Magnetic phase transitions with final ordering: Peculiarities in the critical behavior

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Magnetic crystals containing two subsystems of different types of magnetic ions are considered. It is assumed that the subsystem of the first type of the ions is already ordered at relatively high temperature, while the ordering temperature of the second subsystem is much smaller. We present an analysis, which is performed by use of the Landau-Ginzburg method, of the critical behavior of such double magnetic systems in the vicinity of the ordering temperature of the second subsystem. This temperature is referred to as the temperature of the final ordering. We demonstrate the suppression of the anomalous fluctuations near the point of the phase transition of the final ordering. The peculiarities of the critical behavior in the vicinity of the points of phase transitions of the final ordering in low-dimensional magnets are also discussed.

I. GENERAL CHARACTERISTIC OF THE OBJECTS

There are a large number of double magnetic systems that contain two types of magnetic ions, for example, iron-group, and rare-earth ions. In the case when the ordering temperatures of each subsystem are close ($|T_{c1}-T_{c2}| \ll T_{c1}$) a picture of phase transitions may be quite unusual.[1,2] In this paper we study an opposite situation when one of the magnetic ion subsystems is ordered at a relatively high temperature T_{c1} while the ordering temperature of the second subsystem T_{c2} is much lower, i.e.,

$$T_{c1} \gg T_{c2} \ . \tag{1}$$

This relationship is of a principal importance for the further analysis. It is exactly the cause of the unusual static and dynamic critical behavior in the vicinity of the temperature T_{c2}. This temperature we define as the temperature of *final ordering*.

Practically, all rare-earth orthoferrites, ferrites garnets, and a number of related materials belong to this class of magnetic compounds. As a rule, in such systems the final ordering temperature T_{c2}, which characterizes the rare-earth subsystem, is by three orders of magnitude smaller than the ordering temperature of the iron subsystem. A similar situation takes place in some tetragonal antiferromagnets that are the base compounds for the high-temperature superconducting materials. The systems of R_2CuO_4 type (where R is a rare-earth ion) belong to these compounds. In these compounds the rare-earth subsystem also plays the role of the second magnetic subsystem that orders at low temperatures. Magnetic ordering in the first subsystem at $T=T_{c1}$ implies a conventional ordering-type phase transition, which we do not consider here. In this paper we deal with the phase transition at the critical temperature T_{c2} ($\ll T_{c1}$) only.

Within the temperature range of the ordering of the second subsystem the magnetic sublattices of the first subsystem are practically saturated, thus they can only change their orientation with respect to the crystal axes. Physically this means that the phase transition at $T=T_{c2}$ has a dual nature: for the second subsystem it is the phase transition of the ordering type and at the same time it is the spin-orientation phase transition for the already ordered first subsystem. It is well known that the character of the critical behavior corresponding to these types of transitions is quite different.[3-5] For the first phase transition, pronounced critical fluctuations take place because the Ginzburg parameter may be of the order of unity. For the spin-reorientation phase transitions, such fluctuation phenomena are practically absent. As a rule, the Ginzburg parameter is of the order of $10^{-6}-10^{-9}$.[3,4] In the case under consideration the nature of the magnetic phase transition is a combination of two phenomena, therefore the character of critical anomalies in the vicinity of the point T_{c2} may be quite unusual. In the present study we would like to provide the analysis of these anomalies.

It is evident from the above that the symmetry of magnetic subsystems coupling plays a crucial role in this problem. Omitting the cases that are exotic from the point of view of symmetry one can distinguish three different situations:

(i) The saturated sublattices of the first magnetic subsystem do not reorient due to the appearance of a spontaneous magnetic ordering in the second subsystem at

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temperatures below T_{c2}. For the second subsystem the phase transition at $T=T_{c2}$ is of the pure ordering type and the first subsystem plays no role in the formation of a critical behavior.

(ii) The symmetry of the effective field resulting from spin-spin interactions in the first subsystem is just the same as the symmetry of the spontaneous magnetic ordering arising in the second subsystem at low temperatures. In this situation the critical point T_{c2} is completely absent since the magnetic sublattices of the second subsystem are weakly ordered by the exchange field of the first subsystem in the whole temperature region $T<T_{c1}$. There is a close analogy between this case and the effect of the disappearance of the phase-transition point between paramagnetic and ferromagnetic phases in an external magnetic field.

