



























spaser synchronization with an external field – the Arnold tongue. Our semiclassical model also predicts the possibility of loss compensation by a spaser operating below threshold.

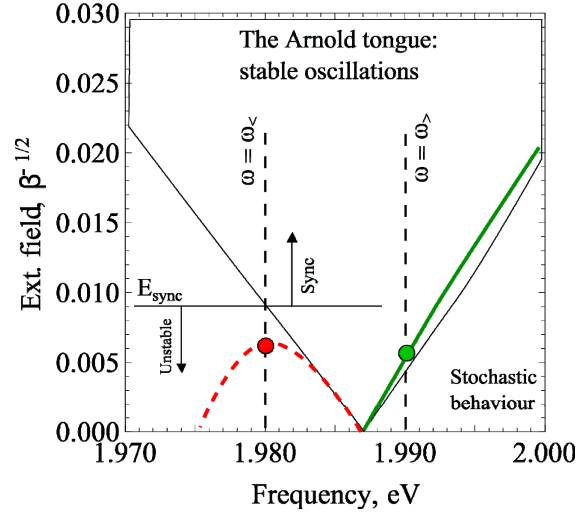


Fig. 4. Stable (filled) and unstable regions of a driven above-threshold spaser for  $D_0 = 0.25$ . The region of stability corresponds to steady-state solutions with  $\text{Re} \Lambda < 0$ . The two vertical lines correspond to the frequencies  $\omega_>$  and  $\omega_<$ , which are also used in Fig. 2. The red and green circles correspond to those in Fig. 2 (c) and 2(d). The red circle, in which  $\text{Im}[d] = 0$ , lies in the instability region, so loss compensation does not occur, while the green circle lies in the stable region and compensation does take place. The red and green curves show the compensation curves lying in unstable and stable regions, respectively, along which the imaginary part of dipole moment turns to zero.

Our model also reveals inconsistencies in linear models of nanolasers. In particular, the authors of [39] consider a metal-coated nanolaser and report a lasing turn-off above the threshold. This result is in disagreement with the experimental observation of spasing in core-shell nanolasers and with general theory of lasers [44]. The authors make a suggestion that the on/off behavior of lasing in coated nanoparticles is caused by detuning of the resonance when the gain is added. However, in our study we show that the spasing frequency does not depend on gain and is a function of nanolaser geometry only (see Eq. (12)).

Using our model, it is interesting to look at the discussion concerning the possibility of loss compensation in plasmonic systems with gain (see [52] and comments to this work). In paper [52], Stockman argues that in a resonant plasmonic structure Ohmic losses are compensated for by gain when spasing occurs. Indeed, this argument is valid for a closed system in which there is no incoming and outgoing radiation. In this case, loss compensation and lasing simply coincide.

In an open system coupled with the radiation, it is necessary to compensate for both Ohmic and radiation losses for spasing to occur. In this case, as we show above for a spaser operating below threshold, lossless scattering of an incoming wave may occur when the system does not spase. This happens because the magnitude of dipole oscillations is smaller than that in the above-threshold spaser and the pumping energy is, therefore, sufficient to compensate for the loss. This situation is analogous to the scheme suggested in [9], in which Ohmic losses in the illuminated photonic crystal composed of alternating metallic and dielectric amplifying layers are compensated below the lasing threshold.

### Acknowledgments

The authors would like to thank A. A. Pukhov for helpful discussion. They also are indebted to R. E. Noskov for helpful discussion of Eq. (22). This work was supported by RFBR Grants Nos. 12-02-01093, 13-02-00407 by a PSC-CUNY grant and by the Dynasty Foundation.