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Generalized renormalization scheme in the Ginzburg-Landau-Wilson model

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For the n-vector model of the general type a renormalization-group (RG) scheme in momentum space is developed. Using this scheme we obtain a generalized *exact* equation of the renormalization group. The main idea of this approach is not to allow the zero-order critical Hamiltonian to move away from the critical surface under the influence of the RG transformation. The equation found differs from the known ones in that all redundant operators are excluded. On the basis of the developed approach we argue that critical exponents are functionals of some function introducing the cutoff into the phase-transition theory.

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I. INTRODUCTION

The renormalization group (RG) has proved to be the most useful technique for the theoretical investigation of the condensed state in the critical region (for review see, e.g., Refs. [1–4]). The theory not only elucidated the essence of the phenomenon, but also allowed one to calculate critical asymptotics; the accuracy of these calculations is in some cases higher than that of experiments [5–8]. The explanation of some obscure effects, as well as the prediction of a number of phenomena later discovered experimentally (see, e.g., Ref. [9]), is another important merit of the theory.

Basically there are two distinct kinds of approaches in the momentum-space RG theory: (i) the approaches that use different types of perturbation theory [10–14], and (ii) the approximation schemes [15–18] that exploit the exact RG equation obtained by Wilson and Kogut [19] (see also Refs. [20,21]). These approaches, as well as ones based on a perturbation theory, have led to very good values of critical exponents [22]. Another important advantage of using exact RG equations is that they may be useful for a foundation of perturbation methods used in RG theory.

In this paper we obtain a generalized exact RG equation. This equation contains an arbitrary function which can be used to simplify the initial Ginzburg-Landau-Wilson functional, eliminate redundant operators, and introduce a small parameter for the equation. The obtained equation can serve as a basis for constructing new approximation schemes in the theory of critical phenomena. One of these schemes, using the exponent η as a small parameter, was developed by authors in Ref. [23].

II. GENERAL RELATIONS

The Wilson functional RG equation for the Landau-type Hamiltonians H can be schematically written as [19]

$$\frac{dH}{dl}=\hat{R}\{H\}\ , \tag{2.1}$$

where $\hat{R}$ stands for some nonlinear operator performing RG transformations. The Hamiltonian H at the critical point of a phase transition is determined by

$$\hat{R}\{H^*\}=0\ , \tag{2.2}$$

and is called a *fixed-point Hamiltonian.* This term, however, has a much wider meaning because not every fixed-point Hamiltonian describes critical behavior. We will use the term *critical Hamiltonian,* which is more restrictive. Every critical Hamiltonian belongs to the set of fixed-point Hamiltonians, but the converse is not true. We may move from the critical point by choosing $H=H^*+\Delta H$ using two distinct ways. If ΔH has nonzero projection on the relevant direction [21], the transformation (2.1) will lead away from the critical point with increasing l. In the other case ΔH will eventually tend to zero. In the latter case a subset of Hamiltonians is introduced which, after undergoing an RG transformation, will turn into critical ones. Traditionally, this subset is called the *critical surface.* The term should be interpreted as a surface in the space defined by all the parameters needed to close an RG transformation. For instance, the most general Ginzburg-Landau-Wilson functional for a translationally invariant isotropic system is

$$H_l[\vec{\phi}]=\sum_{k=0}^{\infty}2^{1-2k}\int_{q_1,q_{\bar{1}};\dots;q_k,q_{\bar{k}}} g_k(\mathbf{q}_1,\mathbf{q}_{\bar{1}};\dots;\mathbf{q}_k,\mathbf{q}_{\bar{k}}) \times(2\pi)^d\delta\left[\sum_{i=1}^{k}(\mathbf{q}_i+\mathbf{q}_{\bar{i}})\right] \times\prod_{i=1}^{k}[\vec{\phi}(\mathbf{q}_i)\cdot\vec{\phi}(\mathbf{q}_{\bar{i}})]\ , \tag{2.3}$$