(iii) The emergence of magnetic ordering in the second subsystem leads to the same change of the magnetic symmetry as a spin-reorientation in the first subsystem. In this case both the magnetic final ordering and the change of orientations for entirely ordered sublattices are two manifestations of the single phase transition at $T=T_{c2}$. Consequently, these phenomena will accompany each other. This situation is of primary interest.

Which of the three above-mentioned cases is realized in the given double magnetic system is determined not only by the arrangement of magnetoactive ions but also by the type of magnetic ordering in both the first and the second subsystems. In particular, all three situations mentioned above occur in rare-earth orthoferrites.

Situations (i) and (ii) may be considered as the trivial limiting cases of situation (iii). We shall treat them in this way in the next section where we start from the case (iii).

The classification of phase transitions connected with final ordering given above is purely symmetrical and does not exhaust all qualitatively different situations. Exchange interactions providing the appearance of long-range magnetic ordering both in the first and in the second subsystems may be anisotropic in space. In the above-mentioned tetragonal antiferromagnets R_2CuO_4 the exchange interactions between neighboring CuO_2 planes are by three or four orders of magnitude smaller than the exchange interactions taking place inside each plane, in particular. At the same time the condition (1) is fulfilled quite well. Naturally, in the case when the magnetic order in a magnetic subsystem is quasi-two-dimensional or quasi-one-dimensional, the character of the critical behavior will be quite different from the case of rare-earth orthoferrites or ferrites garnets. In the latter the exchange integrals in different crystallographic directions are comparable with each other. These cases we consider in Sec. III.

II. SUPPRESSION OF ANOMALOUS FLUCTUATIONS

The three configurations of a double magnetic system are shown in Fig. 1. The sublattice magnetization vectors of the first and second subsystems are denoted by $\mathbf{S}_1$ and $\mathbf{S}_2$, respectively. The directions of these vectors are

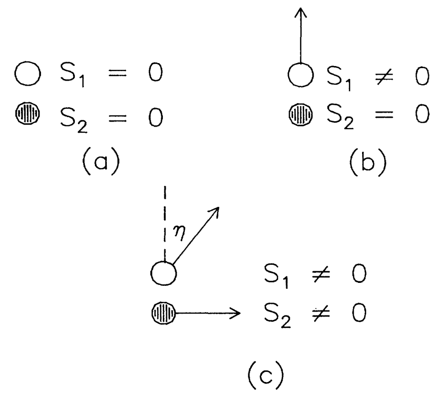

FIG. 1. Magnetic state of a double magnetic system at different temperatures: (a) $T>T_{c1}$, paramagnetic phase; (b) $T_{c2}<T<T_{c1}$, only the first subsystem is ordered; (c) $T<T_{c2}\ll T_{c1}$, $\mathbf{S}_1$ deviates from the original symmetry direction and $\mathbf{S}_2$ comes to be nonzero. The direction of $\mathbf{S}_2$ as shown is arbitrary.

chosen arbitrarily. The character of the magnetic arrangement in the subsystems may be both ferromagnetic and antiferromagnetic and the condition (1) is assumed to be valid.

The phase transition at $T=T_{c2}$ consists in the appearance of magnetic ordering in the second subsystem accompanied by the deviation of the magnetization of the first subsystem sublattices from the primary symmetrical direction. Let us choose y axis as a direction of this deviation and consider S_2 as a scalar for the sake of simplicity. From the point of view of symmetry the values S_{1y} and S_2 are transformed as the phase-transition order parameter and are proportional to each other in the vicinity of T_{c2}. In order to simplify notations let us introduce the following scalar fields

$$\eta=S_{1y}/S_1(0)\ ,\quad \varphi=S_2/S_2(0)\ , \tag{2}$$

where $S_1(0)$ and $S_2(0)$ are the magnitudes of magnetizations at $T=0$ of the first and the second subsystem, respectively. One can write the nonequilibrium thermodynamic potential (Landau-Ginzburg functional) which describes the phase transition at $T=T_{c2}$ as follows:

