Measuring the transport mean free path using a reference random medium

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The transport mean free path of microwave radiation in a random sample is determined from measurements of transmission through a composite sample of the medium with unknown scattering characteristics and a random medium of variable thickness and known scattering parameters. The method can be applied at optical frequencies by use of a ceramic wedge as the medium of variable thickness.

Determining the transport mean free path and the internal reflection coefficient for electromagnetic waves in random media is of fundamental and practical interest.[1] These properties are important in statistical studies of wave propagation and localization. They can also be used to infer dynamical information regarding complex systems, and they determine the opacity of important scattering media such as paints, the atmosphere, and biological tissues. Until recently the internal reflection of waves inside the sample at is boundaries has been neglected in descriptions of photon propagation. However, because of the large angles at which scattered waves may strike the internal surface of the sample, the average reflection coefficient of these waves may be quite large. This can result in substantial discrepancies between theory and experiment.[2,3] Recently we demonstrated that accurate measurements of all basic microscopic parameters can be achieved in both the microwave[4] and visible[5] parts of spectrum if scattering at the boundary is properly taken into account. One can obtain these parameters by measuring the total transmission through and/or reflection from a slab of a random medium for different thicknesses and for different reflection conditions. However, it is often not possible to vary the thickness of the sample under investigation or to modify the reflection coefficient at the boundary. In this Letter we propose a simple nondestructive method for determining the basic microscopic parameters in a random slab. We measure the relative transmission in samples composed of two media juxtaposed along the longitudinal direction as a function of the thickness of one or another of the media. Because the interfaces do not change with thickness, the influence of internal reflection largely factors out.

Let us consider a sample composed of two adjacent slabs of random media with total thickness $L = L_1 + L_2$ measured along the z direction. If conditions of weak scattering are satisfied, then the steady-state photon intensities $I_{1,2}$ inside the slabs obey the diffusion equation

$$\nabla^2 I_{1,2}(\mathbf{r}) - \alpha_{1,2} I_{1,2}(\mathbf{r}) = \frac{1}{D_{1,2}} Q(\mathbf{r}), \tag{1}$$

where α_i is the absorption coefficient, $D_i = {}^1\!/_3 \nu_i \ell$ is the diffusion coefficient, ℓ_1 is the mean free path, ν_i is the transport velocity of the ith medium, and $Q(\mathbf{r})$ is the source function. We have found that one can accurately describe reflection and transmission by replacing the incoming coherent flux by a source of isotropic radiation at a depth $z = z_p$, with a strength equal to the incident flux.[6] For the case of a plane wave incident upon the slab, the source function can then be written as $Q(\mathbf{r}) = q\nu\delta(z - z_p)$, where q is the source intensity. To solve Eq. (1) we need to specify the boundary conditions. Because there is no incoming diffusive flux through the boundaries in our model, the only flux toward the interior of the slab is the reflected part of the outgoing flux. This gives two boundary conditions in the form[6]

$$\begin{aligned} J_+^{(1)}(x, y, z = 0^+) &= -\mathcal{R}_1 J_-^{(1)}(x, y, z = 0^+), \\ J_-^{(2)}(x, y, z = L^-) &= -\mathcal{R}_2 J_+^{(2)}(x, y, z = L^-), \end{aligned} \tag{2a}$$

where $J_+^{(i)}$ and $J_-^{(i)}$ are diffusive fluxes in the ith medium in the positive and negative directions, respectively. The condition at the interface between the slabs can be written as

$$\begin{aligned} &J_+^{(1)}(x, y, z = L_1) - J_-^{(1)}(x, y, z = L_1) \\ &\qquad = J_+^{(2)}(x, y, z = L_1) - J_-^{(2)}(x, y, z = L_1), \\ &J_+^{(2)}(x, y, z = L_1) = (1 - \mathcal{R}_{12}) J_+^{(1)}(x, y, z = L_1) \\ &\qquad + \mathcal{R}_{12} J_-^{(2)}(x, y, z = L_1), \end{aligned} \tag{2b}$$

where $\mathcal{R}_{12}$ is the reflection coefficient between the two media.

