Effect of polarization upon light localization in random layered magnetodielectric media

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Light propagation in a random system of isotropic magnetodielectric layers is studied numerically and analytically. It is shown that if the values of permittivity and permeability are randomly distributed, whereas the characteristic impedance does not change throughout the system, the Lyapunov exponent (the inverse localization length) grows with the angle of incidence and does not depend on the polarization of the incident wave. This independence appears only on the ensemble averaging because in any specific realization the transmission coefficients for *s*- and *p*-polarized light are different. The numerical results confirm analytical analysis.

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I. INTRODUCTION

The light propagation in heterogeneous (both periodic and disordered) media is one of the fundamental problems of electrodynamics, continuously attracting the interest of physicists (see reviews[1–4] and references therein). The most intriguing phenomena observed by light propagation in such systems are the formation of band gaps and localization of light.

The phenomenon of localization was first predicted by Anderson in solid state physics.[5] It has been shown[5–9] that in any one-dimensional disordered system an electron is always localized. It means that the electron's wave function exponentially decreases from a localization center. The characteristic length scale of this decrease is called the localization length L_{loc}.

The Schrödinger and Maxwell equations can be recast as a wave equation. This similarity implies that light should also be localized in a one-dimensional disordered system.[2,3,10–16] Moreover, an absence of the interaction among photons brings the hypothetical quantum problem of a single body into reality in multilayered systems.

In real disordered three-dimensional system the interaction between waves and scatterers is so complicated that both theoretical study and computer simulations are extremely difficult. The solution of the problem requires a series of approximations that are not always justified, making it difficult to relate theoretical predictions to experimental observations. Light propagation in one-dimensional (1D) systems is a more manageable problem that can be exactly solved, for example, by the transfer matrix method.[17] Moreover, results in 1D can provide insight into the problem of wave localization in general and are suitable for testing various ideas. In particular, it has recently been shown[15] that in 1D systems the formation of band gaps in periodic systems and the localization of light in random systems are tightly connected, namely they are determined by the Bragg reflection. From the point of view of developing optical devices, one-dimensional systems play a special role because many devices, such as dielectric mirrors, filters, antireflective layers, etc., are 1D.

Even though the Schrödinger and Maxwell equations are similar, light localization in one-dimensional systems differs from localization of electrons.[1–4,13–16,18–21] The main distinction between them arises from the vector nature of the electromagnetic wave. For oblique incidence, the reflection coefficients for *s*- and *p*-polarized plane waves are different even for isotropic materials[17] so the localization properties of light should be different for different polarizations.[22–24] Indeed, it was shown in Ref. 22 that localization lengths depend upon the polarization for oblique incidence in a disordered binary system of nonmagnetic dielectric layers. For a binary system at the Brewster angle corresponding to the light transmission from one component to another, the *p*-polarized wave becomes delocalized. It is often convenient to use the inverse localization length, called the Lyapunov exponent γ, which in the case of the absence of localization is equal to zero. At the same time, the localization length (as well as γ) for the *s*-polarized wave does not exhibit any peculiarity. However, the transmission coefficient falls off exponentially, which can be regarded as localization in the direction perpendicular to the layers' interfaces.[22]

The localization length can be defined as the length scale at which the transmission coefficient t statistically decreases when the thickness of the system increases[1–3,11,12,22]:

$$L_{\mathrm{loc}} = \frac{1}{\gamma} = \lim_{L\to\infty} \left(L / \langle \ln |t| \rangle \right). \tag{1}$$

In the present paper we study a general case of a random system of magnetodielectric layers for which the Brewster phenomenon may exist not only for p but also for the *s*-polarization or may not exist at all. Special attention is paid to the case when the Brewster phenomenon may be observed for both polarizations. We demonstrate that even though the light scattering is polarization dependent the localization length may not depend upon polarization.

II. BREWSTER PHENOMENON IN MAGNETODIELECTRICS

Let us consider the problem of the plane wave propagation through a layered structure. The wave is incident from a

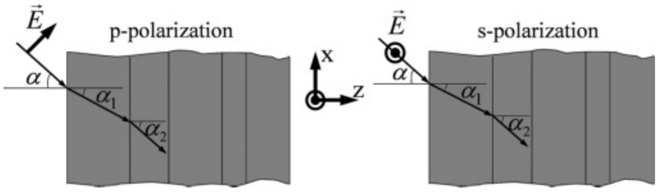

FIG. 1. s- and p-polarized waves incident on the layered structure.

vacuum at an angle α. For a given polarization (s or p) the problem of an oblique incidence can be reduced to the problem of the normal incidence. Each layer will be characterized by the "refraction index" $n_j = \sqrt{\varepsilon_j \mu_j}\cos\alpha_j$ and "surface" admittance Y_j. The latter depends on the polarization and differs from the characteristic admittance $y_j = \sqrt{\varepsilon_j/\mu_j}$. The meaning of the surface admittance of the plane wave is the ratio of the tangent to the surface components of magnetic and electric fields. For the s- and p-polarized waves, the surface admittances are[25] $(Y_j)_s = y_j \cos\alpha_j$ and $(Y_j)_p = y_j/\cos\alpha_j$, respectively, where α_j is the propagation angle in the jth layer (see Fig. 1).

