Wave localization in generalized Thue-Morse superlattices with disorder

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In order to study an influence of correlations on the localization properties of classical waves in random superlattices we introduce a generalized random Thue-Morse model as a four-state Markov process with two parameters that determine probabilities of different configurations. It is shown that correlations can bring about a considerable change in the transmission properties of the structures and in the localization characteristics of states at different frequencies. [S1063-651X(97)09810-3]

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I. INTRODUCTION

The effects of correlation on localization properties of electrons and classical waves in one-dimensional (1D) disordered systems recently have attracted a great deal of attention. For the canonical Anderson model [1] with uncorrelated diagonal disorder, it is a well-established fact that almost all states in 1D systems are localized, ensuring the absence of transport through such systems. Correlation between, for example, random values of energy at different sites was proven to change this situation dramatically. This was shown in Ref. [2], where the so-called random dimer model was introduced. In this model, the same value of energy was randomly assigned to pairs of consecutive sites, which introduced ''rigid'' correlations between energies at consecutive sites. It was shown that in such a model $\sqrt{N}$, where N is the number of sites, states remain delocalized. These delocalized states appear in the vicinity of certain resonant values of energy. The random dimer model is in some aspects analogous to classical wave propagation through a random superlattice constructed from different layers with fixed thickness stacked at random. It was shown in Refs. [3–5] that in the superlattice with two randomly positioned layers there exist two resonance frequencies at which the transmissivity of the system becomes equal to one. In both random dimer and random superlattice models, the dimers or layers themselves were assumed to be distributed randomly without correlation. It is interesting, however, to study how some additional correlations between different blocks of these models affect the localization properties. For the dimer model this question was addressed in Refs. [6–8]. The first of these papers dealt with the effects of thermally induced correlations on the localization length of a random dimer harmonic chain. In Ref. [7] a dependence of the localization length upon the correlation radius of a Markov sequence of the product of random matrices was studied and in Ref. [8] fluctuations of the Lyapunov exponent in the system of finite size were investigated.

The transmission coefficient and localization length of acoustic waves in random correlated superlattices were considered in Ref. [9]. Correlations in that paper were introduced by constructing the superlattice according to three different Markov processes. The Hendricks-Teller (HT) model, the randomized Markov versions of Fibonacci, and Thue-Morse (TM) sequences were considered. The Hendricks-Teller model is a version of a dichotomous process, which is known to result in a stochastic structure with an exponential correlation function. The main feature of Fibonacci and TM superlattices compared to the HT model is the presence of short-range order. It was found in Ref. [9] that the frequency dependence of the transmission coefficient is quantitatively different for the first two models and the last one. One can assume that this difference is due to the difference in the short-range structure of the systems.

In the present paper we proceed with a detailed study of the effects of the short-range correlations on the localization properties of 1D random systems. For the sake of concreteness, we deal with scalar wave propagation through a random superlattice. Our results, however, can be applied to dimer models as well. We consider a random superlattice constructed from two layers A and B with different characteristics (dielectric constants, for instance, in the case of electromagnetic wave propagation) stacked at random according to the rules described in the first section of the paper. These rules introduce a generalized Markov Thue-Morse model. This model can be reduced to the canonical random TM model considered in Ref. [9] by selecting proper probabilities. Our model can also be reduced to the HT model with exponential correlations, so we will be able to investigate an interplay between ''soft'' exponential correlations and a more ''rigid'' short-range order introduced by Thue-Morse-like rules.

II. THE MODEL AND ITS STATISTICAL PROPERTIES

We consider the propagation of classical waves through one-dimensional random media. This model corresponds to propagation of elastic or electromagnetic waves through a layered medium that is random in the direction of propagation of waves and homogeneous in the transversal direction. For the case of normal incidence, the vector nature of waves can be neglected since no conversion between different polarizations occurs and one can consider the scalar wave equation

