ISSN 1064-2269, Journal of Communications Technology and Electronics, 2018, Vol. 63, No. 7, pp. 821–827. © Pleiades Publishing, Inc., 2018.
Original Russian Text © E.V. Kuznetsov, A.M. Merzlikin, A.A. Zyablovsky, A.P. Vinogradov, A.A. Lisyansky, 2018, published in Radiotekhnika i Elektronika, 2018, Vol. 63, No. 7, pp. 743–749.

Suppression of Cross Coupling in Plasmon Waveguides

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Received March 24, 2017

Abstract—Suppression of cross coupling between dielectric plasmon waveguides by inserting an additional waveguide between two main waveguides has been demonstrated. It has been found that the cross-sectional dimensions of the system can be less than two microns, i.e., several hundreds of times smaller than the cross-sectional dimensions of a system made with the use of a dielectric fiber. It has been determined that modes propagating in these waveguides are the sum of one of the symmetric modes and one antisymmetric mode of the coupled system, and the cross coupling is suppressed by matching the wave numbers of these modes. Analytical results are confirmed by numerical simulation.

DOI: 10.1134/S1064226918070094

INTRODUCTION

Surface plasmon waveguides are key components of nanoplasmonic optic devices [1–9]. These waveguides can be used as interchip [10] and intrachip [11, 12] plasmon connections. An advantage of plasmon waveguides consists in their small size and high operating frequencies (up to optical frequencies). This is achieved by means of subwavelength localization of the electromagnetic field. Main drawbacks of plasmon waveguides are the attenuation due to the ohmic loss and the cross coupling between waveguides. The attenuation can be compensated by the use of an active medium [13–18]. The cross coupling arises due to tunneling of the signal from one waveguide to another. As in other waveguides, the surface plasmon wave leaks through the physical boundary of the waveguide. As a result, if two waveguides are placed close to each other, the energy transfer between them or, in other words, cross coupling, takes place.

For miniaturization of devices and increase of the transmission capacity of optical transmission lines, it is desirable to obtain high density of plasmon waveguides. However, high density inevitably leads to an increase in the cross coupling. In [19], an original method for suppressing the cross coupling was proposed: an additional waveguide was placed between the signal-carrying waveguides and, with the use of adiabatic elimination, decoupling between the initial waveguides and the additional one was demonstrated. The proposed method allows us to control the additional waveguide; however, it does not allow us to control the isolation between the lateral (initial) waveguides.

In this study, the possibility of isolation of plasmon waveguides with the use of an additional waveguide placed between them is investigated. The additional waveguide modifies the dispersion relation for the system eigenmodes. Its parameters are selected so as to equate wave numbers of the symmetric and antisymmetric modes in the signal-carrying waveguides. In this case, any linear combination of these modes propagates without changes. In particular, this is true for excitation of such a linear combination when the signal propagates in only one waveguide.

1. SUPPRESSION OF ENERGY TRANSFER BETWEEN WAVEGUIDES WITH THE HELP OF AN ADDITIONAL WAVEGUIDE

Using the theory of coupled modes, let us consider propagation of an electromagnetic wave in a system consisting of two coupled waveguides with identical wave numbers β. Let the z axis be directed along the waveguides and the coupling constant be denoted by

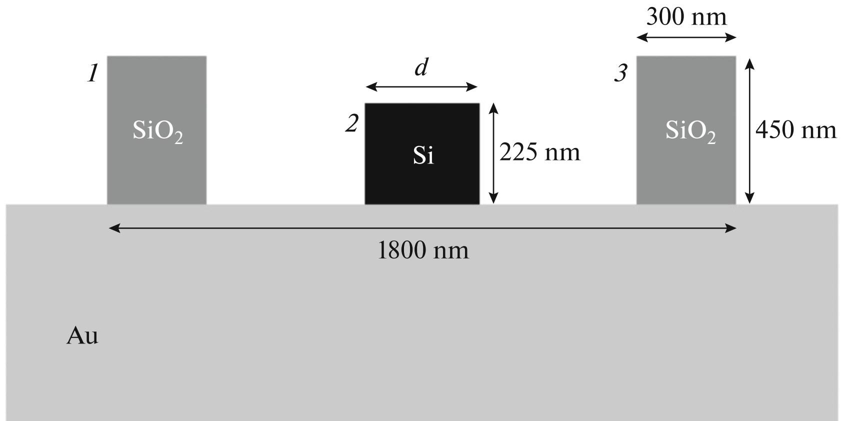

Fig. 1. Layout of the system of plasmon waveguides: (*1*, *3*) signal-carrying waveguides and (*2*) additional waveguide placed between the signal-carrying waveguides.