$$W[\eta,\varphi]=\int d^3x\,\{W_1[\eta(\mathbf{x})]+W_2[\varphi(\mathbf{x})]+W_{12}[\eta(\mathbf{x}),\varphi(\mathbf{x})]\}\ , \tag{3}$$

where

$$W_1[\eta(\mathbf{x})]=\frac{a}{2}\eta^2(\mathbf{x})+\frac{b}{4}\eta^4(\mathbf{x})+\frac{\alpha}{2}[\nabla\eta(\mathbf{x})]^2\ , \tag{4}$$

$$W_2[\varphi(\mathbf{x})]=\frac{A}{2}\varphi^2(\mathbf{x})+\frac{B}{4}\varphi^4(\mathbf{x})+\frac{\beta}{2}[\nabla\varphi(\mathbf{x})]^2\ , \tag{5}$$

$$W_{12}[\eta(\mathbf{x}),\varphi(\mathbf{x})]=-\lambda\eta(\mathbf{x})\varphi(\mathbf{x})\ . \tag{6}$$

The presence of the bilinear coupling of order parameters φ and η in Eq. (6) reflects the combined (from the symmetry point of view) character of the magnetic phase transition at $T=T_{c2}$. Since the parameters a, b, and α characterize the totally saturated first magnetic subsystem, they are assumed to be temperature independent. The order parameter η coincides with the angle of rotation of the sublattice magnetization of the first subsystem. This angle describes the rotation of magnetization with respect to the original symmetrical direction. Therefore, the values a and b are the anisotropy constants of the first subsystem, while the parameter α is of the exchange nature and is of the order of

$$\alpha/d^2 \propto T_{c1}/d^3 , \tag{7}$$

where d is the interatomic distance. Let us consider the conventional situation when exchange interactions in the first subsystem exceed the magnetic anisotropy energy. This implies that

$$\alpha/d^2 \propto T_{c1}/d^3 \gg a,b . \tag{8}$$

The ratio of the exchange to the anisotropy energy is of the order of 10^2-10^4 for the first subsystem in all above-mentioned materials. This inequality as well as the relationship given in Eq. (1) are of a principal importance for our further analysis.

The parameters A, B, and β in (5) describe the second magnetic subsystem, which is ordered at low temperature $T \ll T_{c2}$, and are the only parameters depending directly on the temperature. They do not exceed $T_{c2}/d^3 (\ll T_{c1}/d^3)$. The same concerns the parameter λ that characterizes coupling between magnetic subsystems. It is easy to show that a large value of λ is incompatible with condition (1). Thus, λ is of the same order as the parameters of the second subsystem. As a result of this discussion we have the ultimate condition that defines the hierarchy of interactions in the system,

$$\alpha/d^2 \propto T_{c1}/d^3 \gg a,b,A,B,\lambda,\beta/d^2 . \tag{9}$$

This inequality is always satisfied for all above-mentioned double magnetic systems.

The physical meaning of the inequality (9) is as follows: The isotropic exchange interaction in the first subsystem is the most powerful spin-spin interaction in the system under consideration. It provides a large value of T_{c1} and is responsible for the stiff fixation of the mutual orientation of the spins in the first subsystem at $T=T_{c2}$. However, the orientation of sublattice magnetizations with respect to crystallographic axes may be easily changed since the anisotropy interactions in the first subsystem, characterized by parameters a and b, are much smaller than the exchange one. This is the reason for the profound influence of the first magnetic subsystem on the formation of T_{c2} and on the character of the critical behavior in the vicinity of T_{c2}.