Indeed, for the case of normal incidence (say, along the z axis, as shown in Fig. 1) the Maxwell equations in the jth layer can be reduced to the wave equation[26]

$$\Delta F + k_0^2 n_j^2 F = 0,$$

where $F = E$ or H, $k_0 = \omega/c$, ω is the wave frequency, c is the speed of light in vacuum, and $n_j = \sqrt{\varepsilon_j \mu_j}$ is the refraction index of the jth layer. The solution of this equation has the form

$$\begin{aligned} E &= A_j e^{ik_0 n_j z} + B_j e^{-ik_0 n_j z}, \\ H &= y_j A_j e^{ik_0 n_j z} - y_j B_j e^{-ik_0 n_j z}. \end{aligned} \tag{2}$$

One must also satisfy the boundary conditions making the tangential field components continuous at the interface. For the oblique incidence, as it follows from the Maxwell equations, the tangential components of electric and magnetic fields propagating from left to right are related to each other as

$$\frac{H_\parallel}{E} = \sqrt{\frac{\varepsilon_j}{\mu_j}}\frac{k_{jz}}{k_j} = \sqrt{\frac{\varepsilon_j}{\mu_j}}\cos\alpha_j = (Y_j)_s \tag{3a}$$

$$\frac{H}{E_\parallel} = \sqrt{\frac{\varepsilon_j}{\mu_j}}\frac{k_j}{k_{jz}} = \sqrt{\frac{\varepsilon_j}{\mu_j}}\frac{1}{\cos\alpha_j} = (Y_j)_p. \tag{3b}$$

The refraction index becomes

$$(n_j)_s = (n_j)_p = \sqrt{\varepsilon_j \mu_j}\cos\alpha_j. \tag{3c}$$

For a wave propagating from right to left, the sign of the impedance is negative and the general solution has the form of (2) with substitution (3). Note, that in contrast to the normal incidence, the surface admittance of the vacuum Y_v is not equal to unity. Moreover, it is different for different polarizations:

$$(Y_v)_s = \cos\alpha, \quad (Y_v)_p = \frac{1}{\cos\alpha}. \tag{3d}$$

In the case of a normal incidence ($\alpha = \alpha_1 = \alpha_2 = 0$), the field has tangential components only and the surface and characteristic impedances coincide.

It turns out that in the case of the oblique incidence (Fig. 1), one can obtain a system of equations containing the tangential field components only. Each of these components satisfies the wave equation $\Delta F + k_0^2 \varepsilon_j \mu_j \cos^2\alpha_j F = 0$.[26] The solution of this equation has the form of (2) (except for the insignificant factor $e^{ik_\parallel x}$, where $k_\parallel$ is the tangential component of the wave vector; this factor is omitted below).

For the description of the electromagnetic wave propagation through a layered medium one can use the T-matrix formalism (see, e.g., Ref. 17). As it follows from the above discussion, one can obtain the T matrix for the tangential components of the fields for the oblique incidence by substituting admittances and refraction indices in the T matrix describing the normal incidence $T(Y_j, Y_v, n_j)$ for the respective values in (3).

For nonmagnetic dielectrics, the equation for the Brewster angle,[17,26]

$$\frac{k_\parallel}{\sqrt{\varepsilon_1 k_0^2 - k_\parallel^2}} = \tan(\alpha_1) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}},$$

can be rewritten as an equality of the surface admittances for the p-polarized wave for neighboring layers:

$$\begin{aligned} (Y_1)_p &= \sqrt{\varepsilon_1}\left[\sqrt{1 - \left(\frac{k_\parallel}{k_0\sqrt{\varepsilon_1}}\right)^2}\right]^{-1} \\ &= \sqrt{\varepsilon_2}\left[\sqrt{1 - \left(\frac{k_\parallel}{k_0\sqrt{\varepsilon_2}}\right)^2}\right]^{-1} = (Y_2)_p\,. \end{aligned}$$

This means that the p-polarized light incident on a two-component layered media at the Brewster angle is delocalized, it propagates without reflection from interfaces.[26] At the same time, admittances of different layers for the s-polarized wave are not equal to each other for any angles:

$$(Y_1)_s = \sqrt{\varepsilon_1 - \left(\frac{k_\parallel}{k_0}\right)^2} \neq \sqrt{\varepsilon_2 - \left(\frac{k_\parallel}{k_0}\right)^2} = (Y_2)_s.$$

This inequality represents the fact that in nonmagnetic dielectrics the Brewster angle exists for the p-polarized light only.

For magnetic materials ($\mu_j \neq 1$) the situation is different. Here the functional dependence of that admittance on $k_\parallel$ is more complicated and the Brewster angle can exist for both p- and s-polarized waves.[27] However, as shown in the Appendix, it cannot exist for both polarizations in the same system (with the exception of the special case of equal characteristic admittances of all the layers). At the interface between two media characterized by dielectric permittivities ε_1 and ε_2 and magnetic permeabilities μ_1 and μ_2, the Brewster angles are defined by the systems of equations

$$\text{for } s-\text{polarization} \quad \begin{cases} \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\alpha_1 = \sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\alpha_2 \\ \sqrt{\varepsilon_1\mu_1}\sin\alpha_1 = \sqrt{\varepsilon_2\mu_2}\sin\alpha_2 \end{cases}, \qquad \text{for } p-\text{polarization} \quad \begin{cases} \sqrt{\frac{\varepsilon_1}{\mu_1}}\frac{1}{\cos\alpha_1} = \sqrt{\frac{\varepsilon_2}{\mu_2}}\frac{1}{\cos\alpha_2} \\ \sqrt{\varepsilon_1\mu_1}\sin\alpha_1 = \sqrt{\varepsilon_2\mu_2}\sin\alpha_2 \end{cases}. \tag{4}$$

In particular, when the characteristic admittances for all layers are the same (i.e., magnetic permeability is proportional to dielectric permittivity $\mu_j = \text{const}\cdot\varepsilon_j$), the Brewster angle coincides with the normal $\alpha_1 = \alpha_2 = 0$ and is realized for all polarizations.