$$\frac{d^2E}{dx^2}+k_0^2\epsilon(x)=0, \tag{1}$$

where $k_0 = \omega/c$ is the wave vector of the wave with the frequency ω propagating with speed c in a homogeneous medium surrounding a disordered material. The parameter $\epsilon(x)$ describes a superlattice composed of two different layers with the same thickness d, so that $\epsilon(x)$ takes two different values ϵ_1 and ϵ_2 for each of the layers. These layers are stacked together at random according to the following rules. If a layer is the first one in a sequence of similar layers, then the probability for the second like layer to appear is p. If two like layers already appear in a sequence, the probability for the third consecutive like layer to occur is equal to q. These rules introduce a four-state Markov process with the conditional probabilities $P(AB|B) = P(BA|A) = p$ and $P(AA|A) = P(BB|B) = q$. The conventional TM Markov superlattice considered in [9] corresponds to $p = 1/2$, $q = 0$. This choice of parameters p and q ''forbids'' the occurrence of blocks of the same layers with a length of more than 2. Another interesting realization of this model, which in a sense is opposite to the TM model, arises if one takes $p = 1$ and $q = 1/2$. In this case blocks with the length less than 2 are forbidden. One should not confuse, however, this case with a simple dimerization of layers. In the latter case only blocks with even numbers of like layers can appear, which is obviously equivalent to doubling of layers' thicknesses. In the model proposed here blocks with odd number of like layers and blocks with even numbers of layers can occur. For $p = q$ the model reproduces the properties of the so-called dichotomous process (two-state Markov process), with p being the transition probability from one state to another. This is proven to result in a exponential correlation function with the correlation length $l_{\exp} = (-\ln|2p-1|)^{-1}$. Hence we can conclude that the proposed model exhibits two kinds of correlations. First, we have short-range correlations determined by the specific short-range order presumed in the model with fixed correlation length $l_{\mathrm{sh}} = 2d$ and with the degree of correlation being proportional to the difference $|p-q|$. For $q \neq 1/2$ we have exponentially decreasing correlations of the HT kind. Figures 1(a)–1(c) present the results of numerical calculations of the correlation function $K(r_1 - r_2) = \langle \epsilon(r_1)\epsilon(r_2)\rangle - \langle(\epsilon)^2\rangle$, where angular brackets denote averaging over different realizations of the random function $\epsilon(r)$. For numerical averaging we use 10 000 realizations of a superlattice constructed in accord with the rules described above. Calculations were carried out for superlattices of different lengths and with different choices of the starting point. We found that correlation functions do not depend upon the size of the system or upon the starting point. The insets in these figures present the Fourier transforms $S(q)$ of the corresponding correlation functions. These results demonstrate that the system indeed has both short-range and long-range correlations. (We use the term ''long-range correlations'' to refer to *exponential* correlations, which can have a correlation radius larger than the layer's thickness. Short-range correlations at the scale of several thicknesses of a layer cause oscillations of the correlation functions and corresponding maxima on their Fourier transforms. In the system with short-range correlations only the function $S(q)$ takes zero value at $q = 0$ [Fig. 1(a)]. These correlations disappear when $|p-q| \rightarrow 0$. Long-range correlations exist for $q \neq 1/2$ only and are responisble for the exponential tail of the correlation functions. They reduce the maximum of the function $S(q)$