κ. The field in the waveguides is described by the equation

$$-i\frac{d}{dz}\begin{pmatrix}u_1(z)\\u_2(z)\end{pmatrix}=\begin{pmatrix}\beta & \kappa\\ \kappa^* & \beta\end{pmatrix}\begin{pmatrix}u_1(z)\\u_2(z)\end{pmatrix}, \quad (1)$$

where u_1 and u_2 are the field amplitudes in these waveguides. The eigenmodes of the system can be found as eigenvectors of matrix $\begin{pmatrix}\beta & \kappa\\ \kappa^* & \beta\end{pmatrix}$

$$\begin{pmatrix}u_1\\u_2\end{pmatrix}_+=\begin{pmatrix}\kappa/|\kappa|\\1\end{pmatrix},\ \begin{pmatrix}u_1\\u_2\end{pmatrix}_-=\begin{pmatrix}-\kappa/|\kappa|\\1\end{pmatrix}, \quad (2)$$

and their wave numbers can be found as the eigenvalues of the same matrix:

$$\beta_+=\beta+|\kappa|,\ \ \beta_-=\beta-|\kappa|. \quad (3)$$

One of the modes is reflection symmetric, and the other mode is reflection antisymmetric. Thus, if the phase difference between the modes is zero (π), then the field exists only in the first (second) waveguide.

The energy is completely transferred from one waveguide to the other when the phase difference between the eigenmodes changes by π. This occurs when the lengths of the waveguides satisfy the conditions

$$\mathrm{Re}(\beta_+-\beta_-)L_{\mathrm{CC}}=2|\kappa|L_{\mathrm{CC}}=\pi. \quad (4)$$

In the case of two waveguides, the only way to increase the cross-coupling (CC) length L_{CC} is to decrease $|\kappa|$. This can be attained either by increasing the distance between the waveguides, which decreases the system's transmission capacity per unit area, or by restructuring the system.

Let us now discuss the effect of the loss on the cross coupling in the proposed system. In a system of two coupled lossy waveguides, the wave numbers assume imaginary parts:

$$\beta_+=\beta+|\kappa|+i\gamma_+,\ \ \beta_-=\beta-|\kappa|+i\gamma_-. \quad (5)$$

If losses in both waveguides are identical, then the imaginary parts of the symmetric and antisymmetric modes are also identical: $\gamma_+=\gamma_-$. As a result, the cross-coupling length $L_{\mathrm{CC}}=\pi/(2\,\mathrm{Re}(\beta_+-\beta_-))$ is independent of the loss.

If the imaginary parts of the symmetric and antisymmetric modes differ, then the amplitudes of the fields in the first and second waveguides are identical:

$$\begin{aligned}u_1(z)&=\exp(i\beta_+z)\\&\times(c_++c_-\exp(-2i|\kappa|z-(\gamma_--\gamma_+)z)),\\u_2(z)&=\exp(i\beta_+z)\\&\times(c_+-c_-\exp(-2i|\kappa|z-(\gamma_--\gamma_+)z)),\end{aligned} \quad (6)$$

where c_+ and c_- are the initial amplitudes of the symmetric and antisymmetric eigenmodes. As a result, oscillations in the waveguides occur at $|(\gamma_--\gamma_+)z|\ll 1$. Otherwise, the field distribution is similar to that of the symmetric or the antisymmetric mode and the field amplitudes decrease as the wave propagates. The layout proposed in this study can be used at $L_{\mathrm{CC}}\ll|\gamma_--\gamma_+|^{-1}$.

In order to increase L_{CC} without increasing the distance between the waveguides, we place additional linear waveguide *2* between signal-carrying waveguides *1* and *3* (Fig. 1). It is assumed that the signal-carrying waveguides have identical wave numbers β_1 and the coupling constant of these waveguides is κ_{13}. The wave number of the additional waveguide is β_2, and the coupling constant of the additional waveguide and the signal-carrying waveguides is κ_{12}.

