ISSN 1064-2269, Journal of Communications Technology and Electronics, 2015, Vol. 60, No. 1, pp. 87–96. © Pleiades Publishing, Inc., 2015.
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Magnetically Controlled Vertically Emitting Laser with Anisotropic Pumping

A. A. Zyablovskii[a, b], A. V. Dorofeenko[a, b, c], A. A. Pukhov[a, b, c], A. P. Vinogradov[a, b, c], E. S. Andrianov[a, b], A. B. Granovskii[d], and A. A. Lisyanskii[e]

[a]*All-Russia Research Institute of Automatics, Sushchevskaya ul. 22, Moscow, 127055 Russia*
[b]*Moscow Institute of Physics and Technology (State University), Institutskii per. 9, Dolgoprudnyi, Moscow oblast, 141700 Russia*
[c]*Institute of Theoretical and Applied Electrodynamics, Russian Academy of Sciences, Izhorskaya ul. 13, Moscow, 125412 Russia*
[d]*Moscow State University, Moscow, 119991 Russia*
[e]*Queens College, the City University of New York, Kissena Blvd. 65-30, Queens, NY 11367-1597*
e-mail: adorofeenko@yandex.ru

Received March 21, 2014

Abstract—A vertically emitting Faraday laser that is pumped by quantum wires or strongly anisotropic quantum dots is considered. As distinct from a similar laser on a quantum well or isotropic quantum dots, such a laser is extremely sensitive to external magnetic field (up to switching-off by magnetic field). This circumstance can be used for development of a rapidly tunable source of coherent radiation.

DOI: 10.1134/S1064226915010131

INTRODUCTION

Lasers generate coherent radiation in various optical systems. Systems for optical data transfer necessitate miniaturization of components, and conventional lasers are inapplicable in such systems. The problem can be solved using distributed-feedback (DFB) lasers in which a photonic crystal with a defect mode [1–4] serves as a cavity. The development of the technology of DFB lasers has led to the construction of surface-emitting lasers with a vertical cavity [5].

A conventional surface-emitting laser with a vertical resonator represents a 1D photonic crystal (Bragg mirror) with a resonant cavity [5–12]. The generated radiation in such a device is directed perpendicularly to the layer surface. The active medium is placed in the resonant cavity or Bragg mirror.

The single-mode lasing is easily implemented in a vertical-cavity laser owing to the smallness of the cavity [10, 13].

In the absence of the anisotropy of cavity and amplifying medium, the modes of the vertical-cavity surface emitting laser (VCSEL) are degenerate with respect to polarization. However, the mode competition leads to the linear polarization of VCSEL with a random orientation of polarization depending on technological fluctuations (e.g., minor wedging of the layers of photonic crystals that is caused by technological reasons). A desired direction of the VCSEL polarization can be obtained using anisotropic cavity [14–16] or pumping by quantum wires or anisotropic quantum dots [5, 17–19]. The VCSEL polarization is sensitive to anisotropy and alternative weak effects (e.g., magnetooptical effect). Such a sensitivity is convenient for practical purposes, since it allows the control of lasing using external static magnetic field.

Lasers with an external magnetic field have a long history of study. Most results have been obtained for the Zeeman lasers in which the magnetooptical effect is observed in the amplifying medium [20, 21]. In the Faraday laser [22], the amplifying and magnetic media are spatially separated. A Faraday VCSEL that is pumped by a quantum well in the presence of the external magnetic field exhibits a transition to generation of elliptical polarization [23, 24].

In this work, we study the steady-state lasing of the VCSEL that contains anisotropic and gyrotropic (magnetooptical) layers and the active medium with linear anisotropy. For example, the amplification can be performed with the aid of two layers of perfectly anisotropic quantum wires that provide x- and y-polarizations, respectively.

In general, an anisotropic cavity has two modes with different polarizations and Q factors. In the presence of pumping at which the first mode is lower and the second mode is higher than the generation threshold, the magnetooptical interaction may lead to the total suppression of lasing (i.e., the laser can be switched off by the magnetic field). Note that such an effect is not observed in the magnetooptical lasers on quantum wells [23–25] that are described with the aid of the San Miguel–Feng–Moloney (SFM) model of

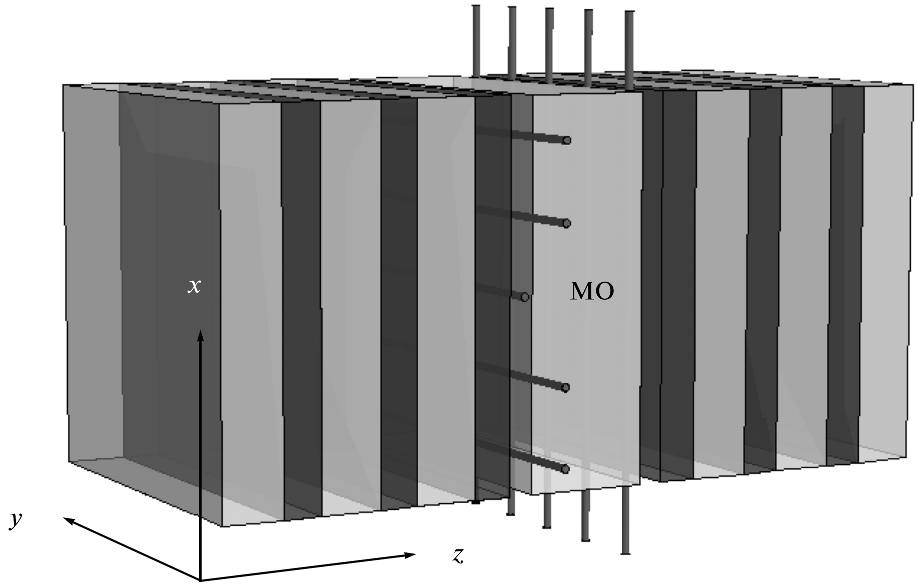

Fig. 1. Structure of the system. Quantum wires are located in the cavity between photonic crystals, and the active medium is located between the quantum wires.

the active medium [26–29]. In accordance with such a model, the amplification in the VCSEL is provided by two systems of carriers in the quantum well that generate radiation with right- and left-hand circular polarizations, rather than the linear polarization (as in the system under study).

