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Influence of internal reflection on correlation of intensity fluctuation in random media

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The correlation function of electromagnetic-intensity fluctuations is calculated in the diffusive regime in the presence of absorption using the Langevin approach. The results show that reflection of the boundary may cause drastic changes in the behavior of the correlation function. In the case of low absorption the spatial-correlation function undergoes a crossover from a quadratic to a linear falloff with separation as the reflection increases. The autocorrelation function for a medium with highly reflecting boundaries varies with frequency shift as $(\Delta\omega)^{-1}$ instead of $(\Delta\omega)^{-1/2}$ for boundaries with low reflection.

I. INTRODUCTION

The diffusion approximation works surprisingly well in most cases as a description of wave propagation through random media.[1,2] This approximation is valid in the case of weak disorder when the wavelength of the radiation is much smaller than the elastic mean free path l. It provides a clear and physically meaningful explanation of many aspects of wave propagation in random media. However, some experimental results reveal systematic deviation from theoretical predictions. It was only recently realized that such discrepancies arise because internal reflection at the boundaries of the random medium, which is present in almost every experiment due to the index mismatch, is not taken into account. Lagendijk *et al.*[4] suggested that inclusion of surface reflection into the diffusion propagator can lead to better agreement between experiment and theory for pulsed radiation propagating in dense random media. Freund *et al.*[5,6] have demonstrated that taking internal reflection into account substantially reduces the discrepancies between theoretical predictions and measurements of the optical memory effect. Garcia *et al.*[7] measured the effect of boundary reflection on the time-of-flight distribution and the transmission coefficient in a random sample. The angular correlation functions in the case of high-index mismatch were measured by Zhu *et al.*[8] They also found that by including the effect of internal reflection it is possible to obtain agreement between experiment and theory. The role of internal reflection in coherent backscattering was demonstrated by Saulnier and Watson.[9] In measurements of the intensity inside and the transmission through a random slab with different reflectors at the boundary, Garcia *et al.*[10] determined the photon mean-free-path and demonstrated an excellent agreement between theory and experiment.

Usually diffusion theory considers boundaries of a random medium as perfectly absorbing. However, it is incorrect to take zero intensity at the boundaries for diffusing photons inside the medium.[1,3] To resolve this problem, an extrapolation length beyond the boundary at which the intensity drops to zero is generally introduced. If this length, which is of the order of l for zero boundary reflection, is much less than the thickness of the random media L, then internal reflection can be neglected and the intensity of the boundary can be assumed to be zero. However, when the reflection coefficient $\mathfrak{R}$ is large, the extrapolation length can become comparable and even greater than L.[7,10]

The main goal of the present paper is to study the effect of internal reflection upon the spatial and the spectral intensity-intensity correlation functions. Correlation of classical waves has been the subject of intensive experimental[7,11–20] and theoretical[20–33] study in the last few years. Feng *et al.*[26] have shown that the intensity-intensity correlation function $C(\mathbf{r},\mathbf{r}')$ consists of three parts: short range C_1, long range C_2, and "infinite range" C_3. It has now been established that the short-range correlation function[14,15,21,28,30] exhibits exponential decay with increasing separation between points $\Delta r=|\mathbf{r}-\mathbf{r}'|$ or frequency shift $\Delta\omega=|\omega-\omega'|$. The long-range contribution C_2 also decays but as a power law rather than exponentially.[22,28] Long-range correlation as a function of frequency has been observed in microwave[13,17–19] and optical[16] measurements. The C_3 term is found to be somewhat analogous to the constant background.[19] The effect of absorption on long-range correlations was studied in recent papers by Pnini and Shapiro[31] and by Kogan and Kaveh.[32] They found that in agreement with experiments by Genack *et al.*,[17] these correlations continue to exist even for distances much larger than absorption length. Absorption modifies but does not completely destroy long-range correlation in the intensity. The frequency-correlation function in the presence of absorption was also derived by de Boer *et al.*[20] for different incident-beam profiles.

In recent theoretical studies of correlation phenomena, the Langevin approach proved to be useful for obtaining long-range spatial and spectral correlation functions. It was first proposed by Zyuzin and Spivak for a study of the conductance fluctuations[24] and then applied to classical waves.[25] This method was later successfully used by a number of authors[20,28,31,32] for photon intensity correlations in random media. According to this approach, the fluctuating part of the intensity $\delta I=I-\langle I\rangle$ can be found as a solution of the diffusion equation with a random source. In all the above-mentioned studies of intensity correlations it was assumed that boundaries are total-

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ly absorbing. In the present study we show that taking account of the internal reflection leads to qualitative changes in both spatial and spectral correlation functions. These changes are due to the "surface" term in the correlation function which is due to fluctuations on the surface of the media. We show that in the case of low internal reflection, the surface term becomes small and we recover previous results.[31,32] In the case of high internal reflection, however, the surface term dominates the correlation function and we obtain completely new behavior for both spatial and spectral correlation functions.