Let us consider this system in the approximation of coupled modes. Let the z axis be directed along the waveguides. In this case, we can write the equation determining the dependence of the fields on the coordinate as

$$-i\frac{d}{dz}\begin{pmatrix} u_1(z) \\ u_2(z) \\ u_3(z) \end{pmatrix} = \begin{pmatrix} \beta_1 & \kappa_{12} & \kappa_{13} \\ \kappa_{12}^* & \beta_2 & \kappa_{12} \\ \kappa_{13}^* & \kappa_{12}^* & \beta_1 \end{pmatrix}\begin{pmatrix} u_1(z) \\ u_2(z) \\ u_3(z) \end{pmatrix}, \qquad (7)$$

where u_1, u_2, u_3 are the amplitudes of the fields in the respective waveguides. The system of these waveguides is symmetric; therefore, all system eigenmodes are either symmetric or antisymmetric. The field of the antisymmetric mode must be zero in the central waveguide and must have different signs in the lateral waveguides. There is only one field distribution that satisfies these conditions. Two other modes are symmetric. These modes are orthogonal to each other; therefore, one of these modes has maxima in the lateral waveguides and the other mode has a maximum in the central waveguide. Below, the symmetric mode with maxima in the lateral waveguides is indexed by +1, the other symmetric mode is indexed by +2, and the antisymmetric mode is simply marked by the minus sign (−). Our goal is to find the waveguide parameters at which wave numbers of antisymmetric mode U_- and first symmetric mode U_{+1} coincide. In this case, according to Eq. (4), $L_{CC} = \infty$.

In the system under consideration, wave numbers β_{+1}, β_{+2} and β_- have the form

$$\beta_{+1} = \frac{1}{2}\Big(\beta_1 + \beta_2 + \kappa_{13} - \sqrt{\beta_1^2 - 2\beta_1\beta_2 + \beta_2^2 + 8\kappa_{12}^2 + 2\beta_1\kappa_{13} - 2\beta_2\kappa_{13} + \kappa_{13}^2}\Big), \qquad (8)$$

$$\beta_{+2} = \frac{1}{2}\Big(\beta_1 + \beta_2 + \kappa_{13} + \sqrt{\beta_1^2 - 2\beta_1\beta_2 + \beta_2^2 + 8\kappa_{12}^2 + 2\beta_1\kappa_{13} - 2\beta_2\kappa_{13} + \kappa_{13}^2}\Big), \qquad (9)$$

$$\beta_- = \beta_1 - \kappa_{13}. \qquad (10)$$

Below, all wave numbers are normalized to the wave vector in free space $\beta_0 = \omega/c$. At the first step, in order to estimate the possibility of elimination of the cross coupling, we use parameters in the range observed in the experiments [20, 21]. We assume that the wave numbers in the lateral waveguides are $\beta_1 = \beta_3 = 2$. Coupling constants of the waveguides κ_{12} and κ_{13} exponentially decrease as the distance between the waveguides increases. The coupling constant of the first and second (or the second and third) waveguides κ_{12} must be several times higher than κ_{13}. The theory of coupled modes, which we use, is the theory of perturbations with respect to coupling constants κ_{12} and κ_{13}. The theory is applicable if $\kappa_{12}, \kappa_{13} \ll \beta_1$. We assume that $\kappa_{12} = 10^{-2}\beta_1 = 2\times10^{-2}$ and $\kappa_{13} = \kappa_{12}/4 = 5\times10^{-3}$, whereas wave number of the central waveguide β_2 will be varied. For these parameters, in the absence of the central waveguide, the cross-coupling length $L_{CC} = \pi/2|\kappa_{13}| = 50\lambda_\sigma$, where $\lambda_\sigma = 2\pi c/\omega$ is the wavelength in free space. The cross-coupling length limits the waveguide length for which data transmission is possible. In our case, the waveguide length cannot exceed $50\,\lambda_\sigma$ since, at this distance, the signal is completely transferred from the first waveguide to the third one.

