Cubic anisotropic magnets with a random field

D. Nicolaides[a)]
Natural Sciences and Mathematics, Bloomfield College, Franklin Street, Bloomfield, New Jersey 07003

A. A. Lisyansky
Department of Physics, Queens College of CUNY, Flushing, New York 11367

The critical behavior at phase transitions of m-component random magnets with cubic anisotropy is studied within the context of an exactly solvable model. The presence of a random field suppresses any phase transition for spatial dimensions $d \leqslant 4$. If one or more of the components of the random field are zero, but with at least one nonzero component, a new second-order phase transition in the anisotropic phase occurs. In the pure case second-order transitions could happen only into the isotropic phase. The critical temperature of this new, random-field-induced second-order transition decreases with increasing randomness.
[S0021-8979(99)75908-6]

The last 20 years have generated a lot of intense research on the subject of random systems with frozen-in disorder.[1–7] In this work the critical behavior at phase transitions of m-component random magnets with cubic anisotropy is studied within the context of an exactly solvable model which considers interactions of fluctuations with equal and opposite momenta. The model allows one to investigate systems with complex symmetry which are hard to study within the framework of the renormalization group theory.[8–10] In the absence of randomness and when the coupling constant of the uniform cubic anisotropy is negative, the model explicitly finds a fluctuation-induced first-order phase transition into the anisotropic phase.[10] Below, it will be shown that a new second-order transition replaces the first order which is present in the pure case. The critical temperature of this new, random-field-induced second-order transition decreases with increasing randomness, which is measured by the field–field correlator. Also, the presence of a random field suppresses any phase transition for spatial dimensions $d \leqslant 4$.

Let us see all these by starting with a system described by the Ginzburg–Landau–Wilson functional

$$H=\frac{1}{2}\int d^dx\Bigg[\tau|\mathbf{S}(\mathbf{x})|^2+c(\nabla\mathbf{S}(\mathbf{x}))^2+u|\mathbf{S}(\mathbf{x})|^4 + v\sum_{\alpha=1}^{m} S_\alpha^4(\mathbf{x})-\mathbf{h}\cdot\mathbf{S}(\mathbf{x})-\mathbf{h}(\mathbf{x})\cdot\mathbf{S}(\mathbf{x})\Bigg], \qquad (1)$$

where $\mathbf{S}(\mathbf{x})$ is an m-component order parameter, $\tau \propto T-T_c$, $\mathbf{h}$ and $\mathbf{h}(\mathbf{x})$ are m-component constant and random fields, respectively. The free energy averaged with respect to a distribution of the random field can be evaluated with the help of the replica method. Assuming that $\mathbf{h}(\mathbf{x})$ is a δ-correlated random variable, $\langle h_\alpha(\mathbf{x})h_\beta(\mathbf{x})\rangle = B_\alpha\delta_{\alpha\beta}\delta(\mathbf{x}-\mathbf{x}')$, and $\varphi(\mathbf{x})$ is a replicated $m\times n$-component vector order parameter, we derive an effective functional $H_{\text{eff}}[\varphi]$,

$$H_{\text{eff}}=\frac{1}{2}\int d^dx\Bigg\{\tau|\varphi(\mathbf{x})|^2+c(\nabla\varphi(\mathbf{x}))^2 + \sum_{i=1}^{n}\Bigg[u|\varphi_i(\mathbf{x})|^4+v\sum_{\alpha=1}^{m}\varphi_{i\alpha}^4(\mathbf{x})-\mathbf{h}\cdot\varphi_i(\mathbf{x})\Bigg] - \sum_{\alpha}^{m}B_\alpha\Bigg(\sum_{i=1}^{n}\varphi_{i\alpha}(\mathbf{x})\Bigg)^2\Bigg\}. \qquad (2)$$

The model can calculate the free energy exactly if we split interaction terms in Eq. (2) as follows:

$$\int d^dx|\varphi_i(\mathbf{x})|^4 \to \frac{1}{V}\Bigg(\sum_{\alpha}^{m}\hat{a}[\varphi_{i\alpha}(\mathbf{x})]\Bigg)^2,$$

$$\int d^dx\varphi_{i\alpha}^4(\mathbf{x}) \to \frac{1}{V}\hat{a}^2[\varphi_{i\alpha}(\mathbf{x})], \qquad (3)$$

$$\hat{a}[\varphi_{i\alpha}(\mathbf{x})]=\int d^dx\varphi_{i\alpha}^2(\mathbf{x}).$$