II. BASIC EQUATIONS

We consider a wave equation for a scalar monochromatic field $\psi_\omega(\mathbf{r})$ of frequency ω,

$$\{\nabla^2+k^2[1+\varepsilon(\mathbf{r})]+ik/l_a\}\psi_\omega(\mathbf{r})=0 , \tag{1}$$

where l_a is an absorption length, $k=\omega/c$, c is a speed of propagation in the random medium, and $\varepsilon(\mathbf{r})$ is the fluctuating part of the refraction index. $\varepsilon(\mathbf{r})$ is assumed to be a Gaussian random variable with zero mean,

$$\langle\varepsilon(\mathbf{r})\rangle=0;\quad \langle\varepsilon(\mathbf{r})\varepsilon(\mathbf{r}')\rangle=\varepsilon\delta(\mathbf{r}-\mathbf{r}') . \tag{2}$$

We start our calculations from the Bethe-Salpeter equation for the field-field correlation function[28]

$$\begin{aligned}\langle\psi_\omega(\mathbf{r})\psi^*_{\omega'}(\mathbf{r}')\rangle=&\langle\psi_\omega(\mathbf{r})\rangle\langle\psi^*_{\omega'}(\mathbf{r}')\rangle\\&+\int d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4\tilde{G}_\omega(\mathbf{r},\mathbf{r}_1)\tilde{G}^*_{\omega'}(\mathbf{r}',\mathbf{r}_2)\\&\times U(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\langle\psi_\omega(\mathbf{r}_3)\psi^*_{\omega'}(\mathbf{r}_4)\rangle ,\end{aligned} \tag{3}$$

where $\tilde{G}_\omega$ is the average Green's function of Eq. (1), U is the irreducible four-point vertex function, and the integration is taken over the scattering volume occupied by the disordered medium. The function $\tilde{G}_\omega(\mathbf{r},\mathbf{r}')$ in Eq. (3) has been calculated for infinite medium by many authors[21,22,28] and has the form

$$\tilde{G}_\omega(\mathbf{r},\mathbf{r}')=\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\exp\langle ik|\mathbf{r}-\mathbf{r}'|-|\mathbf{r}-\mathbf{r}'|/2\tilde{l}\rangle , \tag{4}$$

where $\tilde{l}=(l^{-1}+l_a^{-1})^{-1}$. The term $\langle\psi_\omega\rangle\langle\psi^*_{\omega'}\rangle$ in Eq. (3) is exponentially small when both $\mathbf{r}$ and $\mathbf{r}'$ are taken in the bulk of the medium and is usually neglected.[20,28] An alternative approach is to consider this term as a source function of photon intensity $Q_{\omega\omega'}(\mathbf{r})$ located near the input boundary at a distance of the order of the mean free path. In the weakly scattering regime, $kl\gg1$, in the leading order in $(kl)^{-1}$, the irreducible vertex function $U(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)$ can be taken as

$$U(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=\frac{4\pi}{l}\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4) . \tag{5}$$

The Bethe-Salpeter equation (3) for the field-field autocorrelation function $\langle I_{\omega\omega'}(\mathbf{r})\rangle=\langle\psi_\omega(\mathbf{r})\psi^*_{\omega'}(\mathbf{r})\rangle$ can then be reduced to the differential equation[28]

$$\nabla^2\langle I_{\omega\omega'}(\mathbf{r})\rangle-(\alpha^2-2i\beta)\langle I_{\omega\omega'}(\mathbf{r})\rangle=-\frac{1}{D}Q_{\omega\omega'}(\mathbf{r}) , \tag{6}$$

where $D=\frac{1}{3}cl$ is the diffusion coefficient, $\alpha=(ll_a)^{-1/2}$, and $\beta^2=(\omega-\omega')/2D$. This equation is valid as long as $l/l_a\ll1$ and $\beta l\ll1$, otherwise $\langle I_{\omega\omega'}(\mathbf{r})\rangle$ changes with $\mathbf{r}$ too rapidly for the above expansion to hold. If $\omega'=\omega$, Eq. (6) reduces to the conventional diffusion equation for the photon intensity $\langle I_\omega(\mathbf{r})\rangle=\langle|\psi_\omega(\mathbf{r})|^2\rangle$,

$$\nabla^2\langle I_\omega(\mathbf{r})\rangle-\alpha^2\langle I_\omega(\mathbf{r})\rangle=-\frac{1}{D}Q(\mathbf{r}) . \tag{7}$$

In order to solve Eq. (6) we first need to specify the boundary conditions and second, to define the source function $Q_{\omega\omega'}(\mathbf{r})$. We consider a random sample located along the z axis between $0<z<L$ with reflection coefficients $\Re$ at the input and output boundaries. We assume that the coherent radiation incident on the sample becomes randomized within a distance z_p, which is of the order of mean free path. We replace the incoming coherent flux by a source of diffusive radiation at the plane $z=z_p$ with a strength equal to the incident flux. z_p is viewed as an adjustable parameter of the theory. For the sake of the simplicity we consider a *plane wave* incident on the sample surface. In this case, the source function can be written as $Q_{\omega\omega'}(\mathbf{r})=q\delta(z-z_p)$, where q is a constant.