and the vertices $g_k(\mathbf{q}_1,\mathbf{q}_{\bar{1}};\dots;\mathbf{q}_k,\mathbf{q}_{\bar{k}})$, which are invariant with respect to permutations among pairs of momenta $(\mathbf{q}_i,\mathbf{q}_{\bar{i}})$ and $(\mathbf{q}_j,\mathbf{q}_{\bar{j}})$ and elements within pairs, are the requisite space defining parameters. In Eq. (2.3) $\vec{\phi}$ is an n-component vector,

$$\delta(\mathbf{q})=(2\pi)^{-d}\delta_{q,0}V, \int_q = V^{-1}\sum_q = \int \frac{d^dq}{(2\pi)^d} ,$$

and V is the volume of the d-dimensional system.

By considering a small deviation from the fixed-point Hamiltonian one can distinguish different subsets in the Hamiltonian space defined by Eq. (2.3). Equation (2.1) gives for a linear deviation

$$\frac{\partial \Delta H}{\partial l}=\hat{L}\Delta H , \tag{2.4}$$

where $\hat{L}$ is formally defined as the operator $\delta\hat{R}\{H\}/\delta H$ taken at the point $H=H^*$. The solution of Eq. (2.4) can be obtained with the help of the eigenvectors O_λ of the operator $\hat{L}$

$$\Delta H=\sum_\lambda \mu_\lambda e^{\lambda l}O_\lambda , \tag{2.5}$$

where λ are the eigenvalues for the eigenvectors O_λ, and μ_λ are the expansion coefficients called the scaling fields [24].

Depending on the sign of λ the eigenvectors are classified as relevant ($\lambda>0$), irrelevant ($\lambda<0$), and marginal ($\lambda=0$). The relevant direction mentioned above is defined (for critical behavior) by the only eigenvector having positive λ. Unfortunately, conventional definitions of a particular RG transformation allow the appearance in Eq. (2.5) of more than one eigenvector having $\lambda>0$. Only the one defining the critical exponent ν is physically meaningful, the others should be treated as redundant with no physical meaning. Wegner [21] suggested some criteria to distinguish the physical operators. According to Wegner, those operators whose eigenvalues are dependent on a particular choice of the RG transformation should be treated as redundant. These operators must be excluded from the RG procedure with the help of some additional conditions. In this paper we show that there is a formulation of the exact RG equation which leaves no room for additional conditions and, therefore, it should not contain redundant operators.

Let us assume that $d>4$. Then any of the Hamiltonians

$$H_0[\vec{\phi}]=\tfrac{1}{2}\int_q G_0^{-1}(q,\Lambda)|\vec{\phi}(\mathbf{q})|^2 , \tag{2.6}$$

with the propagator G_0 defined by

$$G_0(q,\Lambda)=q^{-2}S(q^2/\Lambda^2) ,$$

are critical provided the function $S(x)$ is monotonous with $S(x=0)=1$ and $\lim_{x\to\infty}S(x)x^m=0$, for any m. The function $S(x)$ provides a momentum cutoff on a momentum Λ. Hence there should be such a renormalization procedure for which Eq. (2.2) is satisfied independently of a particular choice of the function $S(x)$. Now, let us decrease the dimension to $d<4$; then Hamiltonians (2.6) are no longer critical, but they are still fixed-point Hamiltonians for the chosen definition of the RG transformation. Therefore, when trying to find critical Hamiltonians for $d<4$, one should write

$$\frac{d(H_0+H_I)}{dl}\equiv\frac{dH_I}{dl}=\hat{R}\{H_0+H_I\}\equiv\bar{R}\{H_I\} . \tag{2.7}$$

Until now the operator $\hat{R}$ has not been defined. For practical purposes its definition is not important. Below we find the operator $\bar{R}$.

III. RG EQUATION

To derive an RG equation, one has, first of all, to perform an integration over short-wave modes in a partition function. For this purpose, let us note that the partition function of a system with the functional $H=H_0+H_I$ can be written in the form

$$Z=\int D\vec{\phi}\exp(-H[\vec{\phi}])=Z_0\langle\exp(-H_I[\vec{\phi}])\rangle_{0,\Lambda} \equiv Z_0\langle w[\vec{\phi}]\rangle_{0,\Lambda} , \tag{3.1}$$

where

$$Z_0=\int D\vec{\phi}\exp(-H_0[\vec{\phi}]) , \tag{3.2}$$

and averaging $\langle\ \rangle_{0,\Lambda}$ is performed with the Gaussian functional at a given value of Λ.