1. EQUATION OF THE DYNAMICS OF THE FARADAY LASER

We consider a Faraday laser based on a photonic crystal with a defect that represents magnetooptical medium (e.g., yttrium–iron garnet). The 1D photonic crystal contains a system of layers that are perpendicular to the z axis. We assume that the permittivity of layers is anisotropic and the anisotropy axis is parallel to the x axis. With allowance for the gyrotropic properties of the magnetooptical layer, the system in general is described using the permittivity tensor

$$\hat{\varepsilon}(z) = \begin{pmatrix} \varepsilon_x(z) & ig(z) & 0 \\ -ig(z) & \varepsilon_y(z) & 0 \\ 0 & 0 & \varepsilon_z(z) \end{pmatrix}.$$

The amplifying medium represents two layers of quantum wires with mutually perpendicular orientations parallel to the x and y axes, respectively [30]. The magnetooptical medium is placed between the layers of quantum wires in the resonator cavity (Fig. 1). Thus, we consider surface-emitting Faraday laser (see Introduction).

For simplicity, we consider the total anisotropy and assume that the layers of quantum wires interact only with the x- and y-polarized electromagnetic fields, respectively [30–34].

We consider the following dynamic variables: amplitudes a_x and a_y, of the x and y components of electric field, respectively; polarizations σ_x and σ_y of the x- and y-oriented quantum wells, respectively; and population inversions D_x and D_y of the quantum wells.

The dynamics of lasing is described using the following system of equations (see Appendix):

$$\begin{gathered} \partial_\tau a_x + (1/Q_x + i\Delta_x/\omega_0)a_x + 4\pi\langle g\rangle a_y = -2\pi i\sigma_x, \\ \partial_\tau a_y + (1/Q_y + i\Delta_y/\omega_0)a_y - 4\pi\langle g\rangle a_x = -2\pi i\sigma_y, \\ \partial_\tau\sigma_x + \sigma_x/(\omega_0 T_2) = ia_x D_x, \\ \partial_\tau\sigma_y + \sigma_y/(\omega_0 T_2) = ia_y D_y, \\ \partial_\tau D_x + \left(D_x - D_x^{(0)}\right)\Big/(\omega_0 T_1) = i\left(a_x^*\sigma_x - a_x\sigma_x^*\right), \\ \partial_\tau D_y + \left(D_y - D_y^{(0)}\right)\Big/(\omega_0 T_1) = i\left(a_y^*\sigma_y - a_y\sigma_y^*\right), \end{gathered} \tag{1}$$

where T_1 and T_2 are relaxation times of the population inversion and polarization, respectively, and Q_x and Q_y are Q factors of the eigenmodes in the absence of the external magnetic field $(g = 0)$, when the modes have linear x and y polarizations. For $g = 0$, we also determine eigenfrequencies Δ_x and Δ_y of the x- and y-polarized modes, respectively, that are calculated as detunings from transition frequency ω_0 of amplifying medium.

Note that the polarizations and eigenfrequencies of modes depend on the magnetooptical interaction $(g \neq 0)$ and the laser pumping. Such effects are taken into account in system of equations (1).

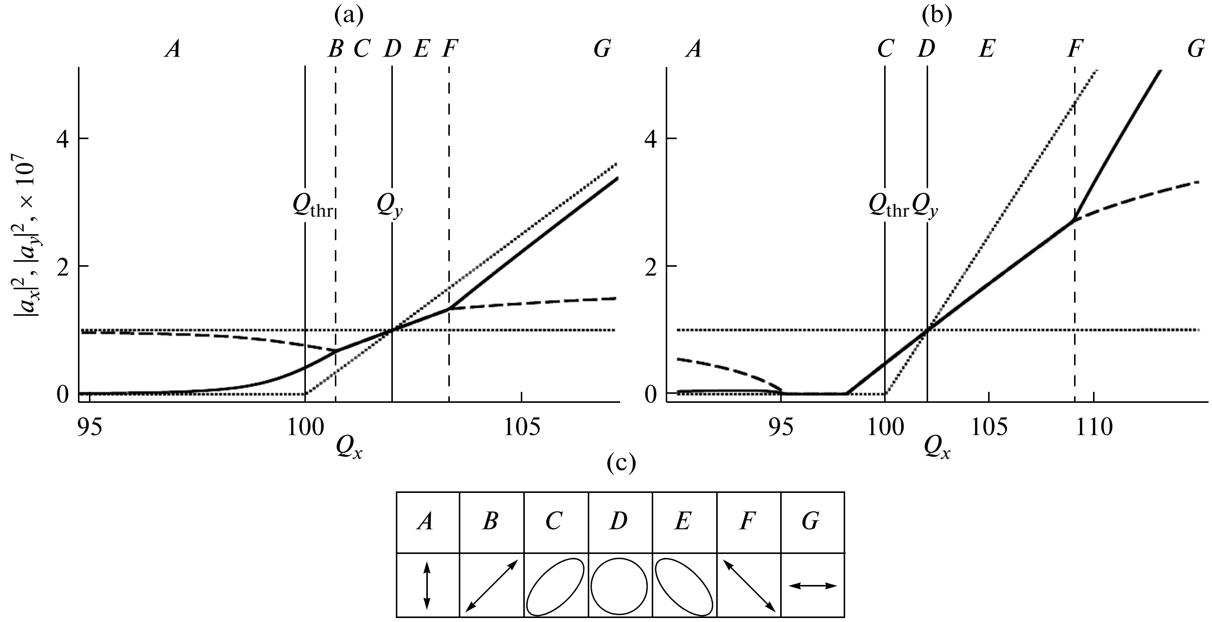

Fig. 2. Plots of generation intensity for (solid line) x and (dashed line) y field components vs. quantity Q_x, for the fixed Q_y = $1.02Q_{\rm thr}$, $\omega_0 T_1 = 10^4$, $\omega_0 T_2 = 10^2$, $\Delta_x = \Delta_y = 0$ at (a) weak ($g = 0.25\times10^{-5}$) and (b) strong ($g = 1.25\times10^{-5}$) magnetooptical interaction. The dotted line corresponds to the absence of interaction ($g = 0$), Panel (c) shows the notation (*A*–*G*) for polarizations.

2. STEADY-STATE LASING OF THE FARADAY LASER. MODE LOCKING AND ARNOLD TONGUE

We consider the regimes of lasing using system of equation (1) with variable magnetooptical constant g and Q factors Q_x and Q_y. For simplicity, we use identical pumping parameters of both systems of quantum wires $\left(D_x^{(0)} = D_y^{(0)} = D^{(0)}\right)$ and eigenfrequencies of the cavity modes that are equal to the transition frequency in the amplifying medium $(\Delta_x = \Delta_y = 0)$.