Since we assume that there is no incoming flux through the boundaries, the only flux going from the boundary towards the inside of the slab is the reflected part of the outgoing flux. This gives boundary conditions in the form[7,8]

$$\begin{aligned}&J_+(z=0^+)=-RJ_-(z=0^+) ;\\&J_-(z=L^-)=-RJ_+(z=L^-) ,\end{aligned} \tag{8}$$

where J_+ and J_- are diffusive fluxes in the positive and negative z directions, respectively. Using the relationship between the diffusive flux and the photon intensity,[1]

$$J_\pm(\mathbf{r})=\frac{\langle I(\mathbf{r})\rangle c}{4}+\frac{D}{2}\frac{\partial\langle I(\mathbf{r})\rangle}{\partial z} , \tag{9}$$

the boundary conditions of Eq. (8) can be rewritten as[7,8]

$$\begin{aligned}&\left[\frac{1}{z_0}\langle I(z)\rangle-\frac{d}{dz}\langle I(z)\rangle\right]_{z=0^+}=0 ;\\&\left[\frac{1}{z_0}\langle I(z)\rangle+\frac{d}{dz}\langle I(z)\rangle\right]_{z=L^-}=0 ,\end{aligned} \tag{10}$$

where

$$z_0=\frac{2}{3}l\frac{1+\Re}{1-\Re} . \tag{11}$$

The solution of Eq. (6) can be written in the form

$$\langle I_{\omega\omega'}(z)\rangle=(q/D)G_{\Delta\omega}(z,z_p) , \tag{12}$$

where $G_{\Delta\omega}(z,z')$ is the Green's function of Eq. (6) with the boundary conditions of Eq. (10),

$$G_{\Delta\omega}(z,z')=\frac{1}{a(\Delta\omega)}\left[[1+a^2(\Delta\omega)z_0^2]\sinh[a(\Delta\omega)L]+2a(\Delta\omega)z_0\cosh[a(\Delta\omega)L]\right]^{-1}\begin{cases}P_{\Delta\omega}(z)P_{\Delta\omega}(L-z'); & z<z'\\ P_{\Delta\omega}(z')P_{\Delta\omega}(L-z); & z>z'\end{cases} \tag{13}$$

where

$$P_{\Delta\omega}(x)=\sinh[a(\Delta\omega)x]+a(\Delta\omega)z_0\cosh[a(\Delta\omega)x]\ ;$$
$$a^2(\Delta\omega)=\alpha^2-2i\beta^2\ . \tag{14}$$

When $\Delta\omega=0$, Eq. (12) gives diffusive intensity distribution inside the slab and we can check the existence of the extrapolation length by solving the following equations:

$$\langle I_{\omega=\omega'}(z=-z_b)\rangle|_{z<0}=0\ ;$$
$$\langle I_{\omega=\omega'}(z=L-z_b)\rangle|_{z>L}=0\ . \tag{15}$$

We find that when $\Re<\Re_c$, defined as

$$\Re_c=\frac{1-\frac{2}{3}\alpha l}{1+\frac{2}{3}\alpha l}\ , \tag{16}$$

Eqs. (15) have the solution

$$z_b=\frac{1}{2\alpha}\ln\left[\frac{1+\alpha z_0}{1-\alpha z_0}\right]\ . \tag{17}$$

If, however, $\Re>\Re_c$, an extrapolation length does not exist because the intensity extrapolated beyond the boundaries never becomes zero. In the absence of absorption, when $\alpha\to 0$, $z_b=z_0$ and for zero reflection boundary conditions Eqs. (10) give $z_b=2l/3$. This is consistent with transport theory which gives the more precise Milne's result, $z_0=0.7104l$.

III. FIELD-FIELD AUTOCORRELATION FUNCTION

Now we consider the squared field-field autocorrelation function defined as

$$C_1(\Delta\omega,z)=|\langle I_{\omega\omega'}(z)\rangle|^2\ , \tag{18}$$

where $\langle I_{\omega\omega'}(z)\rangle$ is given by Eq. (12). The effect of the internal reflection on the autocorrelation function can be easily seen when analyzing Eqs. (12) and (13). For low surface reflection, when $z_0\sim l$ and $z_0\alpha\ll 1, z_0\beta\ll 1$, we recover the result obtained by many authors,[20,31,32]

$$C_1(\Delta\omega;z>z_p)=\frac{q^2}{c^2}\frac{\cosh[2\gamma_+(L-z)]-\cos[2\gamma_-(L-z)]}{\cosh(2\gamma_+L)-\cos(2\gamma_-L)}\ , \tag{19}$$

where $\gamma_\pm$ are defined as

$$\gamma_\pm^2=\frac{1}{2}[(\alpha^4+4\beta^4)^{1/2}\pm\alpha^2]\ . \tag{20}$$

In the absence of absorption, C_1 decays exponentially with frequency shift $\Delta\omega$. However, this behavior changes if reflection becomes large, $z_0\beta\gg 1$. Assuming that $z_p\alpha\ll 1$ and $z_p\beta\ll 1$ still hold, we obtain

$$C_1(\Delta\omega;z>z_p)=\frac{q^2}{D^2}\frac{\sqrt{\alpha^4+4\beta^4}}{4\beta^4}\times\frac{\cosh[2\gamma_+(L-z)]+\cos[2\gamma_-(L-z)]}{\cosh(2\gamma_+L)-\cos(2\gamma_-L)}\ . \tag{21}$$

Due to the prefactor $(\alpha^4+4\beta^4)^{1/2}/\beta^4$, which in the absence of absorption is $(\Delta\omega)^{-1}$, autocorrelation function falls off much faster than in the case of small reflection.