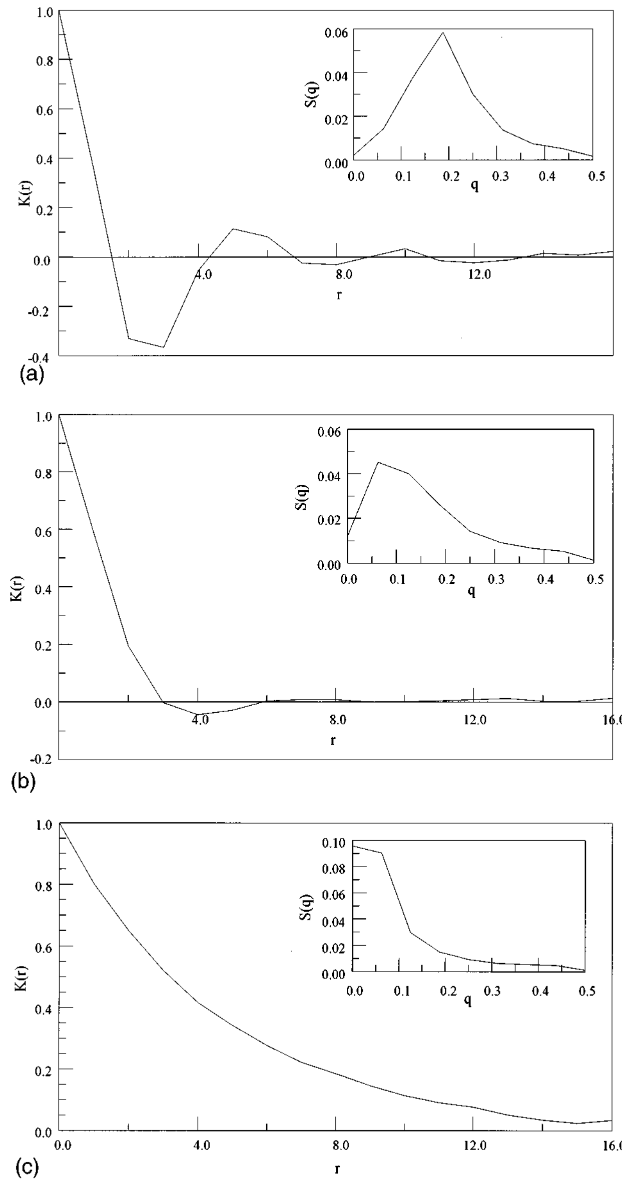


FIG. 1. Correlation function $K(r) = \langle \epsilon(0)\epsilon(r)\rangle - \langle \epsilon^2 \rangle$, for different types of random superlattices. The insets represent the Fourier transforms of the functions $K(r)$. (a) $p = 1$, $q = 1/2$; (b) $p = 1$, $q = 0.6$; (c) $p = q = 0.8$.

and cause its smoother decrease for larger q [Fig. 1(b)]. At $q < 1/2$ these correlations are actually ''anticorrelations'' since they favor the appearance of different blocks at adjacent positions. Two extreme cases with $p = 1$, $q = 0$ and $p = 0$, $q = 1$ correspond to fully ordered periodic structures with periods $2d$ and d, respectively. The case $q = 1$ results in homogeneous structures consisting of one type of block only. It can be either block A or B, whichever block occurs first.

Localization properties of a wave propagating through a random superlattice are determined by scattering from interfaces between blocks of the same layers. Therefore, an important statistical characteristic of the system relevant to wave propagation is the distribution of block lengths $P(n)$, where n is the number of layers of the same kind constituting a block and P is the probability of finding a block of length n. For our model, this function can be shown to be