To estimate the role of fluctuation phenomena near T_{c2}, we employ the Gaussian approximation. The point of the phase transition is defined by the relation

$$A(T)a=\lambda^2 , \quad \text{when } T=T_{c2} , \tag{10}$$

or equivalently,

$$A(T_{c2})=A^*=\lambda^2/a>0 . \tag{11}$$

For the sake of simplicity let us assume that A is the only temperature-dependent parameter that characterizes the ordering subsystem. Taking into account of the temperature dependence of other parameters leads to somewhat complex calculations with essentially similar results. Following the standard procedure of calculation of the specific-heat correction due to fluctuations, one can obtain[6,7]

$$\Delta c_{\mathrm{fl}} = -T\partial^2 F_{\mathrm{fl}}/\partial T^2 , \tag{12}$$

where

$$F_{\mathrm{fl}}=\frac{VT}{2(2\pi)^3}\int_0^{k_0} d^3k \ln|a_k A_k-\lambda^2| ,$$
$$k_0 \propto d^{-1} , \quad a_k=a+\alpha k^2 , \quad A_k=A(T)+\beta k^2 . \tag{13}$$

The most singular part of Δc_{fl} has the form standard for the Gaussian approximation

$$\Delta c_{\mathrm{sing}}=\frac{VT_{c2}^2 a^3(dA/dT)^2}{16\pi(\beta a^2+\alpha\lambda^2)^{3/2}}[A(T)-A^*]^{-1/2} , \quad \text{when } T \geq T_{c2} . \tag{14}$$

To estimate the temperature region for anomalous fluctuations let us compare the singular part of the specific heat (14) with the specific-heat jump Δc calculated in the framework of the Landau theory,

$$\Delta c=\frac{VT_{c2}}{2B}\left[\frac{dA}{dT}\right]^2 . \tag{15}$$

The singular contribution to the specific heat proves to be essential in the following temperature region

$$|T-T_{c2}| \ll \frac{a^6B^2T_{c2}^2}{64\pi^2(\beta a^2+\alpha\lambda^2)^3 dA/dT} . \tag{16}$$

The right-hand side of this inequality is very small due to the large value proportional to α^3 in the denominator. The numerical estimations carried out for orthoferrites and ferrites garnets show that the temperature region in Eq. (16) essential for strongly developed anomalous fluctuations is of the order of 10^{-6}–10^{-9} K.[3,4] It is clear that there is no critical behavior in a real situation. Let us emphasize that from the physical point of view the suppression of critical fluctuations originates in the fact that the isotropic exchange interaction in the first subsystem is much greater than the other magnetic interactions, including those which cause the phase transition at $T=T_{c2}$. We also note that the strong isotropic exchange interaction in the first subsystem does not affect the magnitude of T_{c2} [Eq. (10)], but suppresses fluctuation phenomena.

Now it is necessary to discuss the nature of phase transition at the point T_{c2}. This question is important for understanding the character of the fluctuation phenomena. If $\lambda \neq 0$, then from the standpoint of symmetry it follows that the phase transition at low temperatures has a "combined" nature. It is a spin-reorientation phase transition for the first subsystem and an ordering-type phase transition for the second subsystem. Therefore, it is natural to consider two opposite limiting cases.

(a) The exchange interactions in the second subsystem are negligibly small. This corresponds to

$$\beta=0 . \tag{17}$$

It is clear that in this case the phase transition at T_{c2} manifests itself as a spin-reorientation one, and the spontaneous ordering in the second subsystem cannot take place. The phase transition under consideration is forced only by the interaction W_{12} in the functional (3). When the temperature is decreased this interaction begins to compete with the anisotropy energy in the first subsystem (i.e., with the positive parameter a). The stability condition for the symmetric phase,

$$Aa > \lambda^2 , \quad \text{when} T > T_{c2} , \tag{18}$$

is shown to be violated at the critical point (10). This situation occurs in the majority of rare-earth orthoferrites and ferrites garnets. In accordance with the inequality (16), the temperature region of anomalously developed fluctuations is negligibly small.

(b) The interaction W_{12} between both subsystems is not essential. This is usually the result of the fact that the parameter λ is equal to zero,

$$\lambda=0 , \tag{19}$$

due to the special magnetic symmetry of the system. It is clear that in this case we deal with an ordering-type phase transition in the second subsystem and the first subsystem plays no role. The phase-transition temperature is determined by the condition,

$$A=0 , \quad \text{when } T=T_{c2} \quad (\text{and } \lambda=0) . \tag{20}$$

The strongly developed anomalous fluctuations associated with the ordering-type phase transition should be observed in the vicinity of T_{c2}. Indeed, if $\lambda=0$, from the inequality (16) one can obtain,

$$|T-T_{c2}| \ll \frac{B^2 T_{c2}^2}{64\pi^2\beta^3(dA/dT)} . \tag{21}$$

This expression is just the same as the standard estimation[6,7] for the case of one fluctuating field. The magnitude of the temperature region (21) is not small and may be compared with T_{c2}.