IV. INTENSITY-INTENSITY LONG-RANGE CORRELATION FUNCTION

In this section we consider correlations of the intensity in a tube with a diameter much less than its length L. We may therefore consider the correlation function integrated over the cross section of the tube. According to the Langevin approach, intensity fluctuations $\delta I=I-\langle I\rangle$ obey the diffusion equation[21,28] with a random source $\mathbf{j}_\omega(z)$,

$$(\nabla-\alpha^2)\delta I_\omega(z)=(1/D)\nabla\cdot\mathbf{j}_\omega(z)\ . \tag{22}$$

The correlator of the random function $\mathbf{j}_\omega(z)$ has the form

$$\langle j_\omega^\alpha(z)j_{\omega'}^\beta(z')\rangle=\delta_{\alpha\beta}\frac{\pi l c^2}{3k^2A}|\langle I_{\omega\omega'}(z)\rangle|^2\delta(z-z')\ , \tag{23}$$

where A is the cross section of the tube. In order to solve Eq. (22), we apply the same boundary conditions as for Eq. (7).

The formal solution of Eq. (22) gives

$$\delta I_\omega(z)=-\frac{1}{D}[G_{\Delta\omega=0}(z;L)j_\omega(L)-G_{\Delta\omega=0}(z;0)j_\omega(0)]+\frac{1}{D}\int_0^L dz' j_\omega(z')\frac{d}{dz'}G_{\Delta\omega=0}(z;z')\ . \tag{24}$$

The first term in this expression depends upon values taken on the slab surface. Though it is usually neglected,[28,31,32] it becomes important when surface reflection is large, so we retain it below. As a result, the correlation function splits into two parts: a surface term $C^{(S)}_{\omega\omega'}$ and a volume term $C^{(V)}_{\omega\omega'}$,

$$C_{\omega\omega'}(z_1;z_2)=C^{(S)}_{\omega\omega'}(z_1;z_2)+C^{(V)}_{\omega\omega'}(z_1;z_2)\ ; \tag{25a}$$

$$C^{(S)}_{\omega\omega'}(z_1;z_2)=-\frac{1}{D^2}\int_0^L dz'\left[\left[G_{\Delta\omega=0}(z_2;L)\frac{dG_{\Delta\omega=0}(z_1;z')}{dz'}+G_{\Delta\omega=0}(z_1;L)\frac{dG_{\Delta\omega=0}(z_2;z')}{dz'}\right]\langle j_\omega(L)j_{\omega'}(z')\rangle\right.$$
$$\left.-\left[G_{\Delta\omega=0}(z_1;0)\frac{dG_{\Delta\omega=0}(z_2;z')}{dz'}+G_{\Delta\omega=0}(z_2;0)\frac{dG_{\Delta\omega=0}(z_1;z')}{dz'}\right]\langle j_\omega(0)j_{\omega'}(z')\rangle\right]; \quad (25b)$$

$$C^{(V)}_{\omega\omega'}(z_1;z_2)=-\frac{1}{D^2}\int_0^L dz'dz''\frac{dG_{\Delta\omega=0}(z_1;z')}{dz'}\frac{dG_{\Delta\omega=0}(z_2;z'')}{dz''}\langle j_\omega(z')j_{\omega'}(z'')\rangle . \quad (25c)$$

In Eqs. (25b) and (25c) only terms corresponding to long-range correlation are taken into account. Using Eq. (13), after some tedious algebra we obtain the following expression for the normalized long-range correlation function:

$$C_2(\Delta\omega,R)\equiv\frac{C_{\omega\omega'}(z_1=L-R;z_2=L)}{\langle I(L)\rangle\langle I(L-R)\rangle}=C_2^{(S)}(\Delta\omega,R)+C_2^{(V)}(\Delta\omega,R); \quad (26a)$$

$$C_2^{(S)}(\Delta\omega,R)=\frac{6\pi\alpha z_0^2}{K(\Delta\omega)}\left[\frac{P_{\Delta\omega=0}(L-R)P_{\Delta\omega=0}(L)|P_{\Delta\omega}(z_p)|^2}{P_{\Delta\omega=0}(R)}+\alpha z_0|P_{\Delta\omega}(L-z_p)|^2\right]; \quad (26b)$$

$$C_2^{(V)}(\Delta\omega;R)=\frac{3\pi}{2K(\Delta\omega)}\frac{\alpha^2}{\sqrt{\alpha^4+4\beta^4}}\left[|P_{\Delta\omega}(L-z_p)|^2[I_1(z_p)-J_1(z_p)+I_2(z_p)-J_2(z_p)]\right.$$
$$+|P_{\Delta\omega}(z_p)|^2[I_3(L-z_p)-I_3(R)-J_3(L-z_p)+J_3(R)+I_4(L-z_p)$$
$$-I_4(R)-J_4(L-z_p)+J_4(R)]$$
$$\left.-\frac{|P_{\Delta\omega}(z_p)|^2P_{\Delta\omega=0}(L-R)}{P_{\Delta\omega=0}(R)}[I_5(R)-J_5(R)+I_6(R)+J_6(R)]\right]. \quad (26c)$$