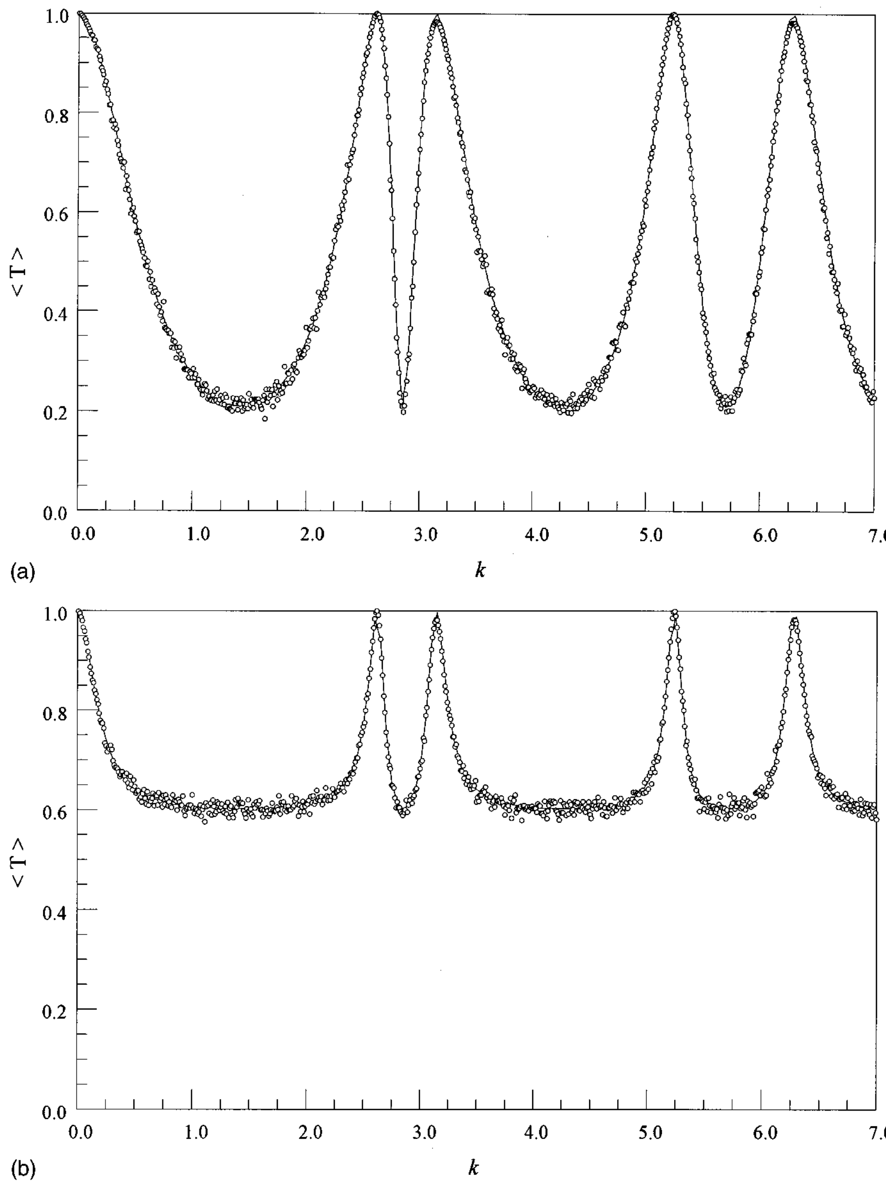


FIG. 2. Average transmission rate for different types of random superlattices. Circles in this figure and in all figures that follow present the results of computer simulations and the solid line shows the theoretical results. (a) $p=q=0.5$ (model without correlations); (b) $p=q=0.8$ (HT model); (c) $p=1$, $q=0.5$ (short-range correlations only); (d) Markov TM model, $p=0.5$, $q=0$ (circles), and generalized TM model, $p=1$, $q=0.5$ (squares); the number of layers is equal to 64; (e) $p=1$, $q=0.8$ (short-range and exponential correlations are present).

$$P(1)=1-p,$$
$$P(n)=p(1-q)q^{n-2} \quad \text{for} \quad n\geqslant 2. \tag{2}$$

In the extreme case $p=1$, $q=1/2$, Eq. (2) takes the form $P(1)=0$, $P(n)=(1/2)^{n-1}$, $n>1$, which is quite similar to the result for an uncorrelated superlattice $P(n)=(1/2)^n$. We show, however, that a seemingly small discrepancy between these two distributions results in a considerable qualitative difference between localization properties of waves propagating in corresponding media. The average lengths of A and B blocks $\langle D_{A,B}\rangle$ are equal to each other,

$$\langle D_A\rangle=\langle D_B\rangle=d\frac{1+p-q}{1-q}, \tag{3}$$

where the total length of the system is assumed to be infinite. This expression diverges at $q\rightarrow 1$, which merely reflects the fact that at $q=1$ the entire superlattice is composed of the same blocks, so the average length of this block is equal to the total length of the system assumed to be infinite.

III. AVERAGE TRANSMISSION RATE AND LYAPUNOV EXPONENT

In order to simulate wave propagation through the system, we make use of the transfer-matrix method. A transfer-matrix connects an amplitude E_n and its first derivative $E_n'=dE/dx$ of the field in nth slab with the corresponding values in the $(n+1)$th slab:

$$\mathbf{u}_{n+1}=\mathbf{T}_n\mathbf{u}_n, \tag{4}$$

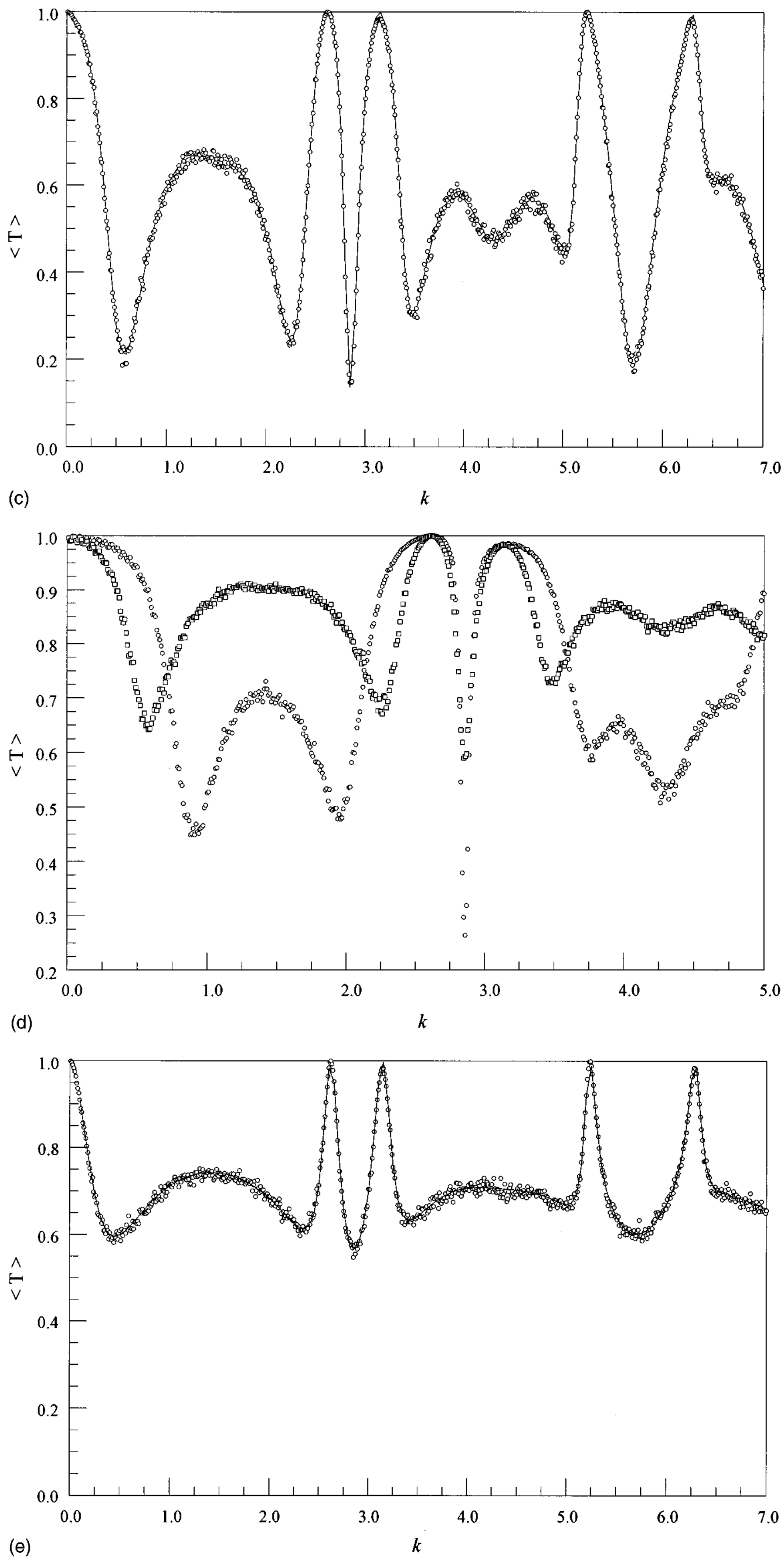
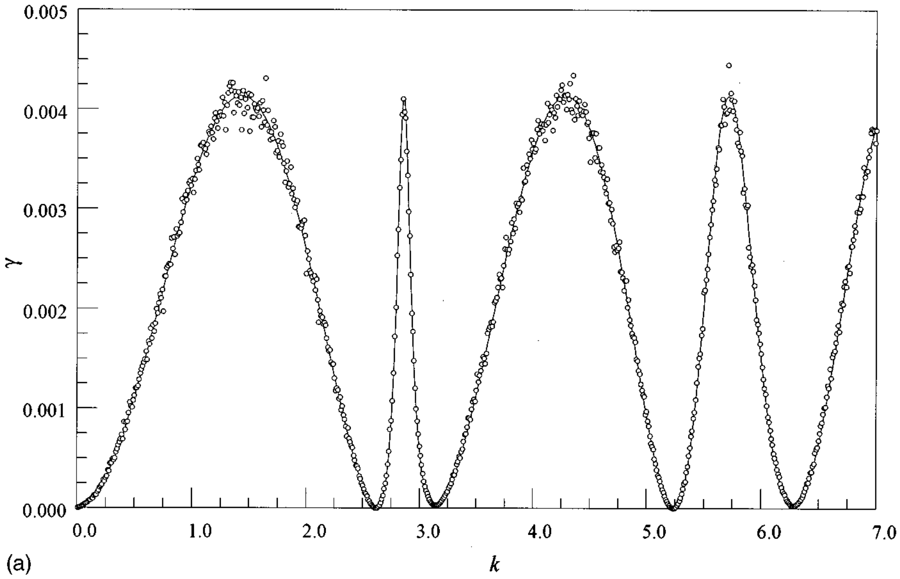