The gyrotropy is absent in the absence of magnetization $(g = 0)$, and the modes with the x and y linear polarizations are independently excited by the corresponding systems of quantum wells (Fig. 1). Note that the lasing threshold depends on the Q factor. For convenience, we fix pumping intensity $D^{(0)}$, so that the lasing takes place when the Q factor exceeds threshold level $Q_{\rm thr}$. We also fix the Q factor of one linearly polarized mode (y-polarized mode) at a level that is higher than the threshold $(Q_y > Q_{\rm thr})$ and consider the dependence of the dynamics on parameter Q_x. The mutual independence of the linear modes leads to the constant amplitude of the generation of the y-polarized mode and the threshold for the x-polarized mode (the dashed curves in Fig. 2).

The laser radiation is elliptically polarized when $Q_x > Q_{\rm thr}$, since the phase difference between independent modes can be arbitrary. In the presence of the magnetooptical interaction between the linearly polarized laser modes, the x- and y-polarized components of the electric field are phase-locked. Thus, we obtain the linear polarization and the polarization direction is determined by the amplitude ratio of the x- and y-polarized components. Letter G in Fig. 2a denotes the corresponding domain of parameters, and Fig. 2b demonstrates the polarization. Note that the generation of the linear polarization in the presence of the magnetooptical interaction that emerges due to relatively strong anisotropy has been reported for gas lasers [21].

When the Q factors of the modes in the absence of the magnetic field are located on both sides of the threshold $(Q_x < Q_{\rm thr} < Q_y)$, the activation of the magnetic field leads to the generation of a relatively small component with the y polarization. In this case, the phase difference of the oscillations of the x- and y-polarized components is π. In this case, we obtain the linear polarization that is almost parallel to the y axis (A in Fig. 2a).

A transition between the above regimes takes place in a narrow region in the vicinity of the point $Q_x = Q_y$, where the oscillation amplitudes are strictly identical and the phase difference ranges from π to 0 (Fig. 2a). Such a scenario corresponds to the transition between orthogonal linear polarizations via elliptical and circular polarizations (Fig. 2c). Such a region of relatively strong interaction in the vicinity of the point of identical parameters increases with an increase in the mag-

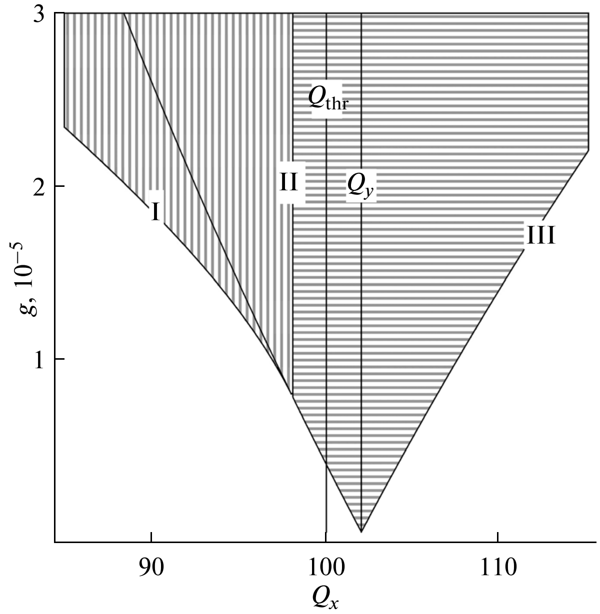

Fig. 3. Generation regimes of the surface-emitting Faraday laser. Unhatched region corresponds to the generation of the linearly polarized radiation (x- and y-polarized field components are phase-matched). The region with horizontal hatching corresponds to the generation of the elliptically polarized radiation with the frequency that is shifted relative to the transition frequency of the active medium (the x- and y-polarized components are matched with respect to amplitude). The region with vertical hatching corresponds to the suppression of lasing. Curves I–III show the boundaries of the regions (see text for details).

netooptical constant and is even extended to the subthreshold region (Fig. 2b). It is seen that the interaction of the super- and subthreshold modes may lead to both excitation of the subthreshold mode and the total suppression of lasing. Figure 3 shows the region of the strong interaction of the autooscillation systems known as the Arnold tongue. The right-hand part of this region corresponds to the generation of the elliptical polarization (circular polarization at $Q_x = Q_y$), and the left-hand part corresponds to the suppression of lasing.

Note that a decrease in the Q factor of the subthreshold mode leads to the transition from the regime in which the laser is switched off to the regime of generation of the superthreshold mode. For practical applications, it is of interest that the laser can be switched off using the activation of the static magnetic field, which corresponds to the vertical displacement in Fig. 3. The switching time is considered in Section 5.

3. ANALYTICAL ANALYSIS OF THE ARNOLD TONGUE

We search for analytical conditions for the transition between the lasing regimes (i.e., the boundaries of the Arnold tongue). For this purpose, we determine the steady-state solutions to system (1). Evidently, the zero solution ($a_x = a_y = \sigma_x = \sigma_y = 0$, $D_x = D_y = D^{(0)}$) is the steady-state solution. The zero steady-state solutions are found using the change of time derivatives $\partial_\tau = i\,\delta\omega/\omega_0$ for quantities a_x, a_y, σ_x, and σ_y, and $\partial_\tau = 0$ for quantities D_x and D_y. We consider the zero frequency detuning $(\Delta_x = \Delta_y = 0)$, and vary only Q factors Q_x and Q_y:

$$i(\delta\omega/\omega_0)a_x + (1/Q_x)a_x + 4\pi\langle g\rangle a_y = -2\pi i\sigma_x, \quad (2a)$$

$$i(\delta\omega/\omega_0)a_y + (1/Q_y)a_y - 4\pi\langle g\rangle a_x = -2\pi i\sigma_y, \quad (2b)$$

$$i(\delta\omega/\omega_0)\sigma_x + \sigma_x/(\omega_0 T_2) = i a_x D_x, \quad (2c)$$

$$i(\delta\omega/\omega_0)\sigma_y + \sigma_y/(\omega_0 T_2) = i a_y D_y, \quad (2d)$$

$$\left(D_x - D^{(0)}\right)\Big/(\omega_0 T_1) = i\left(a_x^*\sigma_x - a_x\sigma_x^*\right), \quad (2e)$$

$$\left(D_y - D^{(0)}\right)\Big/(\omega_0 T_1) = i\left(a_y^*\sigma_y - a_y\sigma_y^*\right). \quad (2f)$$