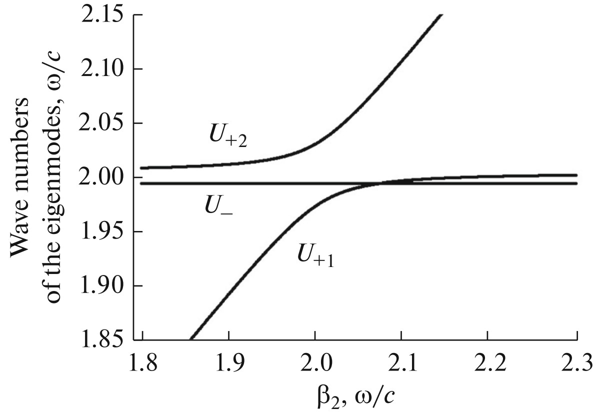

Fig. 2. Wave numbers of the eigenmodes of the three-waveguide system as functions of the wave number in the central waveguide.

Dependences of the wave number of each mode on β_2 are shown in Fig. 2. It can be seen that at $\beta_2 = 2.1102$, wave numbers of modes U_- and U_{+1} coincide and are equal to 1.995. In this case, the eigenmodes of the system have the form

$$U_{+1} = \begin{pmatrix} 0.68 \\ 0.27 \\ 0.68 \end{pmatrix} \quad U_{+2} = \begin{pmatrix} -0.19 \\ 0.96 \\ -0.19 \end{pmatrix}, \quad U_- = \begin{pmatrix} 0.707 \\ 0 \\ -0.707 \end{pmatrix}. \qquad (11)$$

Let us determine the field distribution in the waveguides under the assumption that a signal with the unit amplitude is excited only in the first waveguide. In this case, we obtain the following eigenmodes:

$$c_{+1} = U_{+1}(1) = 0.68, \quad c_{+2} = U_{+2}(1) = -0.19, \quad c_- = U_-(1) = 0.707. \qquad (12)$$

Since the wave numbers of eigenmodes U_- and U_{+1} are identical, the phase difference between them does not change. The field in the first waveguide can be written as

$$|u_1(z)| = |c_{+1}U_{+1}(1) + c_-U_-(1) + \exp(i(\beta_{+2} - \beta_{+1})z)c_{+2}U_{+2}(1)|. \qquad (13)$$

This field reaches minimum values at the points $z_{\min} = (2n+1)\pi/(\beta_{+2} - \beta_{+1})$, where n is an integer. Let us determine the minimum value of u_1 at these points:

$$|u_1(z)| = |c_{+1}U_{+1}(1) + c_-U_-(1) - c_{+2}U_{+2}(1)| = |1 - 2c_{+2}U_{+2}(1)| \approx 0.92. \qquad (14)$$