The following considerations are based on the fact that the averaging over a Gaussian field $\vec{\phi}$ can be replaced by two independent averages over Gaussian fields $\vec{\phi}_1$ and $\vec{\phi}_2$, providing $\vec{\phi}=\vec{\phi}_1+\vec{\phi}_2$ and the sum of the correlators $G_{01}(q,\Lambda_1)=\langle|\vec{\phi}_1(\mathbf{q})|^2\rangle_{0,\Lambda_1}$ and $G_{02}(q,\Lambda_2)=\langle|\vec{\phi}_2(\mathbf{q})|^2\rangle_{0,\Lambda_2}$ is equal to the correlator of the initial field

$$G_0(q,\Lambda)=\langle|\vec{\phi}(\mathbf{q})|^2\rangle_{0,\Lambda}=G_{01}(q,\Lambda_1)+G_{02}(q,\Lambda_2) . \tag{3.3}$$

In other words,

$$\langle w\rangle_0\equiv Z_0^{-1}\int D\vec{\phi}\exp\left[-1/2\int_q G_0^{-1}(\mathbf{q},\Lambda)|\vec{\phi}(\mathbf{q})|^2\right]w[\vec{\phi}]$$

$$=Z_{01}^{-1}Z_{02}^{-1}\int D\vec{\phi}_1 D\vec{\phi}_2\exp\left[-1/2\int_q G_{01}^{-1}(\mathbf{q},\Lambda_1)|\vec{\phi}_1(\mathbf{q})|^2-1/2\int_q G_{02}^{-1}(\mathbf{q},\Lambda_2)|\vec{\phi}_2(\mathbf{q})|^2\right]w[\vec{\phi}_1+\vec{\phi}_2] , \tag{3.4}$$

where

$$Z_{0i}=\int D\vec{\phi}\exp\left[-1/2\int_q G_{0i}^{-1}(\mathbf{q},\Lambda_i)|\vec{\phi}(\mathbf{q})|^2\right] \tag{3.5}$$

and functions $G_0(q,\Lambda)$ and $G_{0i}(q,\Lambda_i)$ satisfy Eq. (3.3).

Let us choose $\vec{\phi}_1$ so that $G_{01}(q,\Lambda_1)=G_0(q,(1-\xi)\Lambda)$, where $\xi \ll 1$. Then the value G_{02} is of the order of ξ,

$$G_{02}(q,\Lambda_2)=G_0(q,\Lambda)-G_{01}(q,\Lambda_1)\simeq\xi\Lambda\frac{\partial G_0(q,\Lambda)}{\partial\Lambda}$$

$$\equiv 2\xi h(q) , \tag{3.6}$$

$$h(q)=q^{-2}\Lambda^2\frac{dS(q^2/\Lambda^2)}{d\Lambda^2} . \tag{3.7}$$

In this case, due to the smallness of ξ, the integration over the field ϕ_2 in Eq. (3.1) can be performed easily,

$$Z=Z_0\langle w[\vec{\phi}]\rangle_{0,\Lambda}=Z_{01}^{-1}Z_{02}^{-1}\int D\vec{\phi}_1 D\vec{\phi}_2\exp\left[-1/2\int_q G_{01}^{-1}(\mathbf{q},\Lambda_1)|\vec{\phi}_1(\mathbf{q})|^2-1/4\xi^{-1}\int_q h^{-1}(q)|\vec{\phi}_2(\mathbf{q})|^2\right]$$

$$\times\left\{w[\vec{\phi}_1]+\frac{1}{2}\frac{\delta^2 w[\vec{\phi}_1]}{\delta\phi_1^\alpha(\mathbf{q})\delta\phi_1^\beta(\mathbf{p})}\delta\phi_2^\alpha(-\mathbf{q})\delta\phi_2^\beta(-p)\right\}=Z_0\langle(1+\xi\hat{L}_A)w[\vec{\phi}]\rangle_{0,\Lambda(1-\xi)} . \tag{3.8}$$