For the sequential elimination of variables in this system, we use the following procedure. Quantities σ_x and σ_y are obtained from Eqs. (2c) and (2d) and substituted in Eqs. (2e) and (2f). Then, we find quantities D_x and D_y and substitute the result in Eqs. (2c) and (2d). The resulting expressions are substituted in Eqs. (2a) and (2b). Thus, we derive the following system of equations for quantities a_x and a_y:

$$i(\delta\omega/\omega_0)a_x + a_x/Q_x + 4\pi\langle g\rangle a_y = \frac{2\pi\omega_0 T_2\left[1 - i\delta\omega T_2\right]a_x D^{(0)}}{1 + (\delta\omega T_2)^2 + 2\omega_0^2 T_1 T_2 |a_x|^2}, \quad (3a)$$

$$i(\delta\omega/\omega_0)a_y + a_y/Q_y - 4\pi\langle g\rangle a_x = \frac{2\pi\omega_0 T_2\left[1 - i\delta\omega T_2\right]a_y D^{(0)}}{1 + (\delta\omega T_2)^2 + 2\omega_0^2 T_1 T_2 |a_y|^2}. \quad (3b)$$

We represent the amplitudes using magnitude and phase ($a_x = |a_x|\exp(i\varphi_x)$ and $a_y = |a_y|\exp(i\varphi_y)$) and obtain

$$Q_x^{-1} + 4\pi\langle g\rangle\left|\frac{a_y}{a_x}\right|\cos(\varphi_y - \varphi_x) = \frac{2\pi\omega_0 T_2 D^{(0)}}{1 + (\delta\omega T_2)^2 + 2\omega_0^2 T_1 T_2 |a_x|^2}, \quad (4a)$$

$$\frac{\delta\omega}{\omega_0} + 4\pi\langle g\rangle\left|\frac{a_y}{a_x}\right|\sin(\varphi_y - \varphi_x) = -\frac{2\pi\omega_0\delta\omega T_2^2 D^{(0)}}{1 + (\delta\omega T_2)^2 + 2\omega_0^2 T_1 T_2 |a_x|^2}, \quad (4b)$$

$$Q_y^{-1} - 4\pi\langle g\rangle\left|\frac{a_x}{a_y}\right|\cos(\varphi_y - \varphi_x) = \frac{2\pi\omega_0 T_2 D^{(0)}}{1 + (\delta\omega T_2)^2 + 2\omega_0^2 T_1 T_2 |a_y|^2}, \tag{4c}$$

$$\frac{\delta\omega}{\omega_0} - 4\pi\langle g\rangle\left|\frac{a_x}{a_y}\right|\sin(\varphi_x - \varphi_y) = -\frac{2\pi\omega_0\delta\omega T_2^2 D^{(0)}}{1 + (\delta\omega T_2)^2 + 2\omega_0^2 T_1 T_2 |a_y|^2}. \tag{4d}$$

In the vicinity of the lasing threshold, the field amplitude is relatively small and the terms $2\omega_0^2 T_1 T_2 |a_{x,y}|^2$ in Eqs. (4a)–(4d) can be neglected in comparison with unity. We subtract Eq. (4d) from Eq. (4b) to obtain

$$4\pi\langle g\rangle\left(\left|\frac{a_y}{a_x}\right| - \left|\frac{a_x}{a_y}\right|\right)\sin(\varphi_y - \varphi_x) = 0. \tag{5}$$

This equation has two solutions.

(i) The solution $\sin(\varphi_y - \varphi_x) = 0$, corresponds to the in-phase oscillations of the x and y field components, the laser radiation is linearly polarized, and Eqs. (4b) and (4d) yield $\delta\omega = 0$, so that the lasing takes place at the transition frequency of the amplifying medium.

(ii) The solution $|a_x| = |a_y| = a$, corresponds to the oscillations with identical amplitudes and different phases that can be found using the difference of Eqs. (4c) and (4a):

$$\cos(\varphi_y - \varphi_x) = \frac{1}{8\pi\langle g\rangle}\frac{Q_x - Q_y}{Q_x Q_y}. \tag{6a}$$

The generation amplitude can be obtained from the sum of Eqs. (4a) and (4b). Note that the term $2\omega_0^2 T_1 T_2 |a|^2$ cannot be neglected, since it is responsible for a decrease in the population inversion by the field and determines the generation amplitude. Thus, we obtain

$$a = \sqrt{\frac{2\pi}{\omega_0 T_1}\left(\frac{Q_x Q_y}{Q_x + Q_y}\right)D^{(0)} - \frac{1}{2\omega_0^2 T_1 T_2^{\text{eff}}}}, \tag{6b}$$

where quantity $T_2^{\text{eff}} = T_2/(1 + \delta\omega^2 T_2^2)$ is introduced with allowance for detuning $\delta\omega$ of the generation frequency from the transition frequency in the given case. To find detuning $\delta\omega$, we eliminate quantity $(\varphi_y - \varphi_x)$ in Eqs. (4b) and (6a). With disregard of quadratic terms $|a_{x,y}|^2$ and $\delta\omega^2$, we obtain

$$\delta\omega = \frac{4\pi\omega_0}{1 + 2\pi\omega_0^2 T_2^2 D^{(0)}}\sqrt{1 - \frac{1}{64\pi^2\langle g\rangle^2}\left(\frac{Q_x - Q_y}{Q_x Q_y}\right)^2}. \tag{6c}$$

In accordance with Eq. (6a), phase difference $(\varphi_y - \varphi_x)$ is not zero and the solution is elliptically polarized. The circular polarization is obtained for $Q_x = Q_y$.

It is seen that the second solution to Eq. (5) exists only under conditions that follow from Eqs. (6a) and (6b):

$$\left|\left(\frac{Q_x - Q_y}{Q_x Q_y}\right)\right| < 8\pi|\langle g\rangle|, \tag{7a}$$

$$\left(\frac{Q_x + Q_y}{Q_x Q_y}\right) < 8\pi\omega_0 T_2^{\text{eff}} D^{(0)}. \tag{7b}$$

Outside the above domain of parameters (at relatively large parameter $\langle g\rangle$ or significant anisotropy $(Q_x - Q_y)/Q_x$), the above solutions are supplemented with the solution in which the x- and y-polarized modes oscillate at different frequencies.

Thus, three steady-state regimes are possible for the laser under study:

(i) zero solution $a_x = a_y = \sigma_x = \sigma_y = 0$ and $D_x = D_y = D^{(0)}$;

(ii) solution with linear polarization at $\delta\omega = 0$;

(iii) solution with elliptical polarization with $|a_x| = |a_y|$ and $\delta\omega \neq 0$, that is transformed into the solution with circular polarization at $Q_x = Q_y$.