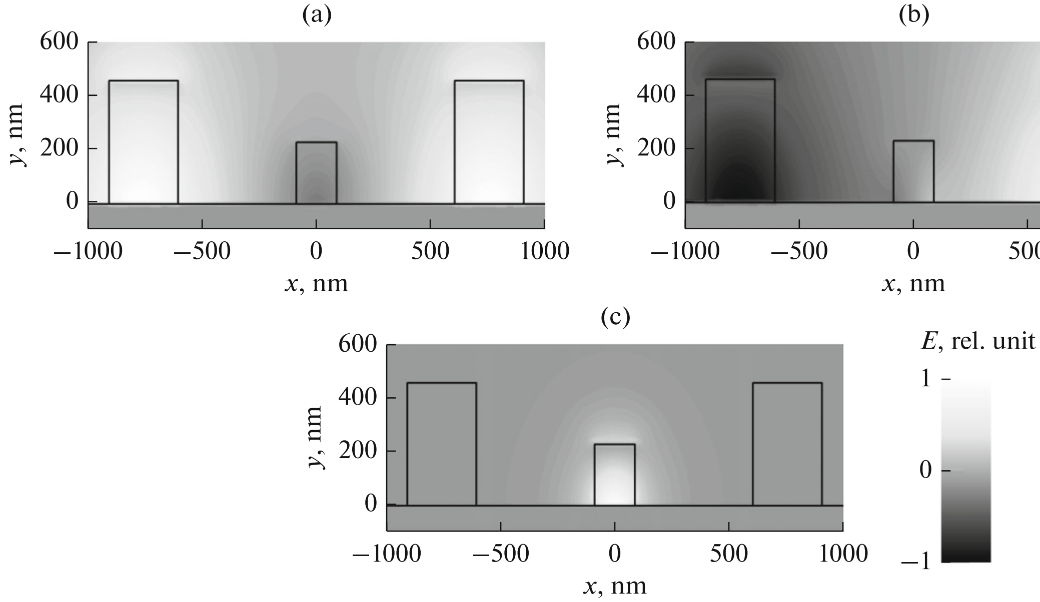

Fig. 3. Distribution of the electric field of eigenmodes at (a) $\beta_1 = 1.0856 + 0.0025i$, (b) $\beta_2 = 1.0856 + 0.0027i$, and (c) $\beta_3 = 2.4636 + 0.0288i$.

The field in the third waveguide is determined by the expression

$$|u_3(z)| = |c_{+1}U_{+1}(3) + c\_U_-(3) + \exp(i(\beta_{+2} - \beta_{+1})z)c_{+2}U_{+2}(3)|. \tag{15}$$

This expression has a maximum at the same points $z_{\min}$, where u_1 is minimal. At these points, the amplitude of u_2 has the form

$$|u_2(z)| = |c_{+1}U_{+1}(3) + c\_U_-(3) - c_{+2}U_{+2}(3)| = |2c_{+2}U_{+2}(3)| \approx 0.07. \tag{16}$$

The maximum field strength in the third waveguide is $\approx 6\times10^{-3}$. Thus, the strength of the induced signal in our system is at a level of 0.6% of the strength of the carrier signal. In other words, the cross coupling is practically eliminated in this system.

In the presence of loss, the situation in the three-waveguide system is similar to that in the two-waveguide system: if the loss level is the same in all waveguides, the cross-coupling length does not change. Indeed, for wave numbers $\beta_1 \to \beta_1 + i\gamma$ and $\beta_2 \to \beta_2 + i\gamma$, Eq. (7) takes the form

$$-i\frac{d}{dz}\begin{pmatrix} u_1(z) \\ u_2(z) \\ u_3(z) \end{pmatrix} = \begin{pmatrix} \beta_1 & \kappa_{12} & \kappa_{13} \\ \kappa_{12}^* & \beta_2 & \kappa_{12} \\ \kappa_{13}^* & \kappa_{12}^* & \beta_1 \end{pmatrix}\begin{pmatrix} u_1(z) \\ u_2(z) \\ u_3(z) \end{pmatrix} + i\begin{pmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} u_1(z) \\ u_2(z) \\ u_3(z) \end{pmatrix}. \tag{17}$$

Let us introduce new variables $u_i(z)' = u_i(z)\exp(-\gamma z)$, where subscript i varies from 1 to 3. These variables satisfy Eq. (7); therefore, we can apply to them the same reasoning as that applied to the lossless system. The losses in the additional waveguide and in the signal-carrying waveguides are different; therefore, the propagation lengths of different modes are also different. The signal propagation length is determined by the shortest propagation length and by the cross-coupling length.

In the context of the theory of coupled modes, the solution for a system of waveguides can be represented in the form of a linear combination of eigenmodes of each waveguide. This theory is the first-order perturbation theory, where the coupling constants of waveguides serve as perturbation parameters. Higher orders of the perturbation theory may noticeably affect the results obtained since even a small difference between the wave numbers of the first symmetric and antisymmetric modes may result in energy transfer between the waveguides. To check the analytical results, we will perform a numerical simulation of signal propagation in the described system.

2. NUMERICAL SIMULATION

Let us consider a system of dielectric surface plasmon waveguides [22, 23] with parameters shown in Fig. 1. The waveguides consist of dielectric strips placed on a 300-nm-thick gold film. At the Telecom wavelength $\lambda = 1.55$ μm, the refractive index of gold is $0.5241 + 10.742i$ [24].