The function $P_{\Delta\omega}(z)$ is defined by Eq. (14), and functions $K(x),I_i(x)$, and $J_i(x)$ are given in the Appendix. Since our expression for $C_2(\Delta\omega;R)$ is rather complicated, we consider below important limiting cases for intensity-intensity spectral and spatial correlation functions separately.

First we consider the spatial correlation function when $\Delta\omega=0$. When absorption is strong, $\alpha L\gg 1,\alpha R>1$, and $\alpha(L-R)>1$, photons are effectively absorbed inside the slab before reaching the boundary at $z=L$. In this case even large surface reflection should not affect the correlation function. Indeed, we find that the surface term

$$C^{(S)}(R)=\frac{6\pi}{k^2lA}\frac{z_0^2}{(z_0+z_p)^2}\frac{z_0}{(1+\alpha z_0)^2} \quad (27)$$

is less than the volume term

$$C^{(V)}(R)=\frac{3\pi}{4k^2lA}\left[L-R+\frac{3}{4}\alpha^{-1}\right],\quad \alpha z_0\ll 1; \quad (28a)$$

$$C^{(V)}(R)=\frac{3\pi}{4k^2lA}\left[L-R-\frac{5}{4}\alpha^{-1}\right],\quad \alpha z_0\gg 1. \quad (28b)$$

Equation (28a) coincides with the corresponding expression of Refs. 31 and 32. The linear decay of $C_2(R)$ in the case of a strong absorption was observed experimentally by Genack *et al.*[17] The dependence of the cross-correlation function upon separation and reflection coefficient is shown in Fig. 1. The dependence of C_2 on $\Re$ demonstrates a maximum at $\Re\simeq\Re_c$, where $\Re_c$ is defined by Eq. (16). This maximum is due to the surface term of the correlation function. This term, as one can see from Eq. (24b), is proportional to the photon flux. The flux is

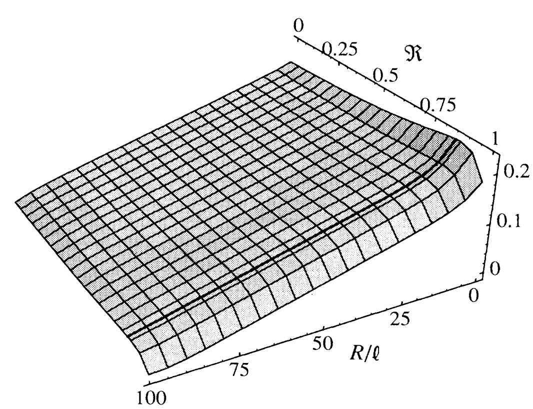

FIG. 1. Long-range spatial correlation function $C_2(R,\Re)$ in the case of strong absorption ($\alpha l=0.1$). In this and subsequent figures all lengths are measured in mean-free-paths and we use following values of parameters $L=100$, $z_p=1,k=8.25,A=20$. For both small ($\alpha z_0\ll 1$) and large ($\alpha z_0\gg 1$) reflection coefficients $C_2(R,\Re)$ is a linear function of R in accordance with Eqs. (34) and (35). C_2 as a function of $\Re$ shows nonmonotonic behavior with a maximum at $\Re=\Re_c$ for any value of R. The dependence $C_2(R,\Re=\Re_c)$ is shown by the thick line.

an increasing function of the reflection coefficient for small reflectivity and it decreases when reflectivity is very high. For zero-frequency shift it reaches maximum at $\Re_c$. Such a maximum always appears in the dependence of the correlation function upon the reflection coefficient.

In the case of the weak absorption, $\alpha L \ll 1, \alpha R \ll 1$, $\alpha z_0 \ll 1$, the correlation function takes the following form:

$$C^{(S)}(R)=\frac{6\pi}{k^2 l A}\frac{z_0^2}{(L+2z_0)^2}\left[\frac{(L+z_0)(L-R+z_0)}{R+z_0}+\frac{(L+z_0-z_p)^2 z_0}{(z_0+z_p)^2}\right]; \tag{29a}$$

$$C^{(V)}(R)=\frac{2\pi}{k^2 l A}\frac{1}{(L+2z_0)^2}\left[\frac{z_0^3(L+2z_0)}{R+z_0}+(L+z_0-z_p)^2\left[L+2z_0-\frac{z_0^3}{(z_p+z_0)^2}\right]-z_0^3-(L+2z_0)(R+z_0)^2\right]. \tag{29b}$$