The third solution exist only under the conditions $8\pi|\langle g\rangle| \geq |(Q_x - Q_y)/(Q_x Q_y)|$ and $(Q_x + Q_y)/(2Q_x Q_y) < 4\pi\omega_0 T_2^{\text{eff}} D^{(0)}$. Below, we demonstrate that the condition

$$8\pi|\langle g\rangle| = |(Q_x - Q_y)/(Q_x Q_y)| \tag{8}$$

corresponds to the boundary I of the domains in Fig. 3 (i.e., the condition for switching off by the magnetic field).

4. LINEAR ANALYSIS OF STABILITY

To determine the parameters of the system that correspond to the above solutions, we analyze the stability. For this purpose, we represent the field amplitude as deviation $\delta a_{x,y}$ from the steady state $a_{x,y}^0$: $a_{x,y} = a_{x,y}^0 + \delta a_{x,y}$. We consider the zero steady-sate solution, eliminate the second-order terms with respect to the deviations from the steady state, and represent Eqs. (3a) and (3b) as

$$\partial_\tau\begin{pmatrix}\delta a_x \\ \delta a_y\end{pmatrix} = \omega_0\begin{pmatrix}2\pi\omega_0 T_2 D^{(0)} - \dfrac{1}{Q_x} & -4\pi\langle g\rangle \\ 4\pi\langle g\rangle & 2\pi\omega_0 T_2 D^{(0)} - \dfrac{1}{Q_y}\end{pmatrix}\begin{pmatrix}\delta a_x \\ \delta a_y\end{pmatrix}. \tag{9}$$

The stability is determined by the eigenvalues of matrix:

$$\lambda_{\pm} = \left(2\pi\omega_0 T_2 D^{(0)} - \frac{Q_x + Q_y}{2Q_x Q_y}\right) \pm \sqrt{\left(2\pi\omega_0 T_2 D^{(0)} + \frac{Q_x - Q_y}{2Q_x Q_y}\right)^2 - \left(4\pi\langle g\rangle\right)^2}. \quad (10)$$

The solution is stable if the real parts of both eigenvalues are negative $\mathrm{Re}(\lambda_{\pm}) < 0$. This condition is represented as

$$\frac{Q_x + Q_y}{Q_x Q_y} > 4\pi\omega_0 T_2 D^{(0)}, \quad (11)$$

if radicand in formula (10) is negative $\left(2\pi\omega_0 T_2 D^{(0)} + \frac{Q_x - Q_y}{2Q_x Q_y}\right)^2 - \left(4\pi\langle g\rangle\right)^2 < 0$. If the radicand is positive, the condition for stability is written as

$$\left(2\pi\omega_0 T_2 D^{(0)} - \frac{1}{Q_x}\right) \times \left(2\pi\omega_0 T_2 D^{(0)} - \frac{1}{Q_y}\right) > -\left(4\pi\langle g\rangle\right)^2. \quad (12)$$

A similar analysis of the stability is possible for the solution with elliptical polarization. However, the parameters remain unchanged: the domain of stability for the solution with elliptical polarization coincides with the domain of existence for solution (7).

We summarize the results of the analytical study of the lasing regimes. In Fig. 3, boundary I that is determined by condition

$$\left(2\pi\omega_0 T_2 D^{(0)} - 1/Q_x\right)\left(2\pi\omega_0 T_2 D^{(0)} - 1/Q_y\right) = -\left(4\pi\langle g\rangle\right)^2$$

(violation of inequality (12)) corresponds to the violation of stability of the zero solution at a real eigenvalue. Boundary II can be determined by either zero amplitude of the steady-state solution (condition $\left(\frac{Q_x + Q_y}{Q_x Q_y}\right) = 8\pi\omega_0 T_2^{\mathrm{eff}} D^{(0)}$, which corresponds to the violation of inequality (7b)) or violation of the stability of the zero solution at a complex eigenvalue (condition $\frac{Q_x + Q_y}{Q_x Q_y} > 4\pi\omega_0 T_2 D^{(0)}$, which corresponds to violation of inequality (11)). Boundary III is determined by the transition of quantity $\left|\cos\left(\varphi_y - \varphi_x\right)\right|$ via unity (condition $\left|\left(\frac{Q_x - Q_y}{Q_x Q_y}\right)\right| = 8\pi\left|\langle g\rangle\right|$, which corresponds to violation of inequality (7a)). The boundaries of domains that result from the analysis of the steady-state solutions and their stability coincide with the boundaries obtained in the numerical simulation of system of equations (1).

Condition (8) shows that the laser can be switched off by the magnetic field in spite of an extremely small magnetooptical constant: we obtain a realistic value of $\langle g\rangle \sim 10^{-5}$, at $Q_{x,y} \sim 100$ and $\left(Q_x - Q_y\right)/Q_{x,y} \sim 0.01$.

5. CHARACTERISTIC SWITCHING TIMES OF THE LASER GENERATION

The on–off switching of the laser generation is interesting for practical applications. The characteristic switching times are found from the linear analysis of the stability of solutions to system (3).

The external magnetic field is needed for the suppression of lasing. When the magnetooptical constant satisfies the condition $4\pi\langle g\rangle > 2\pi\omega_0 T_2 D^{(0)} + \frac{Q_x - Q_y}{2Q_x Q_y}$, the characteristics time of the switching on of the laser generation does not depend on the magnetic field:

$$\tau = \frac{1}{\omega_0}\frac{2Q_x Q_y}{4\pi\omega_0 T_2 Q_x Q_y D^{(0)} - \left(Q_x + Q_y\right)}. \quad (14)$$

When the magnetooptical constant decreases to the minimum level at which the lasing can be switched off, the switching time tends to a finite value. At real parameters of the system, the switching time can be on the order of 10^{-10} s.

The characteristic time of the switching on of the laser generation is determined in the absence of the external magnetic field. It is equal to the maximum of the two times

$$t_+ = \frac{\omega_0 Q_x}{2\pi\omega_0 T_2 D^{(0)} Q_x - 1}, \quad t_- = \frac{\omega_0 Q_y}{2\pi\omega_0 T_2 D^{(0)} Q_y - 1}, \quad (15)$$

which is also on the order of 10^{-10} s.

The estimated switching times are normally greater than the characteristic time of variation in the magnetooptical constant upon on–off switching of the laser generation in the presence of the current modulation of the optical signal [5]. Therefore, the proposed magnetically-controlled laser is a relatively fast device that can be integrated in optical schemes.