The lateral waveguides have identical refractive indices of 1.5277 (SiO_2) [25] (dimensions are shown in Fig. 1). This ensures single-mode operation for each waveguide. The refractive index of the additional

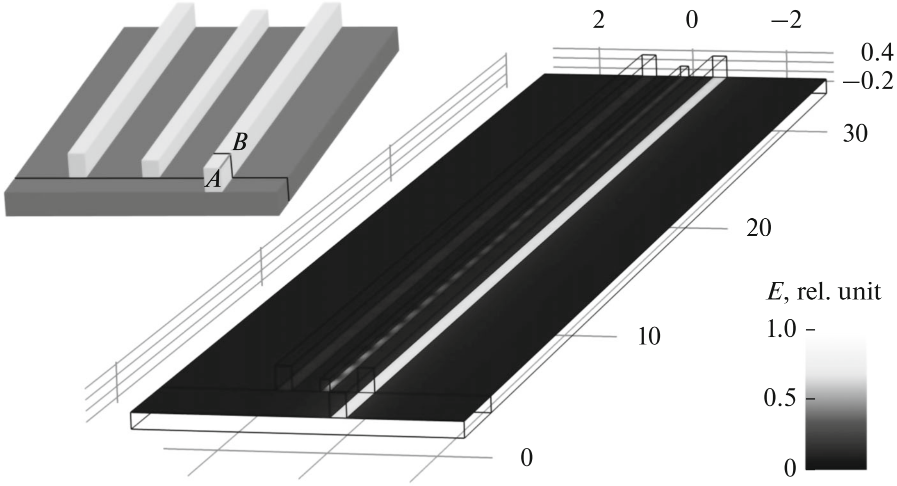

Fig. 4. Distribution of the magnitude of the electric field strength at the interface between the gold film and the waveguide. The inset show the layout of the system, where section AB of the third waveguide is used to excite this waveguide.

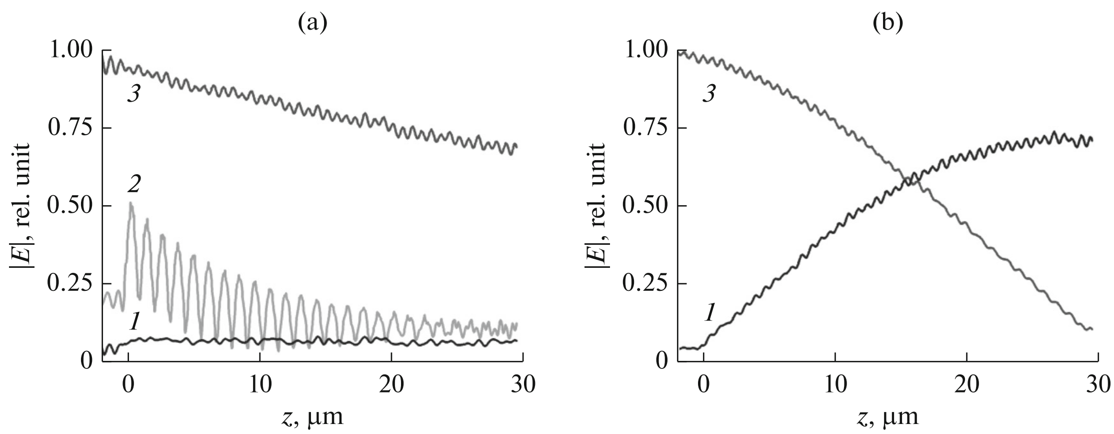

Fig. 5. Distribution of the magnitude of the electric field at the interface between the gold film and (*1*) the right waveguide, (*2*) the central waveguide, and (*3*) the left waveguide (a) with and (b) without the additional waveguide. The origin of the *z* axis is point *B* in the inset in Fig. 4.

waveguide is 3.4757 (Si) [26]. The width of the lateral waveguide is fixed, and the width of the central waveguide is varied.

Let us determine eigenmodes of the system by the finite-element method with a grid size varying from 52 to 260 nm. The closer the element to the boundary of the system, the smaller its size; the farther the element from the boundaries, the larger its size. The width of the calculation area is 6 μm and the height is 3.3 μm. Thus, the size of the system is considerably smaller than the size of the calculation area.

The system supports three eigenmodes: two symmetric modes and one antisymmetric mode (Fig. 3). Numerical simulation shows that, at the width of the second waveguide $d = 175$ nm, real parts of the wave vectors of the symmetric mode with maxima in the lateral waveguides and the antisymmetric mode (Figs. 3a and 3b) are identical and their wave numbers are, respectively, $\beta_1 = 1.0856 + 0.00252i$ and $\beta_2 = 1.0856 + 0.00271i$ at a wavelength of 1.55 μm. The third mode has a maximum in the central waveguide (Fig. 3c).