When internal reflection is weak, $z_0 \ll L, z_0 < L-R$, the surface term can be neglected and we obtain the well-known result,[31,32]

$$C^{(V)}(R)=\frac{2\pi L}{k^2 l A}\left[1-\frac{R^2}{L^2}\right]. \tag{30}$$

In the case of strong internal reflection, $z_0 \gg L, z_0 \gg R$, the surface term dominates.

$$C^{(S)}(R)\approx\frac{3\pi z_0}{k^2 l A}\left[1+\frac{L-R}{z_0}\right], \tag{31a}$$

$$C^{(V)}(R)\approx\frac{\pi z_0}{k^2 l A}\frac{L-2R}{z_0}, \tag{31b}$$

and we find a linear falloff of the spatial correlation function. For highly reflecting boundaries, the degree of correlation increases by the factor of $\sim L/z_0$. The crossover from quadratic to linear decay of the spatial correlation function with increasing reflection coefficient can be seen in Fig. 2, where we plot C_2 as a function of R and $\Re$. We note that for small reflection $\Re_c \to 1$ and the maximum of the correlation function with respect to $\Re$ appears for very high reflection coefficients.

We will now consider the spectral-correlation function $C_2(\Delta\omega; R=0)$. As in the case of the cross-correlation function, for strong absorption, $\alpha L \gg 1$ and $\alpha \gg \beta$, we do not expect a strong dependence upon $\Re$, except for the maximum at $\Re \simeq \Re_c$. This dependence is shown in Fig. 3. Let us assume that the frequency shift $\Delta\omega$ is large enough, $\beta \gg \alpha$ and $\beta L \gg 1$, but limited by the condition $\beta l \ll 1$, so that Eq. (6) is valid. In this case, the correlation function has the form

$$C^{(S)}(\Delta\omega)\approx\frac{3\pi}{k^2 l A}\frac{z_0^3}{(z_0+z_p)^2}\frac{1}{1+2\beta z_0+2\beta^2 z_0^2}; \tag{32a}$$

$$C^{(V)}(\Delta\omega)\approx\frac{3\pi}{k^2 l A}\frac{1}{\beta}\frac{1+2\beta z_p z_0^2/(z_0+z_p)^2}{1+2\beta z_0+2\beta^2 z_0^2}. \tag{32b}$$

Two limiting cases can be distinguished. The first corresponds to small surface reflection when $\beta z_0 \ll 1$. In this case we obtain

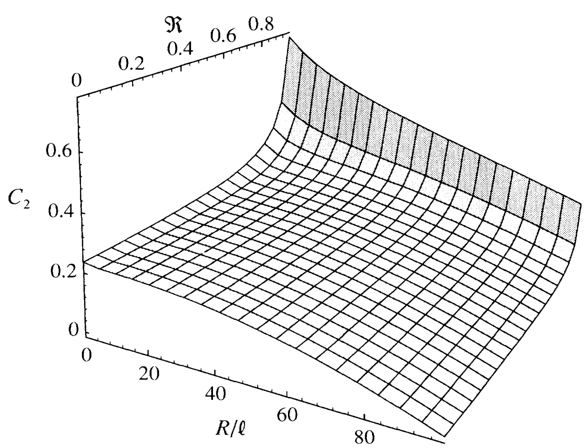

FIG. 2. Long-range spatial correlation function $C_2(R,\Re)$ in the case of the weak absorption ($\alpha l = 0.001$). A crossover from a quadratic falloff in the case of low internal reflection [$\alpha z_0 \ll 1$, Eq. (36)] to a linear falloff in the case of strong internal reflection [$\alpha z_0 \gg 1$, Eq. (37a)] is evident. The maximum of C_2 with respect to $\Re$ is not shown since for small absorption it occurs at a very high reflection coefficient $\Re = \Re_c \simeq 0.999$. The values of parameters L, z_p, k, and A are the same as in Fig. 1.

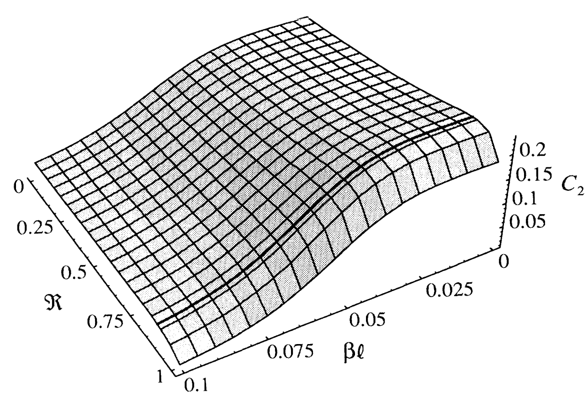

FIG. 3. Dependence of the long-range spectral correlation function $C_2(\beta,\Re)$ upon βl and $\Re$ for the case of strong absorption ($\alpha = 0.1$). It also shows a weak maximum at $\Re \simeq \Re_c$. The dependence $C_2(\beta, \Re = \Re_c)$ is shown by the thick line. The values of parameters L, z_p, k, and A are the same as in Fig. 1.