Equations (3) imply that the model considers interactions of fluctuations with equal and antiparallel momenta, and transfers the φ^4 model into the universality class of the spherical model.[11,12] The model then uses a transformation analogous to that of Hubbard–Stratonovich so the partition function becomes

[a)]Electronic mail: demetris-nicolaides@bloomfield.edu

$$Z=\int\prod_{i=1}^{n}\prod_{\alpha=1}^{m}(D\varphi_{i\alpha\mathbf{q}}dx_{i\alpha}dy_{i\alpha})\exp\Bigg[-\frac{V}{2}\sum_{i=1}^{n}\Bigg(\tau\sum_{\alpha=1}^{m}x_{i\alpha}+u\Bigg(\sum_{\alpha=1}^{m}x_{i\alpha}\Bigg)^2+\nu\sum_{\alpha=1}^{m}x_{i\alpha}^2-\sum_{\alpha=1}^{m}x_{i\alpha}y_{i\alpha}\Bigg)-\frac{1}{2}\sum_{\alpha=1}^{m}\Bigg(\sum_{i=1}^{n}\sum_{\mathbf{q}}|\varphi_{i\alpha\mathbf{q}}|^2(y_{i\alpha}+c\mathbf{q}^2-B_\alpha)-\sum_{i=1}^{n}h_\alpha\varphi_{i\alpha0}-B_\alpha\sum_{\mathbf{q}}\sum_{i\neq j}^{n}\varphi_{i\alpha\mathbf{q}}\sum_{j\neq i}^{n}\varphi_{j\alpha-\mathbf{q}}\Bigg)\Bigg].\qquad(4)$$

After the diagonalization with respect to indices i and j, functional integrals in Eq. (4) can be calculated. Notice that only when $y_{i\alpha}=y_{j\alpha}$ is true, does the degeneracy of the eigenvalues become n as expected by treating the pure cubic system within the replica method. This gives:

$$Z=\int\Bigg(\prod_{i=1}^{n}\prod_{\alpha=1}^{m}\Bigg)dx_{i\alpha}dy_\alpha\exp\Bigg[-\frac{V}{2}F(x_{i,\alpha}y_i,h_\alpha)\Bigg],\qquad(5)$$

$$F(x_i,y,h)=\sum_{i=1}^{n}\Bigg[t\sum_{\alpha=1}^{m}x_{i\alpha}+u\Bigg(\sum_{\alpha=1}^{m}x_{i\alpha}\Bigg)^2+\nu\sum_{\alpha=1}^{m}x_{i\alpha}^2-\sum_{\alpha=1}^{m}x_{i\alpha}y_\alpha\Bigg]-\sum_{\alpha=1}^{m}(1-n)f_d(y_\alpha;c)+\sum_{\alpha=1}^{m}f_d(y_\alpha-nB_\alpha;c)-\sum_{\alpha=1}^{m}\frac{nh_\alpha^2}{y_\alpha-nB_\alpha},\qquad(6)$$

where t is the renormalized trial critical temperature given by $t=\tau+(2mu+2\nu)\,S_d(2\pi)^{-d}\theta_d(\Lambda)$,

$$\theta_d(\Lambda)=\frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{\Lambda^{d-2}}{c(d-2)}, & d\neq 2\\[2mm] \dfrac{1+\ln(c\Lambda^2)}{2c}, & d=2,\end{cases}\qquad(7)$$

$$f_d(y_a;c)=\frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{\pi y_\alpha^{d/2}}{dc^{d/2}\sin(\pi d/2)}\equiv\kappa(c)y_\alpha^{d/2}, & d\neq\text{even}\\[2mm] \dfrac{1}{d}\Big(-\dfrac{y_\alpha}{c}\Big)^{d/2}\ln|y_\alpha|, & d=\text{even}.\end{cases}$$

Λ is the cutoff momentum, and S_d the surface area of a d-dimensional sphere. In the thermodynamic limit, $V\to\infty$, the saddle point of the integrals in Eq. (5) gives an exact solution for the partition function.