III. THE LYAPUNOV EXPONENT IN A RANDOM MAGNETODIELECTRIC LAYERED SYSTEM

To calculate the transmission through the system we use a modified T-matrix formalism.[17,28,29] Between each layer we introduce an auxiliary vacuum layer with zero thickness. The T matrix of such a layer is equal to the unit matrix so that the total T matrix of the system for all polarizations remains unchanged. The advantage of the method is that now we can introduce an independent T matrix for each layer[29]:

$$T_j = \begin{pmatrix} \cos\rho_j + \frac{i}{2}\left(\frac{Y_v}{Y_j} + \frac{Y_j}{Y_v}\right)\sin\rho_j & -\frac{i}{2}\left(\frac{Y_v}{Y_j} - \frac{Y_j}{Y_v}\right)\sin\rho_j \\ \frac{i}{2}\left(\frac{Y_v}{Y_j} - \frac{Y_j}{Y_v}\right)\sin\rho_j & \cos\rho_j - \frac{i}{2}\left(\frac{Y_v}{Y_j} + \frac{Y_j}{Y_v}\right)\sin\rho_j \end{pmatrix},$$

where $\rho_j = k_0 n_j d_j$, d_j is the thickness of the jth layer, and expressions for the surface admittances of the vacuum Y_v and a layer Y_j are given by Eq. (3a) or (3b) dependent on the polarization.

Now, let us calculate the Lyapunov exponent (1) for our system for each of the polarizations. The T matrix of the whole system is

$$T = T_N T_{N-1}\cdots T_2 T_1. \tag{5}$$

It is convenient to use the representation of the T matrix as a product of matrices which depends on $Y_v\,(\alpha)$ or $\cos\alpha_j$ only. Such a product has the form

$$T_j = B_j A_j B_j^{-1},$$

where, for example, the s-polarized wave, we have

$$A_{sj} = \begin{pmatrix} \cos\rho_j + \frac{i}{2}\left(\frac{1}{\cos\alpha_j} + \cos\alpha_j\right)\sin\rho_j & \frac{i}{2}\left(\cos\alpha_j - \frac{1}{\cos\alpha_j}\right)\sin\rho_j \\ \frac{i}{2}\left(\frac{1}{\cos\alpha_j} - \cos\alpha_j\right)\sin\rho_j & \cos\rho_j - \frac{i}{2}\left(\frac{1}{\cos\alpha_j} + \cos\alpha_j\right)\sin\rho_j \end{pmatrix}, \qquad B_j = \begin{pmatrix} Y_v + \sqrt{\frac{\varepsilon_j}{\mu_j}} & Y_v - \sqrt{\frac{\varepsilon_j}{\mu_j}} \\ Y_v - \sqrt{\frac{\varepsilon_j}{\mu_j}} & Y_v + \sqrt{\frac{\varepsilon_j}{\mu_j}} \end{pmatrix}.$$

Since characteristic admittances for all layers are the same, B_j does not depend on j but depends on the polarization. Therefore (5) can be rewritten in the form

$$T = BA_N A_{N-1}\cdots A_1 B^{-1} = BAB^{-1}. \tag{6}$$

As the matrix B is finite and does not depend on the thickness and the realization of the random system, the localization length is fully defined by the matrix A.

Matrix A_{sj} for the s-polarization has the form of the T matrix for the normal incidence of the wave from a vacuum on a layer with the optical thickness ρ_j and the admittance $\cos\alpha_j$. A similar matrix for the p-polarized wave A_{pj} can be obtained from A_{sj} by substitution of $1/\cos\alpha_j$ instead of $\cos\alpha_j$. Therefore these matrices are related as

$$A_{pj} = \left(A_{sj}\right)^T,$$

where the superscript T denotes the transpose. Thus

$$\begin{aligned} A_p &= A_{pN}A_{pN-1}\cdots A_{p1} \\ &= (A_{sN})^T(A_{sN-1})^T\cdots(A_{s1})^T = (A_{s1}A_{s2}\cdots A_{sN})^T. \end{aligned}$$

As the transmission matrix through a model system of layers, the matrix $A_s = A_{sN}A_{sN-1}\cdots A_{s1}$ can be represented as

$$A_s = A_{sN}A_{sN-1}\cdots A_{s1} = \begin{Vmatrix} t - \frac{r_L r_R}{t} & \frac{r_R}{t} \\ -\frac{r_L}{t} & \frac{1}{t} \end{Vmatrix}, \tag{7}$$

where t is the transmission coefficient through the system, r_L and r_R are the reflection coefficients for the wave incident from the left and right, respectively. The matrix $A_{s1}A_{s2}\cdots A_{sN}$ differs from $A_{sN}A_{sN-1}\cdots A_{s1}$ by the inverse order of the model layers. Therefore the former matrix is represented by (7) with r_L and r_R interchanged:

$$A_{s1}A_{s2}\cdots A_{sN} = \begin{Vmatrix} t - \frac{r_L r_R}{t} & \frac{r_L}{t} \\ -\frac{r_R}{t} & \frac{1}{t} \end{Vmatrix}.$$