The operator $\hat{L}_A$ in Eq. (3.8) is defined by the relationship

$$\hat{L}_A=\int_q h(q)\frac{\delta^2}{\delta\vec{\phi}(\mathbf{q})\cdot\delta\vec{\phi}(-\mathbf{q})} . \tag{3.9}$$

The RG transformation will be completed by changing the momentum scale in order to restore the initial value of the parameter Λ: $\mathbf{q}=\mathbf{q}'(1+\xi)$. In order to restore the initial functional H_0 one should also transform the field $\vec{\phi}(\mathbf{q})$,

$$\vec{\phi}(\mathbf{q})=[1+\xi\epsilon_\phi(q')]\vec{\phi}(\mathbf{q}')$$

$$=\left[1+\xi\left[\epsilon_\phi(\mathbf{q})+\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{q}}\right]\right]\vec{\phi}'(\mathbf{q}) . \tag{3.10}$$

If one chooses $\epsilon_\phi(\mathbf{q})=(d+2)/2$ then the functional H_0 will be restored. This results in the following equation

$$w'[\vec{\phi}]=[1+\xi(\hat{L}_A+\hat{L}_B+\hat{L}_V)]w[\vec{\phi}] , \tag{3.11}$$

where

$$\hat{L}_B=\left[\frac{d+2}{2}+\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{q}}\right]\vec{\phi}(\mathbf{q})\cdot\frac{\delta}{\delta\vec{\phi}(\mathbf{q})} , \tag{3.12}$$

and the operator

$$L_V=dV\frac{\partial}{\partial V}$$

appears due to the transformation of the volume of the system.

From Eq. (3.11), using $w=\exp(-H_I)$, we obtain

$$\dot{H}_I[\vec{\phi}]=(\hat{L}_A+\hat{L}_B+\hat{L}_V)H_I[\vec{\phi}]$$

$$-\int_q h(q)\frac{\delta H_I[\vec{\phi}]}{\delta\vec{\phi}(\mathbf{q})}\cdot\frac{\delta H_I[\vec{\phi}]}{\delta\vec{\phi}(-\mathbf{q})} ,$$

or finally,

$$\dot{H}_I[\phi]=dV\frac{\partial H_I}{\partial V}+\int_q\left[\frac{d+2}{2}\vec{\phi}(\mathbf{q})+\mathbf{q}\cdot\frac{\partial\vec{\phi}(\mathbf{q})}{\partial\mathbf{q}}\right]\cdot\frac{\delta H_I[\vec{\phi}]}{\delta\vec{\phi}(\mathbf{q})}$$

$$+\int_q h(\mathbf{q})\left[\frac{\delta^2 H_I[\vec{\phi}]}{\delta\vec{\phi}(\mathbf{q})\cdot\delta\vec{\phi}(-\mathbf{q})}-\frac{\delta H_I[\vec{\phi}]}{\delta\vec{\phi}(\mathbf{q})}\cdot\frac{\delta H_I[\vec{\phi}]}{\delta\vec{\phi}(-\mathbf{q})}\right] . \tag{3.13}$$

Equation (3.13) is an exact RG equation. It differs slightly from the equation obtained in Refs. [19–21]. One can obtain all known results of RG theory using this equation. However, this equation is not convenient for practical uses. As with Wilson's equation, it contains redundant operators that should be properly excluded to obtain results having physical meaning.

The RG equation (3.13) found does not determine the operator $\bar{R}$ [see Eq. (2.7)]. This equation generates terms with the vertex $g_1(\mathbf{q})$ [see Eq. (2.3)] that must be incorporated into the Hamiltonian H_0. Therefore, H_0 changes under the RG transformation, while the choice of H_0 defines the operator $\bar{R}$ and must not undergo any changes with the RG transformation. The latter can be achieved if instead of restoring the functional H_0 one reduces the functional $H_0+\delta H_0$ to H_0, at each step of the RG process with the help of some scale transformation of the field $\vec{\phi}(\mathbf{q})$. A few words should be said about the addition δH_0. This term should include only momentum-dependent parts of the vertex $g_1(\mathbf{q})$ because the term with $g_1(0)\equiv g_{10}$ contains a projection on the relevant direction and, therefore, it cannot be incorporated into the functional H_0.