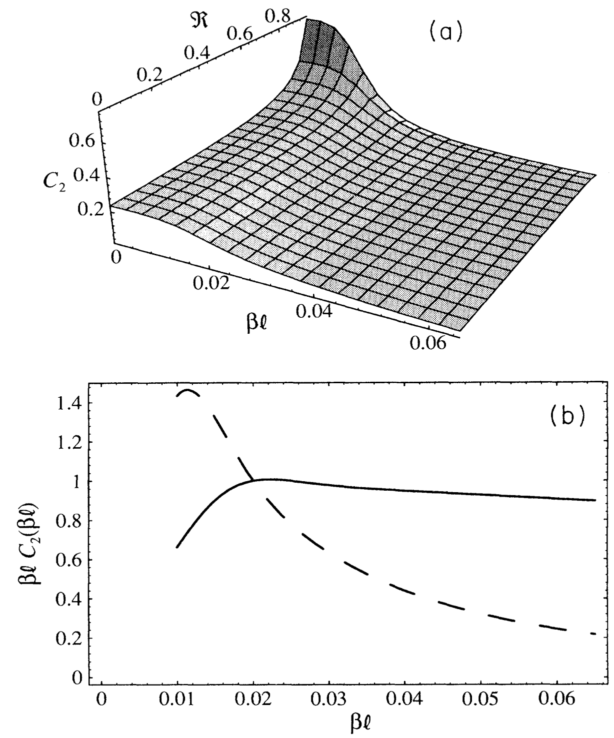

FIG. 4. (a) Dependence of the long-range spectral correlation function $C_2(\beta,\Re)$ upon βl and $\Re$ for the case of weak absorption ($\alpha=0.001$). (b) Functions $\beta l C_2(\beta l,\Re=0)$, shown by the solid line, and $\beta l C_2(\beta l,\Re=\Re_c)$, shown by the dashed line, demonstrate the crossover from the $1/\beta$ dependence of the correlation function for small reflection to $1/\beta^2$ for strong reflection. Both functions are normalized by their values at $\beta=0.2$. The values of parameters L,z_p,k, and A are the same as in Fig. 1.

$$C^{(S)}(\Delta\omega)\approx\frac{3\pi}{k^2lA}\frac{z_0^3}{(z_0+z_p)^2}(1-2\beta z_0)\ ; \tag{33a}$$

$$C^{(V)}(\Delta\omega)\approx\frac{3\pi}{2k^2lA}\frac{1}{\beta}\left[1-2\beta z_0\frac{z_0^2+z_0z_p+z_p^2}{(z_0+z_p)^2}\right]\ . \tag{33b}$$

The volume term is greater than the surface term by a factor $(\beta z_0)^{-1}$ and the leading term in Eq. (33b) gives the well-established result $C_2\propto(\Delta\omega)^{-1/2}$. If we neglect small corrections of the order of βz_0 and βz_p, then Eq. (33b) coincides exactly with the result first obtained by Pnini and Shapiro.[28]

For high surface reflection, $\beta z_0\gg 1$, however, the frequency dependence changes dramatically.

$$C^{(S)}(\Delta\omega)\approx\frac{3\pi}{2k^2lA}\frac{1}{\beta^2 z_0}\left[1-\frac{1}{\beta z_0}\right]\ ; \tag{34a}$$

$$C^{(V)}(\Delta\omega)\approx\frac{3\pi z_0}{4k^2lA}\frac{1}{\beta^3 z_0^2}\left[1-\frac{1}{\beta z_0}\right]\ , \tag{34b}$$

and now the surface term dominates again and decays as $1/\Delta\omega$ rather than as $(\Delta\omega)^{-1/2}$ as occurs in the case of small reflection. The degree of correlation for small β is $\sim L/z_0$ times greater than in the case of high internal reflection. We plot the autocorrelation function $C_2(\Delta\omega;0)$ as a function of the variables $\Delta\omega$ and $\Re$ in Fig. 4.

V. CONCLUSIONS

In conclusion, we have calculated the long-range contribution to the spatial and spectral intensity-intensity correlation functions in the presence of internal reflection. We show that internal reflection can be incorporated into the long-range correlation function using proper boundary conditions. We find that in the presence of internal reflection there is an additional surface term in the long-range correlation function when intensity fluctuations at the sample surface are included. When internal reflection is weak, this term is small in comparison with the volume term and our results exactly coincide with previous results. In the presence of strongly reflecting boundaries, the surface term dominates and we obtain qualitatively different dependences of the correlation functions upon frequency shift and a spatial separation.