So that

$$A_p = (A_{s1}A_{s2}\cdots A_{sN})^T = \begin{Vmatrix} t - \frac{r_L r_R}{t} & -\frac{r_R}{t} \\ \frac{r_L}{t} & \frac{1}{t} \end{Vmatrix}. \tag{8}$$

From Eqs. (7) and (8) one can see that the model systems described by matrices A_s and A_p have identical transmission coefficients, and their reflection coefficients differ by signs only. Now, in the matrix T_s [Eq. (6)] let us express the transmission coefficient t_s of the s-polarized wave via t, r_L, and r_R. After some simplifications we obtain

$$\frac{1}{t_s} = \frac{(1-r_L)(1-r_R)\frac{\varepsilon}{\mu} + 2(1-r_Lr_R)\sqrt{\frac{\varepsilon}{\mu}}Y_v + (1+r_L)(1+r_R)Y_v^2 - t^2\left(Y_v - \sqrt{\frac{\varepsilon}{\mu}}\right)^2}{4t\sqrt{\frac{\varepsilon}{\mu}}Y_v}$$
$$= \frac{\left[\sqrt{\frac{\varepsilon}{\mu}}(r_L-1) - Y_v(r_L+1)\right]\left[\sqrt{\frac{\varepsilon}{\mu}}(r_R-1) - Y_v(r_R+1)\right]}{4t\sqrt{\frac{\varepsilon}{\mu}}Y_v} - t\frac{\left(Y_v - \sqrt{\frac{\varepsilon}{\mu}}\right)^2}{4\sqrt{\frac{\varepsilon}{\mu}}Y_v}. \tag{9}$$

When the thickness of the system is big, the absolute value of the transmission coefficient $|t|$ is exponentially small, the absolute values of the reflection coefficients $|r_R|$ and $|r_L|$ are of the order of unity, and their phases are distributed uniformly in the interval of $[0,2\pi]$.[30] None of the factors in the numerator of the first term of the right-hand side of (9) is equal to zero because this would imply that

$$r = \left(\sqrt{\frac{\varepsilon}{\mu}} + Y_v\right) \Big/ \left(\sqrt{\frac{\varepsilon}{\mu}} - Y_v\right),$$

which is not possible since

$$\left|\left(\sqrt{\frac{\varepsilon}{\mu}} + Y_v\right) \Big/ \left(\sqrt{\frac{\varepsilon}{\mu}} - Y_v\right)\right| > 1.$$

Therefore, we can neglect the second term of the right-hand side of (9), which is proportional to $|t|$. Then

$$\gamma_s = \frac{1}{L}\left\langle \ln\left|\frac{1}{t_s}\right|\right\rangle = \frac{1}{L}\left\langle \ln\left|\frac{1}{t}\right|\right\rangle + \frac{1}{L}\left\langle \ln\left|\sqrt{\frac{\varepsilon}{\mu}}(r_L-1) - Y_v(r_L+1)\right|\right\rangle + \frac{1}{L}\left\langle \ln\left|\left[\sqrt{\frac{\varepsilon}{\mu}}(r_R-1) - Y_v(r_R+1)\right] \Big/ 4\sqrt{\frac{\varepsilon}{\mu}}Y_v\right|\right\rangle. \tag{10}$$

The values of the second and the third logarithms of the right-hand side of (10) are finite, so when the thickness of the system increases, the respective terms vanish. Thus the Lyapunov exponent for the s-polarized wave in the real system is equal to the Lyapunov exponent for the model system of layers with the transmission matrices A_{sj}. Calculating the Lyapunov exponent for the p-polarized wave we have to

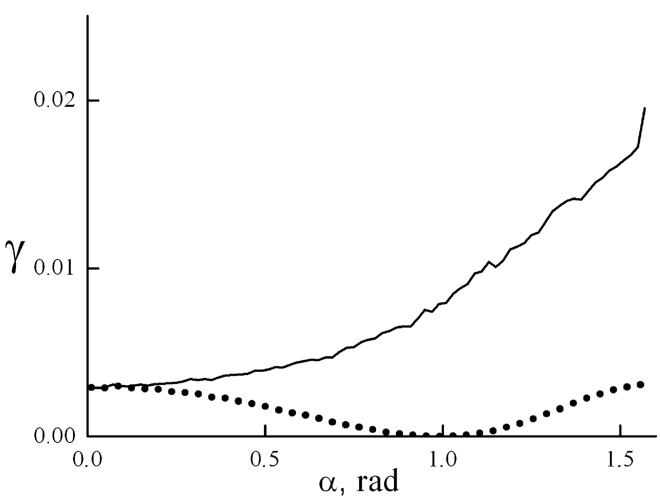

FIG. 2. The dependence of γ, normalized by the average layer's thickness d, on the angle of incidence α, for a two-component layered medium at a frequency $k_0d = 0.8$. The layers of the first and the second types are randomly mixed and their thicknesses are distributed uniformly between $0.8d$ and $1.2d$. The layers' characteristics are $\varepsilon_1 = 1.2$, $\mu_1 = 1$ and $\varepsilon_2 = 1.68$, $\mu_2 = 1$. The solid and dotted curves correspond to s and p polarizations, respectively. The dotted curve is calculated for the ensemble of 500 realizations of the random structure, composed of 50 000 layers. The solid curve is calculated for the ensemble of 2000 realizations of the random structure, composed of 1500 layers.