CONCLUSIONS

We have analyzed a magnetically-controlled surface-emitting laser with a vertical cavity (the VCSEL Faraday laser). Depending on the external magnetic field, three working regimes are possible for the laser: the absence of lasing for both linear polarizations, steady-state lasing with elliptical polarization at a frequency that is shifted from the cavity frequency, and the steady-state lasing with the linear polarization at a frequency that is equal to the cavity frequency. A variation in the magnetic field leads to the transitions between the regimes.

We have considered the magnetically-controlled surface-emitting laser with vertical cavity as a magnetically switched source of coherent radiation in optical systems and determined the on–off switching time.

The estimated times are on the order of 10^{-10} s. Therefore, the proposed magnetically-controlled laser is a relatively compact and fast device that can be integrated in optical schemes.

APPENDIX

Intracavity field $\vec{\mathscr{E}}$ satisfies the equation

$$\frac{\partial^2 \vec{\mathscr{E}}}{\partial z^2} - \frac{\hat{\varepsilon}}{c^2}\frac{\partial^2 \vec{\mathscr{E}}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \vec{\mathscr{P}}}{\partial t^2}. \quad \text{(A.1)}$$

Here, polarization of the active medium results from the averaging of dipole moments of active particles over a small volume: $\vec{\mathscr{P}} = \langle \vec{d} \rangle_{\Delta V}/\Delta V$.

Anisotropy and gyrotropy are considered as perturbations: quantity $\hat{\varepsilon}(z)$ is represented as a sum of isotropic component $\varepsilon_0(z)\hat{I}$ ($\hat{I}$ is the unity matrix) and additional component $\delta\hat{\varepsilon}(z)$. Then expression (A1) is represented as

$$\frac{\partial^2 \vec{\mathscr{E}}}{\partial z^2} - \frac{\varepsilon_0}{c^2}\frac{\partial^2 \vec{\mathscr{E}}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \vec{\mathscr{P}}}{\partial t^2} + \frac{\delta\hat{\varepsilon}}{c^2}\frac{\partial^2 \vec{\mathscr{E}}}{\partial t^2}. \quad \text{(A.2)}$$

Assuming the smallness of quantity $\delta\hat{\varepsilon}$, we find the zero approximation in which the cavity is isotropic. Then, the coordinate representation of expression (A1) is written as

$$\frac{\partial^2 \mathscr{E}_x}{\partial z^2} - \frac{\varepsilon_0}{c^2}\frac{\partial^2 \mathscr{E}_x}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \mathscr{P}_x}{\partial t^2}, \quad \text{(A.3)}$$
$$\frac{\partial^2 \mathscr{E}_y}{\partial z^2} - \frac{\varepsilon_0}{c^2}\frac{\partial^2 \mathscr{E}_y}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \mathscr{P}_y}{\partial t^2}.$$

The right-hand sides of the above equations describe polarization of the active medium $\vec{\mathscr{P}}$. Using transition frequency in the active medium ω_0, we represent the time dependence of polarization $\vec{\mathscr{P}}$ as oscillations at carrier frequency ω_0 with slowly (relative to the carrier frequency) varying amplitude. Let eigenfrequency of unperturbed cavity ω_r be close to frequency ω_0. In the absence of pumping ($\vec{\mathscr{P}} = 0$) the distribution of field E_0 in this mode satisfies the equation

$$\frac{\partial^2 E_0}{\partial z^2} + \varepsilon_0(z)\frac{\omega_r^2}{c^2}E_0 = 0. \quad \text{(A.4)}$$

We use the single-mode approximation owing to the closeness of frequencies ω_0 and ω_r. In this approximation, the solution is represented as a product of coordinate- and time-dependent functions, so that the solution to system (A3) is represented as

$$\vec{\mathscr{E}}^{(0)}(z,t) = \left[a_x(t)\vec{e}_x + a_y(t)\vec{e}_y\right]E_0(z)\exp(-i\omega_0 t)/\sqrt{W}, \quad \text{(A.5)}$$

where $W = \frac{\omega_0}{8\pi c}\int \varepsilon_0(z)E^2 dz$ is the normalization factor, $\vec{e}_x$ and $\vec{e}_y$ are unit vectors, and $a_{x,y}(t)$ are slowly varying amplitudes of the x and y field components. Thus, we disregard the terms containing $\partial^2 a_{x,y}/\partial t^2$. Note that expression (A5) is the zero approximation of Eq. (A1) with respect to anisotropy and gyrotropy.

In the derivation of the equations for $a_{x,y}(t)$, we simultaneously take into account the perturbation of the first order with respect to anisotropic–gyrotropic term $\delta\hat{\varepsilon}$. For this purpose, we introduce unknown functions $\delta E_x(z)$, and $\delta E_y(z)$:

$$\vec{\mathscr{E}}^{(1)}(z,t) = \left[a_x(t)\vec{e}_x\left(E_0(z) + \delta E_x(z)\right) + a_y(t)\vec{e}_y\left(E_0(z) + \delta E_y(z)\right)\right]\exp(-i\omega_0 t)/\sqrt{W}. \quad \text{(A.6)}$$

To simplify the analysis, we derive the equations for $a_{x,y}(t)$ in two stages. First, we derive these equations using the field distribution in the zero approximation (expression (A5)). Then, we demonstrate that first-order corrections $\delta E_x(z)$, and $\delta E_y(z)$ to the field distribution give rise to additional terms that are proportional to the product of small parameter of perturbation theory $\delta\hat{\varepsilon}$ and small parameter of the single-mode approximation $\delta\omega = \omega_r - \omega_0$. We neglect the terms proportional to $\delta\hat{\varepsilon}\delta\omega$ owing to the smallness of $\delta\omega$. Thus, expression (A6) is reduced to expression (A5).

To obtain the equation for time amplitudes, we multiply both sides of Eq. (A2) by quantity $E_0\vec{e}_x$ and subtract Eq. (A4) multiplied by $\mathscr{E}_x$, from the resulting expression. Then, we integrate both sides of the equation with respect to z. Quantity $\vec{\mathscr{E}}^{(0)}$ given by formula (A5) is substituted as $\vec{\mathscr{E}}$ in the resulting equation. Thus, we obtain the following expression:

$$\begin{aligned} &\int\left[E_0\frac{\partial^2 \mathscr{E}_x^{(0)}}{\partial z^2} - \frac{\partial^2 E_0}{\partial z^2}\mathscr{E}_x^{(0)}\right]dz \\ &+ \frac{2i\omega_0 \dot{a}_x}{c^2}\sqrt{W}\exp(-i\omega_0 t) \\ &+ \frac{\omega_0^2 - \omega_r^2}{c^2}a_x\sqrt{W}\exp(-i\omega_0 t) \\ &+ 8\pi\frac{\omega_0^2}{c^2}\sqrt{W}\left(\langle\delta\varepsilon_x\rangle a_x + i\langle g\rangle a_y\right) \\ &\times \exp(-i\omega_0 t) = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\int E_0\left(\vec{\mathscr{P}}\cdot\vec{e}_x\right)dz, \end{aligned} \quad \text{(A.7)}$$

where $\langle\delta\varepsilon_x\rangle = \int\delta\varepsilon_{xx}E^2 dz\Big/\int\varepsilon_0 E^2 dz$, and $i\langle g\rangle = \int\delta\varepsilon_{xy}\times E^2 dz\Big/\int\varepsilon_0 E^2 dz$.

