second-order phase transition. Because the pure case presents a second-order transition, then in the random case with $B_1=0$ and $B_2 \neq 0$ instead of dealing with the exact solution $\varphi_{1+}=0$ from Eq. (16) to find the critical temperature (which finds a very long and cumbersome expression), since in the pure case the order parameter is continuous at the critical temperature $T_{c\text{-}p}$, i.e., $\varphi_{1+}(B_2=0, T=T_{c\text{-}p})=0$, we expand $\varphi_{1+}(B_2 \neq 0, T)$ about the critical temperature of the pure case, $T_{c\text{-}p}$. Then up to order B_2, the critical temperature of the random case is $T_{c\text{-}r} = T_{c\text{-}p} - \tau B_2$ where

$$T_{c\text{-}p} = T_1 - \frac{9g_2\kappa^2(c_2)wT_1}{32} + \frac{9\kappa^2(c_2)w^2T_1^2}{32T_2} + \frac{3|\kappa(c_2)|T_1\sqrt{\frac{9\kappa^2(c_2)w^2(g_2T_2-wT_1)^2}{16} + 4w^2T_2(T_1-T_2)}}{8T_2}$$

$$\tau = \frac{3w^2|\kappa(c_2)|T_1T_2}{\sqrt{9\kappa^2(c_2)w^2(g_2T_2-wT_1)^2 + 64w^2T_2(T_1-T_2)}}. \tag{20}$$

If $t' = T - T_{c\text{-}r}$ then for small t'

$$\phi_{1+}^2(t') = -\frac{2t'}{g_1T_1} + \frac{3w|\kappa(c_2)|(g_1T_1-wT_2)t'}{4g_1^2T_1T_2\sqrt{\left(\frac{9w\kappa^2(c_2)\Delta}{32g_1} + \frac{T_{c\text{-}p}}{T_1} - 1\right)^2\frac{16}{9w^2\kappa^2(c_2)} + \frac{B_2\tau(g_1T_1-wT_2)}{g_1T_1T_2}}} + \frac{3w|\kappa(c_2)|(g_1T_1-wT_2)B_2t'}{8g_1^2T_1T_2\left[\left(\frac{9w\kappa^2(c_2)\Delta}{32g_1} + \frac{T_{c\text{-}p}}{T_1} - 1\right)^2\frac{16}{9w^2\kappa^2(c_2)} + \frac{B_2\tau(g_1T_1-wT_2)}{g_1T_1T_2}\right]^{3/2}}. \tag{21}$$

Equations (20) and (21) reduce to those of the pure problem upon suppression of the random field as expected. Notice that an analog calculation of expansion of $\varphi_{1+}(B_2 \neq 0, T)$ about the critical temperature of the pure case in systems described by $\Delta < 0$ cannot be carried out since the pure case presented a first-order phase transition and the behavior of the order parameter had a discontinuity at the critical point.

IV. CONCLUSION

It has been shown that, despite a fluctuation-induced first-order phase transition which occurs in the pure case of coupled order parameters, the existence of one random field replaces such a transition by a continuous one. The critical temperature decreases with increasing randomness. The order parameter has a steeper slope as the strength of the random field weakens, and it diverges as the random field vanishes, marking the crossover to the first-order phase transition of the pure case. In addition, unlike the pure case where depending on the system a phase transition into either one of the three low-symmetry phases may be possible, the random case is different: for three-dimensional systems if both random fields are nonzero no phase transition takes place. A phase transition occurs when only one random field is nonzero, and in that case the mixed phase is unstable and only phase 1 (or 2 depending on what random field is zero) is stable.

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