Consequently, we have two trivial limiting cases. In the first case the phase transition looks like a spin-orientation one and fluctuation phenomena are absent. In the second case we deal with a phase transition of a pure ordering type, and the typical picture of highly developed critical fluctuations for the system characterized by a small value of the Ginzburg parameter takes place.

The intermediate case, when the low-temperature phase transition is of a truly combined nature, is expected to be the most interesting. Here the parameters λ and β/d^2 may be comparable. Due to the inequality (16), the region of strongly developed critical fluctuations will be smaller than the one in the case (a) of the pure spin-reorientation phase transition. If the symmetry of the system allows an existence of the interaction W_{12}, the presence (or absence) of the term βa^2 in the denominator on the right-hand part of the inequality (16) plays no role since in any case $\alpha\lambda^2 \gg \beta a^2$. Therefore, the role of fluctuation phenomena in the case $\lambda\neq 0$ may seem to be negligibly small irrespective of the exchange interactions in the second subsystem which undergoes ordering. But it is not quite so because the estimations, based on the relations (14–16), take into account only the contribution to the specific-heat singularity at the T_{c2} point. This contribution is really very small for $\lambda\neq 0$. But in the special situation under consideration we should carry out a detailed study of Eqs. (12) and (13) for the fluctuation contribution into the specific heat, not restricting ourselves by the singular part (14) at $T=T_{c2}$. The simplest way to do this is to regard the limiting case

$$\alpha \to \infty , \tag{22}$$

which is naturally connected with the conditions (1) and (9). It reflects the predominance of exchange interactions in the first subsystem over other magnetic interactions in the system.

In the limiting case (22) the contribution of fluctuations into the specific heat turns out to be nonsingular at $T \to T_{c2}$. In fact, using Eqs. (12) and (13) with $\alpha\to\infty$ we obtain,

$$\Delta c_{\text{fl}} = -\frac{VT}{4\pi^2}\frac{d^2}{dT^2}\left[T\int_0^{k_0} dk\, k^2 \ln(A+\beta k^2)\right] . \tag{23}$$

Since only the second subsystem fluctuates in the limiting case (22), parameters characterizing the first subsystem have dropped out from Eq. (23). In other words, it is only the ordered second subsystem that fluctuates in the limiting case $\alpha\to\infty$. But at the same time the presence of the first subsystem considerably affects the location of the critical point T_{c2} [see the Eq. (10)] and "provokes" the phase transition before the point corresponding to the temperature at which the fluctuations of the order parameter $\varphi(x)$ would become anomalously large. Indeed, the expression (23) for the first specific-heat correction exactly coincides with the similar formula for the case of one fluctuating field. On the other hand, according to Eq. (11), the value of the parameter A is nonzero at the critical point (10). Therefore, in the limiting case (22) the fluctuation contribution to the specific heat turns out to be finite at the critical point. However, the specific heat may possess a noticeable maximum caused by fluctuations in the second subsystem.

The contribution Δc_{fl} (23), which increases most strongly in the vicinity of T_{c2}, is caused by the long-wavelength fluctuations in the rare-earth subsystem and is described by the expression:

$$\Delta c_{\text{fl}} \approx \frac{VT_{c2}^2}{16\pi\beta^{3/2}}\left[\frac{dA}{dT}\right]^2 \begin{cases} [A(T)]^{-1/2}, & A(T) > A^* , \\ \frac{1}{4}[-2A(T)+3\lambda^2/a]^{-1/2}, & A(T) < A^* . \end{cases} \tag{24}$$