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APPENDIX

Here we give definitions of functions $K(x)$, $I_i(x)$, and $J_i(x)$ appearing in Eq. (26).

$$K(\Delta\omega)=k^2lAP^2_{\Delta\omega=0}(z_p)\{[(1+\alpha^2z_0^2)^2+4\beta^4z_0^4][\cosh(2\gamma_+L)-\cos(2\gamma_-L)]+4(\gamma_+^2+\gamma_-^2)z_0^2[\cosh(2\gamma_+L)+\cos(2\gamma_-L)]$$
$$+4[\gamma_+z_0(1+\alpha^2z_0^2)+2\beta^2\gamma_-z_0^3]\sinh(2\gamma_+L)+4[\gamma_-z_0(1+\alpha^2z_0^2)-2\beta^2\gamma_+z_0^3]\sin(2\gamma_-L)\}\ , \tag{A1}$$

$$I_1(x)=[1+(\gamma_+^2+\gamma_-^2)z_0^2]\left[(1-\alpha^2z_0^2)\frac{\sinh(2\gamma_+x)}{2\gamma_+}+\frac{1}{4}(1+\alpha^2z_0^2)\left[\frac{\sinh[2(\gamma_++\alpha)x]}{\gamma_++\alpha}+\frac{\sinh[2(\gamma_+-\alpha)x]}{\gamma_+-\alpha}\right]\right.$$
$$\left.+\frac{1}{2}\alpha z_0\left[\frac{\sinh^2[(\gamma_++\alpha)x]}{\gamma_++\alpha}-\frac{\sinh^2[(\gamma_+-\alpha)x]}{\gamma_+-\alpha}\right]\right]\ , \tag{A2}$$

$$I_2(x)=2\gamma_+z_0\left[(1-\alpha^2z_0^2)\frac{\sinh^2(\gamma_+x)}{\gamma_+}+\frac{1}{2}(1+\alpha^2z_0^2)\left[\frac{\sinh^2[(\gamma_++\alpha)x]}{\gamma_++\alpha}+\frac{\sinh^2[(\gamma_+-\alpha)x]}{\gamma_+-\alpha}\right]\right.$$
$$\left.+\frac{1}{2}\alpha z_0\left[\frac{\sinh[2(\gamma_++\alpha)x]}{\gamma_++\alpha}-\frac{\sinh[2(\gamma_+-\alpha)x]}{\gamma_+-\alpha}\right]\right], \tag{A3}$$

$$I_3(x)=[1+(\gamma_+^2+\gamma_-^2)z_0^2]\left[(1-\alpha^2z_0^2)\frac{\sinh(2\gamma_+x)}{2\gamma_+}+\frac{1}{4}(1+\alpha^2z_0^2)\left[\frac{\sinh[2(\gamma_++\alpha)x-2\alpha L]}{\gamma_++\alpha}+\frac{\sinh[2(\gamma_+-\alpha)x+2\alpha L]}{\gamma_+-\alpha}\right]\right.$$
$$\left.-\frac{1}{2}\alpha z_0\left[\frac{\cosh[2(\gamma_++\alpha)x-2\alpha L]}{\gamma_++\alpha}-\frac{\cosh[2(\gamma_+-\alpha)x+2\alpha L]}{\gamma_+-\alpha}\right]\right], \tag{A4}$$

$$I_4(x)=2\gamma_+z_0\left[(1-\alpha^2z_0^2)\frac{\sinh^2(\gamma_+x)}{\gamma_+}+\frac{1}{4}(1+\alpha^2z_0^2)\left[\frac{\cosh[2(\gamma_++\alpha)x-2\alpha L]}{\gamma_++\alpha}+\frac{\cosh[2(\gamma_+-\alpha)x+2\alpha L]}{\gamma_+-\alpha}\right]\right.$$
$$\left.-\frac{1}{2}\alpha z_0\left[\frac{\sinh[2(\gamma_++\alpha)x-2\alpha L]}{\gamma_++\alpha}-\frac{\sinh[2(\gamma_+-\alpha)x+2\alpha L]}{\gamma_+-\alpha}\right]\right], \tag{A5}$$

$$I_5(x)=[1+(\gamma_+^2+\gamma_-^2)z_0^2]\left[[(1+\alpha^2z_0^2)\cosh(\alpha L)+2\alpha z_0\sinh(\alpha L)]\frac{\sinh(2\gamma_+x)}{2\gamma_+}\right.$$
$$+\frac{1}{2}(1-\alpha^2z_0^2)\left[\frac{\sinh[(\gamma_++\alpha)x]\cosh[(\gamma_++\alpha)x-\alpha L]}{\gamma_++\alpha}\right.$$
$$\left.\left.+\frac{\sinh[(\gamma_+-\alpha)z]\cosh[(\gamma_++\alpha)x+\alpha L]}{\gamma_+-\alpha}\right]\right], \tag{A6}$$

$$I_6(x)=2\gamma_+z_0\left[[(1+\alpha^2z_0^2)\cosh(\alpha L)+2\alpha z_0\sinh(\alpha L)]\frac{\sinh(2\gamma_+x)}{2\gamma_+}\right.$$
$$\left.+\frac{1}{2}(1-\alpha^2z_0^2)\left[\frac{\sinh[(\gamma_++\alpha)x]\sinh[(\gamma_++\alpha)x-\alpha L]}{\gamma_++\alpha}+\frac{\sinh[(\gamma_+-\alpha)x]\sinh[(\gamma_++\alpha)x+\alpha L]}{\gamma_+-\alpha}\right]\right], \tag{A7}$$

and functions $J_i(x)$ can be obtained from $I_i(x)$ by substitution of $-i\gamma_-$ for γ_+.

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