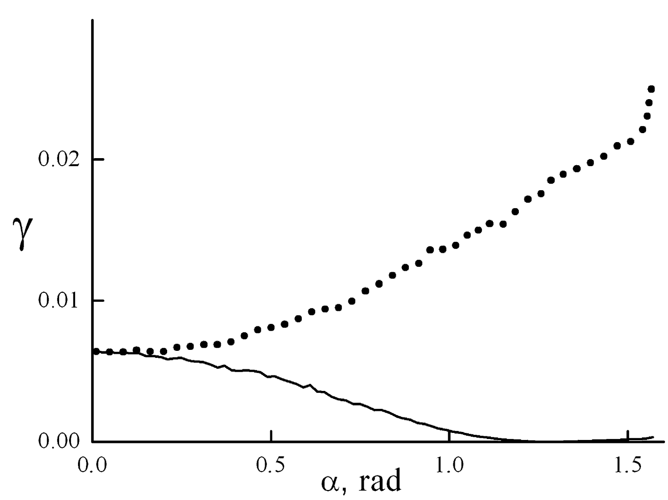

FIG. 3. The same as in Fig. 2 except $\varepsilon_1 = 1.5$, $\mu_1 = 1$ and $\varepsilon_2 = 1.5$, $\mu_2 = 1.6$. The solid curve is calculated for the ensemble of 500 realizations of the random structure, composed of 50 000 layers. The dotted curve is calculated for the ensemble of 4000 realizations of random structure, composed of 700 layers.

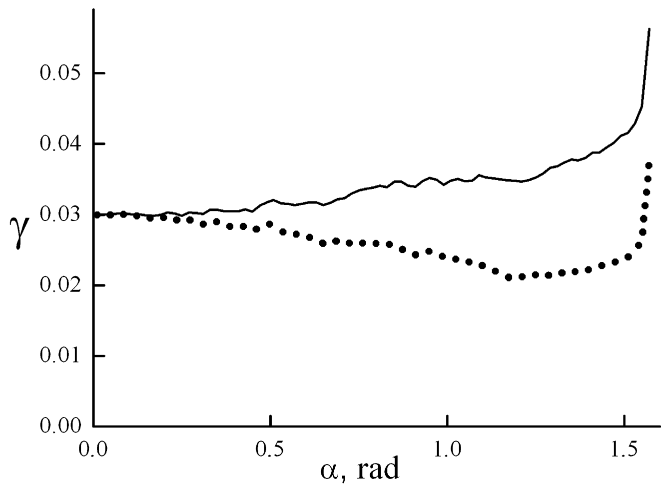

FIG. 4. The same as in Figs. 2 and 3 except $k_0 d = 1.0$, $\varepsilon_1 = 5$, $\mu_1 = 1.08$, and $\varepsilon_2 = 15$, $\mu_2 = 1.1$. The dotted and solid curves are calculated for the ensemble of 5000 realizations of the random structure, composed of 400 layers.

replace r_L, and r_R by $-r_L$ and $-r_R$. As the result, the Lyapunov exponent γ does not change, so that

$$\gamma_s = \frac{1}{L}\left\langle \ln\left|\frac{1}{t}\right|\right\rangle = \gamma_p. \tag{11}$$

Let us note that for the normal incidence, if ε and μ are interchanged, the transmission coefficient does not change. Indeed, in this case, in the Maxwell equations, the fields E and H are substituted by H and $-E$, respectively. Hence, when the admittance Y is substituted by $1/Y$, the transmission coefficient does not change. Therefore in our model systems the transmission coefficients, as well as the localization lengths, are the same. Thus the localization lengths are identical for both s- and p-polarized waves.

Despite the equality of the transmission coefficients in model systems, the reflection coefficients for these model systems have different signs. Therefore the matrices A for different polarizations are different. In addition, matrices B for the s and p polarizations are also different. Consequently, in general, the transmission coefficients for the s- and p-polarized waves for an oblique incidence are different. Nonetheless, the localization lengths are equal. This result is in agreement with the formula for the localization length obtained in the perturbation theory in Ref. 13 and generalized for the oblique incidence in Ref. 31.