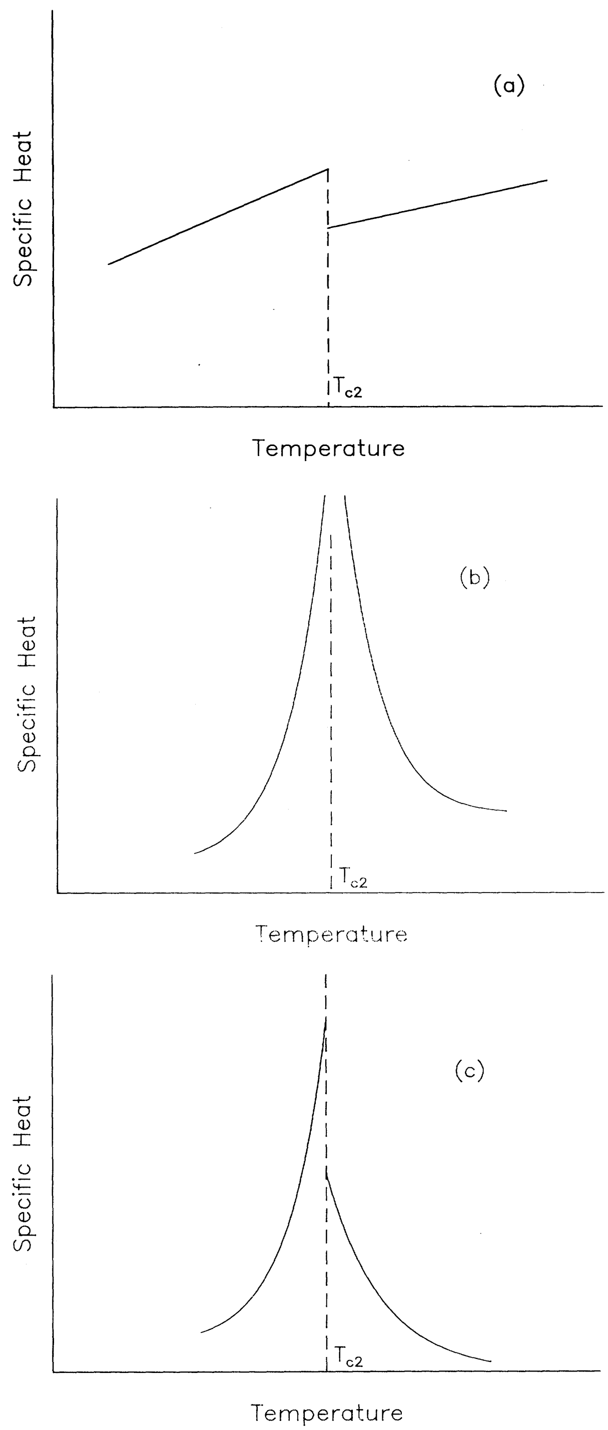

FIG. 2. Critical behavior of the specific heat in the vicinity of T_{c2} for $\alpha\rightarrow\infty$: (a) $\beta=0$, a pure spin-reorientation phase transition; (b) $\lambda=0$, an ordering-type phase transition in the second subsystem. This is a standard dependence for the case of three-dimensional fluctuations; (c) $\lambda\neq0$, $\beta\neq0$, a magnetic phase transition of combined nature. A specific-heat jump, as in the Landau theory, against a background of a pronounced maximum. If we put $\lambda\rightarrow0$ the case (c) transforms into (b). Taking into account the finite value of α leads to the divergence of $c(T)$ near the critical point, but in an extremely narrow temperature region.

The limits of $\Delta c_{\rm fl}$ when $T\rightarrow T_{c2}$ are different on the left and on the right of the critical point,

$$\Delta c_{\rm fl}(T=T_{c2}-0)=4\Delta c_{\rm fl}(T=T_{c2}+0) =\frac{VT_{c2}^2(dA/dT)^2}{4\pi\beta^{3/2}(\lambda^2/a)^{1/2}}\ . \tag{25}$$