The solutions of Eq. (1) with the boundary conditions (2) are

$$\begin{aligned} I_1(z) = &-\frac{3q}{\alpha_1 \ell_1} \theta(z - z_p) \sinh[\alpha_1(z - z_p)] \\ &+ \frac{qS}{\alpha_1} \frac{\cosh[\alpha_1(L_1 - z_p)]}{\cosh[\alpha_1(L_1 + z_1)]} \Bigg\{ \tanh[\alpha_2(L_2 + z_2)] \\ &+ \frac{\alpha_2 \ell_2}{\alpha_1 \ell_1} \tanh[\alpha_1(L_1 - z_p)] + z_{12} \Bigg\} \sinh[\alpha_1(z + z_1)], \end{aligned} \tag{3a}$$

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$$I_2(z) = \frac{qS}{\alpha_1}\frac{\nu_1}{\nu_2}\frac{\sinh[\alpha_1(z_1 + z_p)]}{\cosh[\alpha_1(L_1 + z_1)]\cosh[\alpha_2(L_2 + z_2)]} \times \sinh[\alpha_2(L + z_2 - z)]. \tag{3b}$$

Here we have introduced the designations

$$z_i = \frac{1}{2\alpha_i}\ln\frac{1 + \alpha_i z_{0i}}{1 - \alpha_i z_{0i}}, \quad z_{0i} = \frac{2\ell_1}{3}\frac{1 + \mathcal{R}_i}{1 - \mathcal{R}_i},$$

$$R_{12} = \frac{4}{3}\frac{\mathcal{R}_{12}}{1 - \mathcal{R}_{12}},$$

$$S = \left\{\tanh[\alpha_2(L_2 + z_2)] + \frac{\alpha_2\ell_2}{\alpha_1\ell_1}\tanh[\alpha_1(L_1 + z_1)] + \ell_2 R_{12}\right\}^{-1}. \tag{4}$$

In experiments one usually measures the total transmission T through or the total reflection R from a slab. Using Eqs. (2) and (3), we can write these quantities as

$$T = \frac{1 - \mathcal{R}_2}{q\nu_1}J_+^{(2)}(L^-) = \frac{3S}{2\alpha_1\ell_1}\frac{1 - \mathcal{R}_2}{1 + \mathcal{R}_2}\frac{\sinh(\alpha_2 z_2)\sinh[\alpha_1(z_1 + z_p)]}{\cosh[\alpha_1(L_1 + z_1)]\cosh[\alpha_2(L_2 + z_2)]}, \tag{5a}$$

$$R = \frac{1 - \mathcal{R}_1}{q\nu_1}J_-^{(1)}(0^+) = \frac{3S}{2\alpha_1\ell_1}\frac{1 - \mathcal{R}_1}{1 + \mathcal{R}_1}\frac{\sinh(\alpha_1 z_1)\cosh[\alpha_1(L_1 - z_p)]}{\cosh[\alpha_1(L_1 + z_1)]} \times \left\{\tanh[\alpha_2(L_2 + z_2)] + \frac{\alpha_2\ell_2}{\alpha_1\ell_1}\tanh[\alpha_1(L_1 - z_p)] + \alpha_2\ell_2 R_{12}\right\}. \tag{5b}$$

We now consider measurements by using a random scattering reference sample with known scattering parameters for one of the media. Unknown microscopic parameters of the other medium can then be determined from Eqs. (5) from measurements of transmission or reflection at different thicknesses. Because the reflection coefficient between the adjacent random media $\mathcal{R}_{12}$ is generally small, the term $\propto R_{12}$ can be neglected. This reduces the number of unknown parameters to three: the mean free path ℓ_2, the absorption coefficient α_2, and the reflection coefficient $\mathcal{R}_2$. If absorption in medium 1 is not negligible, the range of thickness L_1 must be chosen such that $\alpha_1(L_1 + z_1) < 1$ for the smallest thickness in order that $\tanh[\alpha_1(L_1 + z_1)]$ not be saturated over the entire range of variation in L_1. There are no restrictions on L_2. In our experiments we measure relative transmission, keeping the thickness of the second medium L_2 constant. Moreover, because we had $\alpha_2(L_2 + z_2) \simeq 2$ in this experiment, $\tanh[\alpha_2(L_2 + z_2)] \simeq 1$, so the expression for the relative transmission through the sample can be simplified as

$$\frac{T(L_1, L_2)}{T(L_1', L_2)} = \frac{\cosh[\alpha_1(L_1' + z_1)] + \dfrac{\alpha_2\ell_2}{\alpha_1\ell_1}\sinh[\alpha_1(L_1' + z_1)]}{\cosh[\alpha_1(L_1 + z_1)] + \dfrac{\alpha_2\ell_2}{\alpha_1\ell_1}\sinh[\alpha_1(L_1 + z_1)]}, \tag{6}$$

and we end up with only one unknown parameter ℓ_2. Note that the penetration depth z_p has dropped out of this expression.

We applied the method described here to measure the mean free path of microwave radiation in a sample of 0.95-cm-diameter alumina spheres

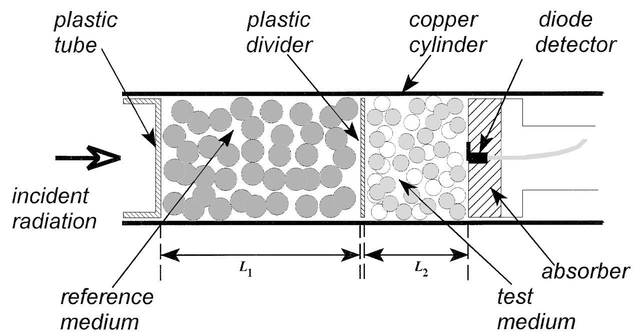

Fig. 1. Schematic of the experimental setup used to measure the total transmission of microwave radiation through the combined sample.