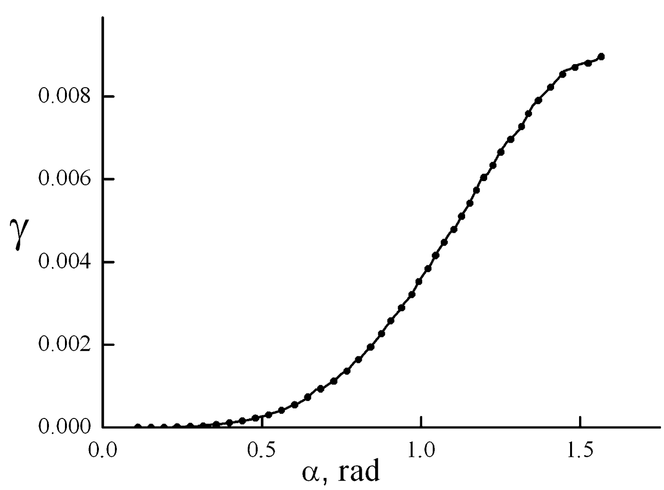

FIG. 5. The dependence of γ normalized by the average layer's thickness d on the angle of incidence α for a two-component layered medium at a frequency $k_0 d = 1.0$. The characteristics of the first and the second layers are $\varepsilon_1 = 2$, $\mu_1 = 1$ and $\varepsilon_2 = 15$, $\mu_2 = 7.5$, respectively. The layers of the first and the second types are randomly mixed and their thicknesses are distributed uniformly between $0.8d$ and $1.2d$. The solid and dotted curves correspond to s and p polarizations, respectively. For the angles between 0.1 and 0.4 radian, both curves are calculated for the ensemble of 400 realizations of the random structure, composed of 100 000 layers, and for the angles between 0.4 and 1.57 radian both curves are calculated for the ensemble of 1000 realizations of the random structure, composed of 30 000 layers.

IV. NUMERICAL CALCULATIONS

The numerical results for the dependence of the Lyapunov exponent on the angle of incidence for different values of dielectric permittivities and magnetic permeabilities for constant frequency are shown in Figs. 2–5. In the case when magnetic permeability is the same for all layers (Fig. 2), for the p-polarized wave there exists the delocalization angle,[22] which is equal to the Brewster angle (4) for the transition from one layer to another. At the same time, for the s-polarized wave, γ grows with the angle of incidence. This is associated with the increase of the surface admittances' contrast when the angle increases [see Eq. (3a)].

In the case when dielectric permittivity is the same for all layers and magnetic permeabilities are different, the situation is exactly the opposite (see Fig. 3). Here the delocalization angle for the s-polarized wave exists. This angle is equal to the

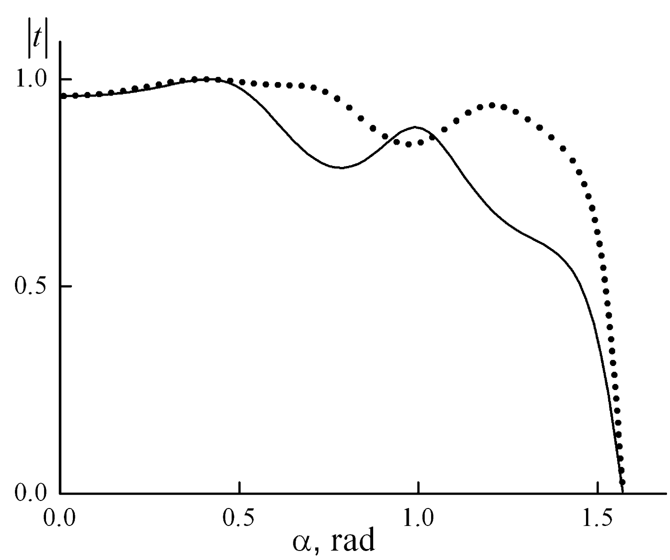

FIG. 6. Transmission coefficients of s- (the solid curve) and p- (the dotted curve) polarized waves through the system of 30 layers as functions of the angle of incidence calculated for a single realization of the random structure. The values of parameters are the same as in Fig. 5.

Brewster angle of the s-polarized wave in magnetic layers (4). The Lyapunov exponent for p-polarized wave is growing with the angle of incidence.

It turns out that for some values of the parameters, the Brewster angle does not exist for both s- and p-polarized waves. In this case γ is not equal to zero at any angle (see Fig. 4).

Hence, generally, our system may have the Brewster angle either for s- or p-polarized waves or the angle does not exist. Only in the case of equal characteristic admittances is the Brewster angle realized for both polarizations (see the Appendix). In this case the Brewster angle is equal to zero (the normal incidence). Let us consider localization in such a system.

When the characteristic admittances for all the layers are the same, for the normal incidence ($\alpha = 0$) the numerical experiment confirms an obvious prediction that there is no localization for both polarizations, so that for the Lyapunov exponents we have $\gamma_s = \gamma_p = 0$.

For the oblique incidence the Lyapunov exponents have nonzero values for both polarizations. However, we find that within the accuracy of the numerical experiment (the errors occur due to the finite size of the system and finite number of realizations) the Lyapunov exponents for s and p polarizations are the same (see Fig. 5). The numerical calculations shown in Fig. 5 are in a good agreement with analytical results (11).

As we discussed in Sec. III, the transmission coefficients for s and p polarizations are different in each realization, as shown in Fig. 6.

The obtained results are also valid for a disordered system of metamaterial layers with negative permittivities in which the surface plasmon resonance may exist (see, e.g., Ref. 31 and references therein). As shown in Fig. 7, in such systems when characteristic admittances do not change the localization lengths for s- and p-polarized waves are identical.