Thus, the fluctuations increase the jump of the specific heat at the temperature T_{c2} predicted by the Landau theory. The formulas (24) and (25) describe the main contribution to the specific heat only in the case, when

$$r_c^{-1}(T=T_{c2})\equiv(\lambda^2/\beta a)^{1/2}\ll k_0\propto d^{-1}\ , \tag{26}$$

where r_c is the fluctuation correlation length in the second subsystem. This condition is fulfilled if the main reason of the phase transition at $T=T_{c2}$ is the ordering of the second subsystem. In other words, from the quantitative point of view this phase transition is of the order-disorder type rather than spin-reorientation one. On the other hand, if λ is too small the fluctuations in the vicinity of T_{c2} will be too large to be adequately described in the framework of the Gaussian approximation. Nevertheless, the conclusion about the finite value of the specific heat at $T=T_{c2}$ for the limiting case $\alpha\rightarrow\infty$ remains valid for any nonzero value of λ.

If spin-spin interactions inside the second subsystem are negligible it is possible to set $\beta=0$ in (23). This limiting case can be adequately described in the framework of the mean-field approximation. It is senseless to distinguish the fluctuation contribution to the thermodynamic values in this case, because this contribution has no singularity at $T\rightarrow T_{c2}$. This particular case was studied in Refs. 3 and 4.

In Fig. 2, three types of temperature dependencies of the specific heat in the vicinity of T_{c2} for the limiting case $\alpha\rightarrow\infty$ are presented. Case (a) corresponds to the phase transition of the ordering type ($\lambda=0$); case (b) corresponds to the spin-reorientation type; and case (c) is attributed to the phase transition of combined nature. All three above-mentioned possibilities are realized in rare-earth orthoferrites at low temperatures.

III. LOW-DIMENSIONAL MAGNETS

In a number of magnetic crystals spin-spin interactions are sharply spatially anisotropic. These compounds are called low-dimensional (quasi-two-dimensional or quasi-one-dimensional) magnetically ordered crystals. A lot of papers are devoted to studies of such objects. In particular, the above-mentioned tetragonal antiferromagnets of R_2CuO_4 type belong to these systems. In these antiferromagnets the temperature of the magnetic ordering of the first subsystem (the copper subsystem) is two or three orders of magnitude greater than the final ordering temperature of the rare-earth subsystem (see Refs. 8–12, and the references therein). In these compounds the energy of the exchange interactions of the copper spins inside the tetragonal planes x-y is two or four orders of magnitude greater than the binding energy between spins belonging to the neighboring x-y planes. Naturally, the character of the critical behavior of low-dimensional magnets differs considerably from the ordinary three-dimensional case. It relates to both of the critical points T_{c1} and T_{c2}. We consider only the low-temperature phase transition at $T=T_{c2} \ll T_{c1}$, because only this one is of the final ordering type.

The most evident is the case when low-dimensional ordering is only the spin arrangement in the second subsystem and ordinary three-dimensional ordering takes place in the first subsystem. The limiting case $\alpha \to \infty$ is not essentially different from the one considered in the previous section. As before, the first subsystem does not take part in the nonuniform fluctuations in the vicinity of T_{c2}. However, it affects considerably the value of T_{c2}. This leads to the phase transition that happens, in accordance to the inequality (11), before the moment when the parameter A turns to zero. To estimate the fluctuation contribution to the thermodynamic parameters it is necessary to substitute the value of $A_k = A + \beta k^2$ in Eq. (23) by

$$A_k = A + \beta k_z^2 \quad \text{or} \quad A_k = A + \beta(k_x^2 + k_x^2) , \tag{27}$$

for one-dimensional and two-dimensional ordering in the second subsystem, respectively. Of course, it intensifies fluctuation effects in the vicinity of T_{c2}, but with respect to analogy with the limiting case $\alpha \to \infty$ the fluctuation corrections accessible to experimental measuring are nonsingular.

Let us consider the inverse situation when the low-dimensional ordering takes place at $T=T_{c1} \gg T_{c2}$. In this case the gradient invariant in Eq. (4), describing the nonuniform exchange interaction in the first subsystem, has to be presented in an anisotropic form. Then it is necessary to substitute the factor $a_k = a + \alpha k^2$ in Eq. (13) by

$$a_k = a + \alpha_{\parallel} k_z + \alpha_{\perp}(k_x^2 + k_y^2) . \tag{28}$$

The inequality $\alpha_{\parallel} \gg \alpha_{\perp}$ corresponds to the one-dimensional ordering in the first subsystem; $\alpha_{\parallel} \ll \alpha_{\perp}$ is attributed to the two-dimensional one.