In this equation, we disregard the quantity proportional to $\dot{a}_x\delta\hat{\varepsilon}$, (i.e., the product of the small parameters of the perturbation theory and single-mode approximation or, more precisely, the approximation of slowly varying amplitudes). The small parameters are independent, and one of them can be significantly greater than another. However, the greater parameter is more significant and we correctly disregard the quadratic terms.

The integral in expression (A7) with the integrand in brackets is reduced to the surface integral (to the values of function at the boundaries of the cavity in the 1D system under study) and provides the contribution to the Q factor that is related to the escape of radiation through the cavity walls [35]. Thus, we obtain

$$\dot{a}_x + \left(\omega_0/Q_x + i\Delta_x\right) a_x + 4\pi\omega_0 \langle g\rangle a_y = -2\pi i\omega_0 p_x \quad (A.8)$$

and a similar expression

$$\dot{a}_y + \left(\omega_0/Q_y + i\Delta_y\right) a_y - 4\pi\omega_0 \langle g\rangle a_x = -2\pi i\omega_0 p_y. \quad (A.9)$$

Here, we introduce the notation for the expression in the right-hand sides

$$p_{x,y} \exp(-i\omega_0 t) = \frac{\omega_0}{c} \int E\left(\mathcal{P} \cdot \vec{e}_{x,y}\right) dz/\sqrt{W}, \quad (A.10)$$

and frequency detuning $\Delta_{x,y} = \omega_r - \omega_0 - 4\pi\omega_0 \langle \delta\varepsilon'_{x,y}\rangle$. Real part of anisotropy $\delta\varepsilon'_{x,y}$ leads to the shift of the frequencies of the two cavity modes from the transition frequency. Imaginary part $\delta\varepsilon''_{x,y}$ contributes to loss $(\omega_0/Q_{x,y})$ and the loss related to the escape of the field from the cavity. Different losses for the x and y components can be due to the wedging of the layers. Note the importance of even small anisotropy that is on the order of the magnetooptical factor $\langle g\rangle \sim 10^{-4}$ for materials similar to YIG.

At the second stage of the derivation of the equations for field amplitude, we demonstrate that the zero approximation (expression (A5)) is sufficient for the first approximation of the perturbation theory with respect to the solution of time equation (A1). For this purpose, we take into account the corrections to the field in the first approximation of the perturbation theory (expression (A6)).

We consider the above procedure involving Eqs. (A2) and (A4) that has led to Eq. (A7). At this stage, we substitute quantity $\vec{\mathcal{E}}^{(1)}$ given by expression (A6) for quantity $\vec{\mathcal{E}}$. When terms $\delta E_x(z)$ and $\delta E_y(z)$ are taken into account, quantity $a_x(t)\frac{\varepsilon_0}{c^2}\omega_0^2 E_0 \delta E_x(z)/\sqrt{W}$ is added to the first term in the integrand and quantity

$$-\varepsilon_0(z)\frac{\omega_r^2}{c^2} a_x(t) E_0 \delta E_x(z)/\sqrt{W}$$

is added to the second term in the integrand. Thus, we obtain

$$a_x(t) \int \left(\frac{\omega_0^2 - \omega_r^2}{c^2} \varepsilon_0 E_0\right) \delta E_x dz/\sqrt{W}.$$

Both frequency detuning $\omega_0^2 - \omega_r^2$ and field variation δE_x are small quantities. We neglect the product of small quantities in comparison with the terms proportional to a single small quantity. Thus, we substantiate the application of expression (A5) for the field in the derivation of the first-order equations for the field amplitude.

To derive the equation for polarization, we employ the equation of [36, 37] for the general dynamics of polarization

$$\frac{\partial \vec{\mathcal{P}}}{\partial t} + \left(\frac{1}{T_2} + i\omega_0\right)\vec{\mathcal{P}} = \frac{i}{\hbar}\mathcal{N}\vec{d}\left(\vec{d} \cdot \vec{\mathcal{E}}\right). \quad (A.11)$$

Here, $\mathcal{N}(t,z)$ is the population inversion of the active medium that is averaged over a small volume (similarly to the polarization), T_2 is the polarization relaxation time, and $\vec{d}$ is the off-diagonal vector matrix element of the dipole moment. To obtain the equation for the polarization amplitudes given by expression (A10), we multiply both sides of Eq. (A11) by quantity $E\vec{e}_x/\sqrt{W}$ and integrate with respect to z. The resulting expression is written as

$$\frac{\partial p_x}{\partial t} + \frac{p_x}{T_2} = \frac{i\omega_0}{\hbar c} \int \mathcal{N}\left(\vec{d} \cdot E\vec{e}_x\right)\left(\vec{d} \cdot \vec{\mathcal{E}}\right) dz/\sqrt{W}.$$

We expand the field in terms of modes (2) in the right-hand side with allowance for new variables

$$D_{\alpha\beta} = \frac{\omega_0}{c}\frac{1}{\hbar\omega_0} \int \mathcal{N}\left(\vec{d} \cdot E\vec{e}_\alpha\right)\left(\vec{d} \cdot E\vec{e}_\beta\right) dz/W \quad (A.12)$$

(subscripts α and β are x or y) to obtain the equation

$$\frac{\partial p_x}{\partial t} + \frac{p_x}{T_2} = i\omega_0\left(D_{xx}a_x + D_{xy}a_y\right) \quad (A.13)$$

and a similar expression

$$\frac{\partial p_y}{\partial t} + \frac{p_y}{T_2} = i\omega_0\left(D_{yx}a_x + D_{yy}a_y\right). \quad (A.14)$$

The equations for population inversion are derived from the equations of [36, 37]:

$$\frac{\partial \mathcal{N}}{\partial t} + \frac{\mathcal{N} - \mathcal{N}_0}{T_1} = \frac{i}{\hbar}\left(\vec{\mathcal{E}}^* \vec{\mathcal{P}} - \vec{\mathcal{E}}\vec{\mathcal{P}}^*\right). \quad (A.15)$$