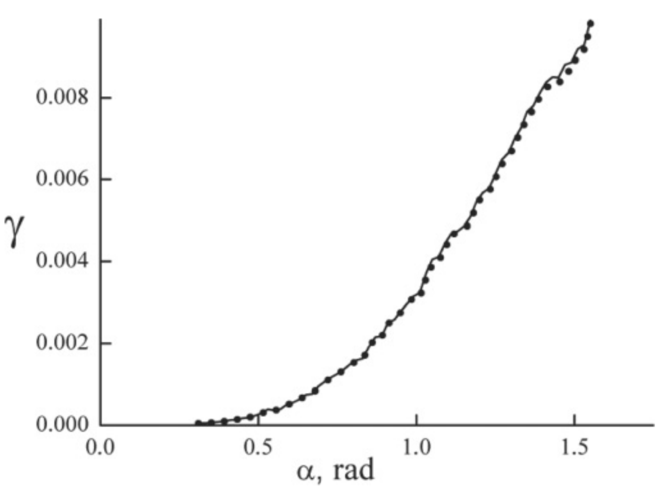

FIG. 7. The dependence of γ normalized by the average layer's thickness d on the angle of incidence α for a random two-component layered medium. The characteristics of the first and the second layer types are $\varepsilon_1 = -2,\ \mu_1 = -1$ and $\varepsilon_1 = 15,\ \mu_1 = 7.5$, respectively, all other parameters are the same as in Fig. 5.

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APPENDIX

In this Appendix we prove that the Brewster angles cannot exist for both s- and p-polarized waves at the same system (if $\sqrt{\varepsilon_1/\mu_1} \neq \sqrt{\varepsilon_2/\mu_2}$). Let us consider an interface between two media characterized by dielectric permittivities ε_1 and ε_2 and magnetic permeabilities $\mu_1$and μ_2, respectively. We assume that $\varepsilon_1 \geqslant 1, \varepsilon_2 \geqslant 1, \mu_1 \geqslant 1, \mu_2 \geqslant 1$, and $\sqrt{\varepsilon_1/\mu_1} \neq \sqrt{\varepsilon_2/\mu_2}$. Let us presume the possibility of the existence of the Brewster angles for both polarizations. Then, from the equality of the surface admittances for the p polarization,

$$\left(\frac{\varepsilon_1}{\mu_1}\right)\left(1-\frac{\sin^2\alpha_p}{\varepsilon_1\mu_1}\right)^{-1} = \left(\frac{\varepsilon_2}{\mu_2}\right)\left(1-\frac{\sin^2\alpha_p}{\varepsilon_2\mu_2}\right)^{-1}, \quad \text{(A1)}$$

one can obtain the expression for the Brewster angle for the p-polarized wave

$$\sin^2\alpha_p = \frac{\varepsilon_1\varepsilon_2(\varepsilon_1\mu_2 - \varepsilon_2\mu_1)}{\varepsilon_1^2 - \varepsilon_2^2}.$$

In these equations $\sin\alpha_p$ denotes the ratio $k_\parallel/k_0$, at which Eq. (A1) holds. We assume that $\varepsilon_1 > \varepsilon_2$, then in order to have $\sin^2\alpha_p > 0$, the inequality

$$\varepsilon_1\mu_2 > \varepsilon_2\mu_1 \quad \text{(A2)}$$

must be satisfied.

Similarly, using equality of surface admittances for the s-polarized waves,

$$\left(\frac{\varepsilon_1}{\mu_1}\right)\left(1-\frac{\sin^2\alpha_s}{\varepsilon_1\mu_1}\right) = \left(\frac{\varepsilon_2}{\mu_2}\right)\left(1-\frac{\sin^2\alpha_s}{\varepsilon_2\mu_2}\right),$$

we obtain an expression for the Brewster angle for the s polarization

$$\sin^2\alpha_s = \frac{\mu_1\mu_2(\varepsilon_1\mu_2 - \varepsilon_2\mu_1)}{\mu_2^2 - \mu_1^2}.$$

In order to satisfy inequality (A2), along with $\varepsilon_1 > \varepsilon_2$, and to have $\sin^2\alpha_s$ be positive, an additional condition must be imposed: $\mu_2 > \mu_1$.

In order for α_s and α_p to exist simultaneously, they must have real values, which means that $\sin^2\alpha_p \leqslant 1$ and $\sin^2\alpha_s \leqslant 1$. This leads to the inequalities $0 < (\varepsilon_1\mu_2 - \varepsilon_2\mu_1) < (\varepsilon_1^2 - \varepsilon_2^2)/\varepsilon_1\varepsilon_2$ and $0 < (\varepsilon_1\mu_2 - \varepsilon_2\mu_1) < (\mu_2^2 - \mu_1^2)/\mu_1\mu_2$ or equivalently $\varepsilon_1^2(\varepsilon_2\mu_2 - 1) < \varepsilon_2^2(\varepsilon_1\mu_1 - 1)$ and $\mu_2^2(\varepsilon_1\mu_1 - 1) < \mu_1^2(\varepsilon_2\mu_2 - 1)$. From the latter two inequalities one obtains $\varepsilon_1\mu_2 < \varepsilon_2\mu_1$, which contradicts inequality (A2). Hence, in a general case when $\varepsilon_1 \geqslant 1, \varepsilon_2 \geqslant 1, \mu_1 \geqslant 1, \mu_2 \geqslant 1$, and $\sqrt{\varepsilon_1/,\mu_1} \neq \sqrt{\varepsilon_2/\mu_2}$, if the Brewster angle exists, then it exists for one of the polarizations only.