According to Eqs. (12) and (13) the fluctuation correction to the specific heat is determined by the expression,

$$\Delta c_{\mathrm{fl}} = \frac{VT^2(dA/dt)^2}{2(2\pi)^3} \int_0^{k_0} \frac{d^3k \, a_k^2}{(a_k A_k - \lambda^2)^2} , \tag{29}$$

that has already been used while obtaining the expressions (14) and (23). But then we used the expression $a_k = a + \alpha k^2$. However, the latter is invalid in the cases when the magnetic ordering in the first subsystem is quasi-one-dimensional or quasi-two-dimensional. Now we will consider just these two cases.

Following the logic of the previous section, the most interesting is the analysis of two contributions to the fluctuation correction to the specific heat. The first contribution is the most singular in the vicinity of T_{c2} and, as a rule, this contribution is analyzed while studying the role of fluctuation at second-order phase transitions. In the case of three-dimensional ordering in the first subsystem the contribution to thermodynamic values singular at $T=T_{c2}$ is negligible [see Refs. 3 and 4 and the inequality (16)], that is why it cannot be observed experimentally in principle. It is connected to the practically total suppression of nonuniform fluctuations in the first subsystem by the large exchange interaction α. In the case of low-dimensional magnetic ordering in the first subsystem, the effect of suppression will not be total just because some diagonal components of tensor $\hat{\alpha}$ are not large, and even in the limit $\alpha_{\parallel} \to \infty$ or $\alpha_{\perp} \to \infty$ the nonuniform states are possible in this subsystem. Let us pick out the most singular part of (29) without a concrete determination of the relation between $\alpha_{\parallel}$ and $\alpha_{\perp}$. Instead of Eq. (14) we have

$$\Delta c_{\mathrm{sing}} = \frac{VT_{c2}^2 a^3 (dA/dt)^2}{16\pi(\beta a^2 + \alpha_{\perp}\lambda^2)(\beta a^2 + \alpha_{\parallel}\lambda^2)^{1/2}} [A(T) - A^*]^{-1/2} , \quad \text{when } T \geq T_{c2} . \tag{30}$$

Comparing Δc_{sing} with the specific-heat jump (15) predicted by the Landau theory, we obtain the following expression for the region of strongly developed fluctuations:

$$|T - T_{c2}| \ll \frac{a^6 B^2 T_{c2}^2}{64\pi^2(\beta a^2 + \alpha_{\perp}\lambda^2)^2(\beta a^2 + \alpha_{\parallel}\lambda^2) dA/dT} . \tag{31}$$

The comparison of the inequality (31) with Eq. (15) for the three-dimensional case shows that in the low-dimensional one (when only one of the components $\alpha_{\parallel}$ or $\alpha_{\perp}$ is large) the region of strongly developed anomalous fluctuations is considerably larger. Nonetheless, as before, the denominator of the right-hand side of inequality (31) contains the very large parameter $\alpha_{\perp}^2$ in the case of quasi-two-dimensional ordering in the first subsystem, or $\alpha_{\parallel}$ in the case of quasi-one-dimensional one.

If one of the parameters $\alpha_{\parallel}$ or $\alpha_{\perp}$ is large enough to neglect the singular contribution to the specific heat (13) it is necessary to consider the limiting case $\alpha_{\perp} \to \infty$ in the quasi-two-dimensional situation or $\alpha_{\parallel} \to \infty$ in the quasi-one-dimensional one in correspondence with the main idea of the previous section. Carrying out a proper limiting transition we obtain the expression (23) for the fluctuation correction to the specific heat connected with the fluctuations in the second subsystem under ordering. The formulae (24)–(26) are also valid. Thus the only fluctuation correction that can be measured experimentally is nonsingular in these cases, and this exactly corresponds to the case of three-dimensional ordering in the first magnetic subsystem.

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