Let us define the function $\epsilon_\phi(q)$ introduced in Eq. (3.10) as

$$\epsilon_\phi(q)=\frac{d+2}{2}-\eta(q) . \tag{3.14}$$

Then instead of Eq. (3.11) we have

$$w'[\vec{\phi}] = \left[1+\xi\left[\hat{L}_A+\hat{L}'_B+\hat{L}_V+\frac{1}{2}\int_q \eta(q)G_0^{-1}(q,\Lambda)|\vec{\phi}(\mathbf{q})|^2 - \frac{V}{2}\int_q \eta(q)\right]\right]w[\vec{\phi}] , \tag{3.15}$$

where the operator $\hat{L}'_B$ is defined as

$$\hat{L}'_B=\int_q \left[\frac{d+2-\eta(q)}{2}\vec{\phi}(\mathbf{q})+\mathbf{q}\cdot\frac{\partial\vec{\phi}(\mathbf{q})}{\partial\mathbf{q}}\right]\cdot\frac{\delta}{\delta\vec{\phi}(\mathbf{q})} . \tag{3.16}$$

Using Eqs. (3.15), (3.1), (3.9), and (3.16) we find

$$\dot{H}_I[\vec{\phi}]=Vd\frac{\partial H_I[\vec{\phi}]}{\partial V}+\frac{V}{2}\int_q \eta(q)-\tfrac{1}{2}\int_q \eta(q)G_0^{-1}(q,\Lambda)|\vec{\phi}(\mathbf{q})|^2 + \int_q \left[\frac{d+2-\eta(q)}{2}\vec{\phi}(q)+\mathbf{q}\cdot\frac{\partial\phi(q)}{\partial\mathbf{q}}\right]\cdot\frac{\delta H_I[\vec{\phi}]}{\delta\vec{\phi}(\mathbf{q})}+\int_q h(q)\left[\frac{\delta^2 H_I[\vec{\phi}]}{\delta\vec{\phi}(\mathbf{q})\cdot\delta\vec{\phi}(-\mathbf{q})}-\frac{\delta H_I[\vec{\phi}]}{\delta\vec{\phi}(-\mathbf{q})}\cdot\frac{\delta H_I[\vec{\phi}]}{\delta\vec{\phi}(\mathbf{q})}\right]\equiv\hat{R}_g\{H_I\} . \tag{3.17}$$

Again we have obtained the exact RG equation. This equation is a generalized form of Eq. (3.13), which enables us to carry out the reduction of $H_0+\delta H_0$ to the H_0. Equation (3.17) contains an arbitrary function $\eta(q)$ that accounts for this.

The following steps are of crucial importance and quite clear. First of all, let us separate terms of zeroth and first order in ϕ^2 in the functional H_I,

$$H_I[\vec{\phi}]=bV+\tfrac{1}{2}\int_q g_1(q)|\vec{\phi}(\mathbf{q})|^2+H' . \tag{3.18}$$

Then, using Eq. (3.17) for the functional (2.3), one obtains the simple equation for a renormalization of the constant b,

$$\dot{b}=db+\tfrac{1}{2}\int_q \eta(q)+n\int_q h(q)g_1(q) . \tag{3.19}$$

The equation for $g_1(q)$ has the form

$$\dot{g}_1(q)=-\eta(q)G_0^{-1}(q,\Lambda)+\left[2-\eta(q)-2q^2\frac{\partial}{\partial q^2}\right]g_1(q) +Q(q)-2h(q)g_1^2(q) , \tag{3.20}$$

where the function $Q(q)$ is given by

$$Q(q)=\int_q h(p)[ng_2(-\mathbf{p},\mathbf{p};-\mathbf{q},\mathbf{q})+2g_2(-\mathbf{p},\mathbf{q};\mathbf{p},-\mathbf{q})] . \tag{3.21}$$