The disorder averaged free energy and the various components of the order parameter, Φ, are given, respectively, by

$$F_{\text{av}}=\lim_{n\to0}\frac{1}{n}F(x_{i\alpha},y_\alpha,h_\alpha),\quad \Phi_\alpha=-\frac{\partial F_{\text{av}}}{\partial h_\alpha}=\lim_{n\to0}\frac{h_\alpha}{y_\alpha(h_\alpha)},\qquad(8)$$

with $x_{i\alpha}$ and y_α defined by the equations $\partial F/\partial x_{i\alpha}=0$ and $\partial F/\partial y_\alpha=0$. This system of $n\times m+m$ equations can be reduced to just m equations for the unknowns y_α. The latter equations should be expanded in powers of n up to the first order. The resulting equations for y_α can be written in terms of Φ_α when Eq. (8) is used,

$$2\nu\Phi_\alpha^2+t-\frac{h_\alpha}{\Phi_\alpha}+\frac{\nu(-1)^{d/2}}{c^{d/2}}\Big(\frac{h_\alpha}{\Phi_\alpha}\Big)^{(d-2)/2}\Big[\frac{1}{d}+\ln\Big(\frac{h_\alpha}{\Phi_\alpha}\Big)\Big]-\frac{\nu(-1)^{d/2}B_\alpha(d-2)}{c^{d/2}}\Big(\frac{h_\alpha}{\Phi_\alpha}\Big)^{(d-4)/2}\Big[\frac{1}{d}+\frac{1}{d-2}+\frac{1}{2}\ln\Big(\frac{h_\alpha}{\Phi_\alpha}\Big)\Big]+2u\sum_{\beta=1}^{m}\Phi_\beta^2+\frac{u(-1)^{d/2}}{c^{d/2}}\sum_{\beta=1}^{m}\Big(\frac{h_\beta}{\Phi_\beta}\Big)^{(d-2)/2}\Big(\frac{1}{d}+\ln\Big(\frac{h_\beta}{\Phi_\beta}\Big)\Big)-\frac{u(-1)^{d/2}(d-2)}{c^{d/2}}\sum_{\beta=1}^{m}B_\beta\Big(\frac{h_\beta}{\Phi_\beta}\Big)^{(d-4)/2}\times\Big[\frac{1}{d}+\frac{1}{d-2}+\frac{1}{2}\ln\Big(\frac{h_\beta}{\Phi_\beta}\Big)\Big]=0,\quad d=\text{even}\qquad(9a)$$

$$2\nu\Phi_\alpha^2+t-\frac{h_\alpha}{\Phi_\alpha}+\nu d\kappa(c)\Big(\frac{h_\alpha}{\Phi_\alpha}\Big)^{(d-2)/2}-\frac{\nu B_\alpha}{2}\kappa(c)d(d-2)\times\Big(\frac{h_\alpha}{\Phi_\alpha}\Big)^{(d-4)/2}+2u\sum_{\beta=1}^{m}\Big(\Phi_\beta^2+\frac{d}{2}\kappa(c)\Big(\frac{h_\beta}{\Phi_\beta}\Big)^{(d-2)/2}-\frac{d(d-2)B_\beta}{4}\kappa(c)\Big(\frac{h_\beta}{\Phi_\beta}\Big)^{(d-4)/2}\Big)=0,\quad d=\text{noneven}.\qquad(9b)$$

It becomes obvious from Eq. (9) that in the limit $h_\alpha\to0$ there are no solutions for Φ_α when $d\leqslant4$. On the other hand, when $d>4$ it is seen that the random field has no effect and the results are those of the pure system.[10] Let us solve Eq. (9b) in the limit $h_\alpha\to0$ for $d=3$. In this case an ordered phase can exist if some of the components of the random field are equal to zero. Say the first l components of $\mathbf{B}$ are zero. Then the possible phases are those which have nonzero components of any number from the set of components between Φ_1 and Φ_l (including Φ_l). It can be shown that the phases that may occur under the proper conditions, stated below, are: the isotropic phase, which is the one having $\Phi_1=\Phi_2=\cdots=\Phi_l\equiv\Phi_l\neq0$ with the rest of the components being zero, and the anisotropic phase with only one nonzero component, say, Φ_1.