To obtain the expressions for quantities $D_{\alpha\beta}$, given by formula (A12), we multiply both sides of Eq. (A15) by quantity $\left(\vec{d} \cdot E\vec{e}_\alpha\right)\left(\vec{d} \cdot E\vec{e}_\beta\right)/W\hbar c$ and integrate with respect to z. Thus, we have

$$\frac{\partial D_{\alpha\beta}}{\partial t} + \frac{D_{\alpha\beta} - D_{\alpha\beta}^{(0)}}{T_1} = \frac{i}{\hbar^2 c} \int \left(\vec{\mathcal{E}}^* \vec{\mathcal{P}} - \vec{\mathcal{E}}\vec{\mathcal{P}}^*\right)\left(\vec{d} \cdot E\vec{e}_\alpha\right)\left(\vec{d} \cdot E\vec{e}_\beta\right) dz/W. \quad (A.16)$$

The right-hand side of this expression is transformed using the expansion of electric field $\vec{\mathcal{E}}(z,t) =$

$a_\gamma(t)\vec{e}_\gamma E(z)\exp(-i\omega_0 t)/\sqrt{W}$ (γ is x or y and the summation over repeated subscripts is assumed):

$$\frac{\partial D_{\alpha\beta}}{\partial t}+\frac{D_{\alpha\beta}-D_{\alpha\beta}^{(0)}}{T_1} = \frac{i}{\hbar^2 c}\int\left(\left(a_\gamma{}^*\vec{\mathcal{P}}-a_\gamma\vec{\mathcal{P}}^*\right)\cdot\vec{e}_\gamma E\right)\left(\vec{d}\cdot E\vec{e}_\alpha\right) \times\left(\vec{d}\cdot E\vec{e}_\beta\right)dz/W^{3/2}. \quad (A.17)$$

Thus, the population inversion is described using three variables D_{xx}, D_{yy}, and D_{xy} (by definition, we have $D_{yx} = D_{xy}$).

Then, we consider the pumping by two layers of quantum wires. In the first layer $z = z_x$, the wires are oriented along the x axis. In the second layer ($z = z_y$), the wires are oriented along the y axis. Evidently, the x- and y-polarized modes are independently pumped in such a system. The polarization is concentrated in the two layers $\left(\vec{\mathcal{P}}\cdot\vec{e}_{x,y}\right)\sim\delta\left(z-z_{x,y}\right)$, so that the integral vanishes in expression (A10):

$$p_{x,y}\exp(-i\omega_0 t) = \frac{\omega_0}{c}E\left(z_{x,y}\right)\left(\vec{\mathcal{P}}\left(z_{x,y}\right)\cdot\vec{e}_{x,y}\right)/\sqrt{W}. \quad (A.18)$$

A similar dependence is typical of quantity $\mathcal{N}$, so that the population inversion is determined by two quantities

$$D_{xx} = \frac{1}{\hbar c}\mathcal{N}(z_x)d_x^2E^2(z_x)/W, \qquad D_{yy} = \frac{1}{\hbar c}\mathcal{N}(z_y)d_y^2E^2(z_y)/W, \quad (A.19)$$

since the third quantity is $D_{xy}\equiv 0$, due to the fact that product $\left(\vec{d}\cdot E\vec{e}_x\right)\left(\vec{d}\cdot E\vec{e}_y\right)=0$ is zero at any point in space. Thus, Eq. (A17) is transformed into

$$\frac{\partial D_{xx}}{\partial t}+\frac{D_{xx}-D_{xx}^{(0)}}{T_1} = i\omega_0\frac{a_x^*p_x-a_xp_x^*}{w_x}, \qquad \frac{\partial D_{yy}}{\partial t}+\frac{D_{yy}-D_{yy}^{(0)}}{T_1} = i\omega_0\frac{a_y^*p_y-a_yp_y^*}{w_y}. \quad (A.20)$$

Here, we use interaction coefficients $w_{x,y} = W(\hbar\omega_0)^2/\left(d_{x,y}E\left(z_{x,y}\right)\right)^2$, that have dimension of energy. Note that subscript γ in expression (A17) is x or y but the term with $\gamma = y$ vanishes in the equation for D_{xx}, since it is proportional to $\delta\left(z-z_y\right)$, and $d_x = 0$ at $z = z_y$.

We assume that $\xi = \sqrt{(w_x+w_y)/2}$ is a unit of energy and polarization. Then, we introduce the notation $a_{x,y} = e_{x,y}/\xi$ and $\sigma_{x,y} = p_{x,y}/\xi$ and employ dimensionless time $\tau = \omega_0 t$, to represent Eqs. (A8), (A9), (A13), (A14), and (A20) as

$$\begin{aligned}
&\partial_\tau a_x+\left(\frac{1}{Q_x}+i\frac{\Delta_x}{\omega_0}\right)a_x+4\pi\langle g\rangle a_y = -2\pi i\sigma_x,\\
&\partial_\tau a_y+\left(\frac{1}{Q_y}+i\frac{\Delta_y}{\omega_0}\right)a_y-4\pi\langle g\rangle a_x = -2\pi i\sigma_y,\\
&\partial_\tau\sigma_x+\frac{\sigma_x}{\omega_0T_2} = i\left(D_{xx}a_x+D_{xy}a_y\right),\\
&\partial_\tau\sigma_y+\frac{\sigma_y}{\omega_0T_2} = i\left(D_{yx}a_x+D_{yy}a_y\right),\\
&\partial_\tau D_{xx}+\frac{D_{xx}-D_{xx}^{(0)}}{\omega_0T_1} = i\frac{w_x+w_y}{2w_x}\left(a_x^*\sigma_x-a_x\sigma_x^*\right),\\
&\partial_\tau D_{yy}+\frac{D_{yy}-D_{yy}^{(0)}}{\omega_0T_1} = i\frac{w_x+w_y}{2w_y}\left(a_y^*\sigma_y-a_y\sigma_y^*\right).
\end{aligned} \quad (A.21)$$

Using quantities $\sqrt{\frac{w_x+w_y}{2w_y}}a_{x,y}$ and $\sqrt{\frac{w_x+w_y}{2w_y}}\sigma_{x,y}$ as the field and polarization amplitudes, respectively, and simplifying the notation ($D_x\equiv D_{xx}$ and $D_y\equiv D_{yy}$,), we derive system of equations (1).

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Translated by A. Chikishev