Now let us separate the part dependent on momenta from the vertex $g_1(q)=g_{10}+g'_1(q)$. Equations for g_{10} and $g'_1(q)$ can be written in the form

$$\dot{g}_{10}=[2-\eta(0)]g_{10}+Q(0)-2h(0)g_{10} , \tag{3.22}$$

$$\dot{g}'_1(q)=-\eta(q)G_0^{-1}(q,\Lambda)-[\eta(q)-\eta(0)]g_{10} +Q(q)-Q(0)-2[h(q)-h(0)]g_{10} +2q^2\frac{\partial g'_1(q)}{\partial q^2}-2h(q)g_1'^2(q) . \tag{3.23}$$

Our objective is to eliminate the generation of the q-dependent part of the vertex g_1 in the RG equation provided that its initial value does not depend on momentum [i.e., $\dot{g}'_1(q)=0$, $g'_1(q)=0$]. Using Eq. (3.23), we can easily find the function $\eta(q)$ which ensures such behavior,

$$\eta(q)=\eta(0)-\frac{Q(q)-Q(0)-2g_{10}[h(q)-h(0)]+\eta(0)G_0^{-1}(q,\Lambda)}{G_0^{-1}(q,\lambda)-g_{10}} . \tag{3.24}$$

In addition, if we demand that

$$\eta(0)=\frac{d}{dq^2}[Q(q)-2h(q)g_{10}]_{q=0} , \tag{3.25}$$

then the expansion of $\eta(q)$ will start from q^4. This completes the procedure of reducing $H_0+\delta H_0$ to the H_0 simply by fixing $\delta H_0\equiv 0$. Now the function $\eta(q)$ depends on all higher vertices and has clear physical meaning: at a stable fixed point (critical behavior) $\eta^*(0)$ is equal to the Fisher exponent, i.e., at the critical point the correlation function $\langle\vec{\phi}(\mathbf{q})\cdot\vec{\phi}(-q)\rangle\propto q^{-2+\eta^*(0)}$.

IV. CONCLUSION

Finally, we have obtained the operator $\bar{R}$ as an operator $R_g\{H_I\}$ but with the function $\eta(q,l)$ defined by the Eqs. (3.24) and (3.25). Insofar as the Hamiltonian H_0 is given, this RG transformation is unique and, therefore, it does not contain any redundant operators. In comparison with the traditional RG equations [19–21,17] in the Wilson approach the developed scheme must seem to be a little cumbersome. This is not really the case because the traditional approach, having a simpler definition of the RG transformation [like the one in Eq. (3.13)], transfers the difficulties to the problem of excluding the redundant operators. In fact, the scheme suggested in this paper was already exploited by the authors to construct a new perturbation theory using $[\eta^*(0)]^{1/2}$ as a small parameter [23].

In conclusion it is interesting to compare some formal aspects of the Wilson approach and the generalized scheme developed here. In the Wilson approach one has to deal with some arbitrary function defining an RG transformation. By the procedure of excluding some of the redundant operators (in the general case we have infinite umber of redundant operators), one may restore this function. In the generalized scheme the RG procedure is defined uniquely but this definition depends on the choice of the function $S(x)$. Therefore, in this scheme all eigenvalues λ of the linear RG operator [see Eq. (2.4)] are functionals of $S(x)$ and universality is lost. The latter accounts for the introduction of a new dimensionless scale x_0. This is the scale on which the function $S(x)$ goes to zero with increasing x. The loss of universality is not a big problem because it can be immediately restored simply by letting $x_0 \to \infty$. In this limit all the RG defining Hamiltonians give the same results. Nonetheless, this argumentation shows that if one calculates exactly the critical exponents for $d=2$ and $n=1$ they might not coincide with the exponents of the Ising model. This accounts for the fact that Ising model keeps H_0 (or x_0) fixed, while traditionally we put $x_0 \to \infty$. The latter, of course, should be considered not as a strictly proved fact, but rather as the authors' supposition.

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