Now, let us assume that only inequalities $\varepsilon_1 \neq 0, \varepsilon_2 \neq 0, \mu_1 \neq 0, \mu_2 \neq 0$ are satisfied. We also assume that signs of ε and μ are the same. Let us suppose that the Brewster angles exist for both s and p polarizations. These angles must be smaller than the angles of the total internal reflection for the light incident on either media from vacuum. This implies

$$\sin^2\alpha_s = \frac{\mu_1\mu_2(\varepsilon_1\mu_2 - \varepsilon_2\mu_1)}{\mu_2^2 - \mu_1^2} < \varepsilon_1\mu_1,$$

$$\sin^2\alpha_p = \frac{\varepsilon_1\varepsilon_2(\varepsilon_1\mu_2 - \varepsilon_2\mu_1)}{\varepsilon_1^2 - \varepsilon_2^2} < \varepsilon_2\mu_2.$$

These inequalities are not compatible because from the first one it follows that $\varepsilon_1\mu_1 < \varepsilon_2\mu_2$, and the second one implies that $\varepsilon_1\mu_1 > \varepsilon_2\mu_2$.

Hence, for the oblique incidence either the Brewster angle does not exist for one of the polarizations or the wave with s or p polarization experiences the total internal reflection when it is incident on the interface from vacuum at the Brewster angle. When $\sqrt{\varepsilon_1\mu_1} = \sqrt{\varepsilon_2\mu_2}$, the Brewster angle is equal to zero [Eq. (3a)] and the phenomenon does not depend on the polarization.

[1] I. M. Lifshitz, S. A. Gredeskul, and L. A. Pastur, *Introduction to the Theory of Disordered Systems* (Wiley, New York, 1989).
[2] C. M. Soukoulis, ed., *Photonic Crystals and Light Localization in the 21th Century* (Kluwer, Dordrecht, The Netherlands, 2001).
[3] P. Sheng, *Introduction to Wave Scattering, Localization and Mesoscopic Phenomena*, 2nd ed. (Springer, Berlin, 2006).
[4] V. V. Shalaev, ed., *Optical Properties of Nanostructured Random Media* (Berlin, Springer, 2010).
[5] P. W. Anderson, Phys. Rev. **109,** 1492 (1958).
[6] N. F. Mott, Adv. Phys. **16**, 49 (1967).
[7] N. F. Mott, Adv. Phys. **50**, 865 (2001).
[8] K. Ishii, Progr. Theor. Phys. Suppl. **53**, 77 (1973).
[9] M. Luban and J. H. Luscombe, Phys. Rev. B **35**, 9045 (1987).
[10] S. John, Phys. Rev. Lett. **53**, 2169 (1984).
[11] H. Furstenberg, Trans. Am. Math. Soc. **108**, 377 (1963).
[12] V. D. Freilikher and S. A. Gredeskul, Progress Opt. **30**, 137 (1992).
[13] F. M. Izrailev and N. M. Makarov, Phys. Rev. Lett. **102**, 203901 (2009).
[14] P. Han and C. Zheng, Phys. Rev. E **77**, 041111 (2008).
[15] A. P. Vinogradov and A. M. Merzlikin, Phys. Rev. E **70**, 026610 (2004).
[16] M. A Kaliteevski, D. M. Beggs, S. Brand R. A. Abram, and V. V. Nikolaev, Phys. Rev. B **73**, 033106 (2006).
[17] M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, 7th ed. (Cambridge University Press, Cambridge, 1999).
[18] A. M. Merzlikin, M. Levy, A. A. Jalali, and A. P. Vinogradov, Phys. Rev. B **79**, 195103 (2009).
[19] S. G. Erokhin, A. A. Lisyansky, A. M. Merzlikin, A. P. Vinogradov, and A. B. Granovsky, Phys. Rev. B **77**, 233102 (2008).
[20] A. M. Merzlikin, A. P. Vinogradov, A. V. Dorofeenko, M. Inoue, M. Levy, and A. B. Granovsky, Physica B **394**, 277 (2007).
[21] K.Yu. Bliokh, Yu. P. Bliokh, V. Freilikher, S. Savel'ev, and F. Nori, Rev. Mod. Phys. **80**, 1201 (2008).
[22] J. E. Sipe, P. Sheng, B. S. White, and M. H. Cohen, Phys. Rev. Lett. **60**, 108 (1988).
[23] X. Du, D. Zhang, X. Zhang, B. Feng, and D. Zhang, Phys. Rev. B **56**, 28 (1997).
[24] W. Deng and Z.-Q. Zhang, Phys. Rev. B **55**, 14230 (1997).
[25] L. M. Brekhovskikh, *Waves in Layered Media*, 2nd ed. (Academic, New York, 1980).
[26] L. D. Landau, L. P. Pitaevskii, and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Vol. **8** (Course of Theoretical Physics), 2nd ed. (Butterworth-Heinemann, London, 1984).
[27] V. G. Veselago, Phys. Usp. **46**, 764 (2003).
[28] P. Yeh, A. Yariv, and C.-S. Hong, J. Opt. Soc. Am. **67**, 423 (1977).
[29] A. P. Vinogradov and A. M. Merzlikin, JETP **94**, 482 (2002).
[30] K. Kim, Phys. Rev. B **58**, 6153 (1998).
[31] D. Mogilevtsev, F. A. Pinheiro, R. R. dos Santos, S. B. Cavalcanti, and L. E. Oliveira., Phys. Rev. B **82**, 081105 (2010).