Let us now consider propagation of the signal excited in the third waveguide of this system. The lengths of the first and second waveguides are 30 μm, whereas the length of the third waveguide, which carries the signal, is 32 μm. Thus, excitation of this waveguide occurs in additional 2-μm-long section AB (see the inset in Fig. 4). The remaining parameters of the system are the same as in Fig. 1. The layout of the system used in the numerical calculations is shown in Fig. 4. The signal in the form of the eigenmode of an (isolated) waveguide is excited in the third 32-μm-long waveguide.

Numerical calculations show that the signal propagates along the right waveguide with a very small energy transfer to the left one. Fields in each waveguide are shown in Fig. 5a.

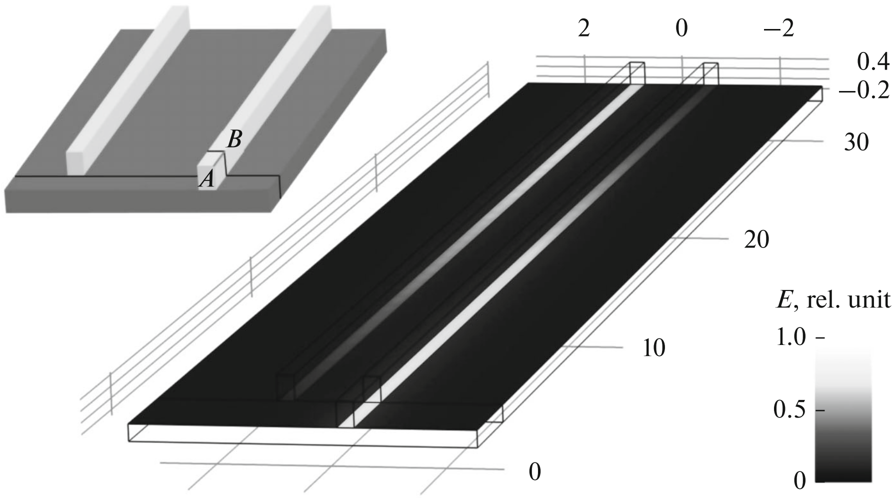

Fig. 6. Distribution of the magnitude of the electric field at the interface between the gold film and the waveguide in the system formed by only two signal-carrying waveguides. The system layout is shown in the inset.

It can be seen in Fig. 5a that the electric field amplitude in waveguide *3* decreases along the z axis, which is mainly due to the ohmic loss in the metal. Besides, the electric field amplitude decreases because the mode excited in section AB is not the exact sum of the symmetric and antisymmetric modes of the three waveguides. At the boundary of transition from one waveguide (AB) to the three waveguides, a certain part of the energy is used to excite modes in the first and second waveguides. As the signals travel along the waveguides, oscillation of the electric field also occurs (which is particularly noticeable in the central waveguide) (see Fig. 5a). This effect is caused by the cross coupling between the right and central waveguides. It can be shown that the length of the cross coupling between these two waveguide $L_{CC} \approx 0.56$ μm. Note that the system operates even if the imaginary parts of the wave numbers are different.

In order to show the contribution of the cental wavegude to suppression of the cross coupling, we calculated a system without an additional waveguide (Fig. 6). It can be seen that, as the wave propagates in the third waveguide, the energy is transferred to the other waveguide (see Figs. 5b and 6).

CONCLUSIONS

It has been shown that the energy transfer between two waveguides can be suppressed by placing between them an additional waveguide with specially selected parameters. The wave excited in one of the waveguides that is the sum of a symmetric and antisymmetric modes corresponds to zero field in the other two waveguides. Since wave numbers of the symmetric and antisymmetric modes coincide, the modes propagate unchanged along the waveguide. The proposed approach makes it possible to significantly increase the density of plasmon waveguides on the chip and increase the transmission capacity of optical communication lines based on these waveguides.

ACKNOWLEDGMENTS

The study was supported by the Presidium of the Russian Academy of Sciences (program no. I.7 "Topical Problems of Photonics, Sounding of Nonuniform Media and Materials").

A.A. Lisyansky acknowledges the support of the National Science Foundation (grant no. DMR-1312707).

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Translated by I. Nikishin