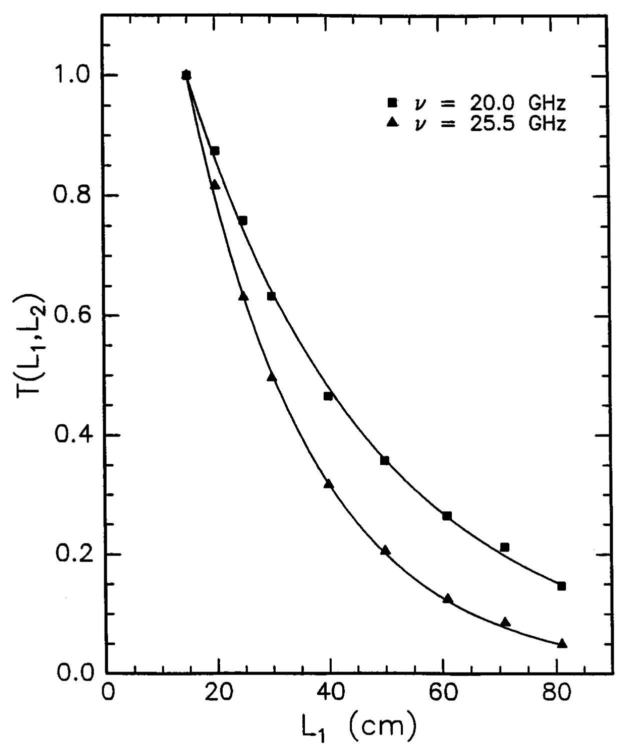

Fig. 2. Relative transmission through a combination of a sample of randomly packed polystyrene spheres of length L_1 and a mixture of alumina and hollow polypropylene spheres for filling fraction $f = 0.30$ at a thickness of $L_2 = 15$ cm as a function of the thickness of the polystyrene sample at frequencies of 20.0 and 25.5 GHz.

Table 1. Input Parameters and Mean Free Paths[a]

ν (GHz)	α_1 (cm^{-1})	ℓ_1 (cm)	α_2 (cm^{-1})	ℓ_2 (cm)
20.0	0.0286	6.40	0.129	1.83
25.5	0.0463	2.60	0.164	1.03

[a]Absorption coefficient α_1 and transport mean free path ℓ_1 are from the reference medium; absorption coefficient α_2 is from the test medium; the transport mean free path ℓ_2 is from the test medium, which is determined from the fit to the data shown in Fig. 2.

with a volume filling fraction $f = 0.30$ and hollow polypropylene spheres of the same diameter with wall thicknesses of 0.2 cm. The frequency was near the second Mie resonance of the alumina spheres. The sample is contained in a 7.3-cm inner-diameter copper tube. The absorption coefficient of this sample was determined from measurements of the exponential attenuation coefficient of transmission, but the transport mean free path was unknown. As a reference medium with known parameters we used a random collection of 1.27-cm polystyrene spheres at a $f = 0.56$.[4] The experimental arrangement is shown in Fig. 1. K-band radiation is produced by an Alfred microwave oscillator emitted from a horn placed 20 cm in front of the combined sample. The incident radiation impinges upon the polystyrene reference medium, whose thickness is varied. The thickness of the alumina/polypropylene test medium was fixed at $L_2 = 15$ cm. The signal is detected by using a Schottky diode detector. The microwave amplitude is modulated at 2 kHz, and the signal is measured by using a lock-in detector. The variation of the transmission through the combined sample with L_1 for two different frequencies is shown in Fig. 2. We fitted Eq. (6) to the data using only a single fitting parameter, ℓ_2. From a comparison of such measurements of the transport mean free path and independent frequency domain measurements of the diffusion coefficient it is possible to determine the transport velocity in the presence of sphere resonances.[7] The values of the input parameters and of the mean free paths determined from the fit to the data shown in Fig. 2 are given in Table 1.

In conclusion, we have derived an expression for the relative transmission and reflection coefficients of a structure composed of two adjoining random media. When the parameters for one of the media are known, it is possible to find the mean free path of the other even though the internal reflection coefficients at the sample's boundaries are not accurately known. Once the mean free path is known the reflection coefficient can be obtained from measurement of transmission.[5] The method is illustrated with microwave measurements but could be readily applied to optical measurements of the propagation properties of a slab by use of a wedged reference sample with known propagation parameters to produce a composite sample of variable length when the slab and the wedge are juxtaposed.

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