Below, the solutions for the order parameters Φ_l and Φ_1 are given for the case when the nonzero components of the random field are equal to one another and have strength B. This does not affect any general result but rather makes the equations somehow simpler:

$$\Phi^2_{m_0\pm}(t)=-\frac{t}{2(\nu+um_0)}-\frac{3u\kappa(c)(m-m_0)}{4(\nu+um_0)^2}\left(3\kappa(c)\nu(\nu+um)\pm\sqrt{9\kappa(c)^2\nu^2(\nu+um)^2+4\nu(\nu+um_0)t}\right)\pm\frac{3\kappa(c)(m-m_0)uB}{2\sqrt{9\kappa(c)^2\nu^2(\nu+um)^2+4\nu(\nu+um_0)t}}. \quad (10)$$

The integer m_0 may take the values $1-l$, giving therefore the appropriate expressions for the anisotropic phase $\Phi_{1\pm}(t)$, and the isotropic phase $\Phi_{l\pm}(t)$.

The pure case is obtained in the limit $B\rightarrow 0$. For this special case it was derived previously[10] that when the coupling constant ν is negative, the anisotropic phase occurs via a fluctuation-induced first-order transition. On the other hand, when ν is positive the phase transition is of the second order but into the isotropic phase. However, the presence of randomness B creates a new second-order phase transition for both positive and negative ν. Explicitly, if $t=t_c+\tau$, where t_c is the critical temperature for the second-order transition for the random case, then

$$\Phi^2_{m_0\pm}(\tau)=-\frac{\tau}{2(\nu+um_0)}\mp\frac{3u\nu\kappa(c)(m-m_0)\tau}{2(\nu+um_0)\sqrt{9\kappa(c)^2\nu^2(\nu+um)^2+4\nu(\nu+um_0)t_c}}\mp\frac{3u\nu\kappa(c)(m-m_0)(\nu+um_0)B\tau}{2(\nu+um_0)(9\kappa(c)^2\nu^2(\nu+um)^2+4\nu(\nu+um_0)t_c)^{3/2}}. \quad (11)$$

Equation (10) equated to zero gives the critical temperature t_c. For positive ν the critical temperature of the isotropic phase is higher than that of the anisotropic phase. Therefore, in Eqs. (10) and (11) $m_0=l$. The reverse is true for negative ν when the anisotropic phase is realized before the isotropic one. Therefore, in Eqs. (10) and (11) $m_0=1$. Since in the pure case with ν positive the transition is of the second kind, one can find a simple analytic expression up to the order of B for the critical temperature in the random case with positive ν. This is

$$t_c=\frac{u(m-m_0)(\nu+u)(\nu+um_0)B}{\nu(-\nu(\nu+um_0)-um_0(um+\nu)+u^2(m-m_0))}. \quad (12)$$

It is easily verified that the critical temperature decreases with increasing B.

For negative ν, the analytic expression of the critical temperature obtained by equating Eq. (10) to zero is very cumbersome. An approximate solution cannot be obtained since in the pure case the existence of the first-order phase transition provides a discontinuity of the order parameter at criticality. However, numerical calculations show that this critical temperature decreases with increasing randomness, as well as the slope of the curve $\Phi_{1+}(\tau)$ vs τ becomes steeper with decreasing randomness. This is expected as in this limit the discontinuity of the first-order transition is recovered.

Parameter $c^{1/2}$ represents the radius of interactions in the original system. Setting $c\rightarrow\infty$ [or $\kappa(c)=0$] we suppress fluctuations. In this case one can see from Eq. (9) that a second-order phase transition is restored and critical exponents become the same as the mean field ones independent of dimensionality or the presence of random fields.

[1] Y. Imry and S.-K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).
[2] G. Grinstein, Phys. Rev. Lett. **36**, 944 (1976).
[3] A. Aharony, Y. Imry, and S.-K. Ma, Phys. Rev. Lett. **37**, 1364 (1976).
[4] A. P. Young, J. Phys. C **10**, L247 (1977).
[5] S. Singh and Jr. Banavar, Phys. Rev. Lett. **55**, 2220 (1985).
[6] D. I. Uzunov, E. R. Korutcheva, and Y. T. Millev, Physica A **129**, 535 (1985).
[7] P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, 1997).
[8] A. A. Lisyansky and D. Nicolaides, Phys. Lett. A **173**, 179 (1993).
[9] D. Nicolaides and A. A. Lisyansky, Phys. Rev. B **53**, 8168 (1996).
[10] Yu. M. Ivanchenko, A. A. Lisyansky, and A. E. Filippov, Phys. Lett. A **119**, 55 (1986).
[11] T. Schneider, E. Stoll, and H. Beck, Physica A **34**, 436 (1975).
[12] Yu. M. Ivanchenko, A. A. Lisyansky, and A. E. Filippov, Theor. Math. Phys. **67**, 413 (1986).