FIG. 2 (*Continued*).



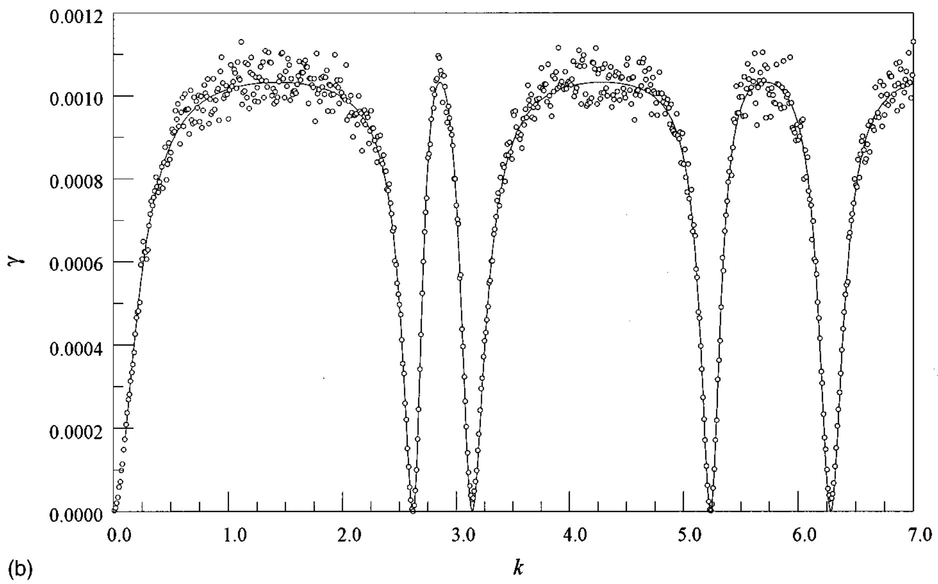


FIG. 3. Lyapunov exponents for models presented in Figs. 2(a)–2(d), respectively.

where $\mathbf{u}_n$ is a vector with components E_n and E_n' and $\mathbf{T}_n$ is the transfer-matrix determined as

$$\mathbf{T}_n=\begin{pmatrix} \cos(k_n d) & \dfrac{1}{k_n}\sin(k_n d) \\ -k_n\sin(k_n d) & \cos(k_n d)\end{pmatrix}, \qquad (5)$$

where $k_n=k_0\sqrt{\epsilon_n}$ is a wave number in the nth layer. The transmission coefficient T is determined by the equation

$$\mathbf{u}_N=\hat{\mathbf{T}}_N\mathbf{u}_0,$$

where

$$\mathbf{u}_N=\begin{pmatrix} t \\ itk_0\end{pmatrix}$$

describes a wave transmitted through the superlattice and

$$\mathbf{u}_0=\begin{pmatrix} 1+r \\ -irk_0\end{pmatrix}$$

corresponds to incident and reflected waves. The transmission rate is defined according to $T=|t|^2$. The matrix $\hat{\mathbf{T}}_N$ is the product of all T matrices corresponding to each layer:

$$\hat{\mathbf{T}}_N=\prod_1^N \mathbf{T}_i .$$

The Lyapunov exponent γ is determined according to

$$\gamma=\lim_{N\to\infty}\ln\hat{\mathbf{T}}_N \qquad (6)$$

and is known to be a self-averaging quantity in the limit of an infinite system. For a system of a finite size this is a random variable. To characterize the statistical properties of the Lyapunov exponent one can use its statistical momenta

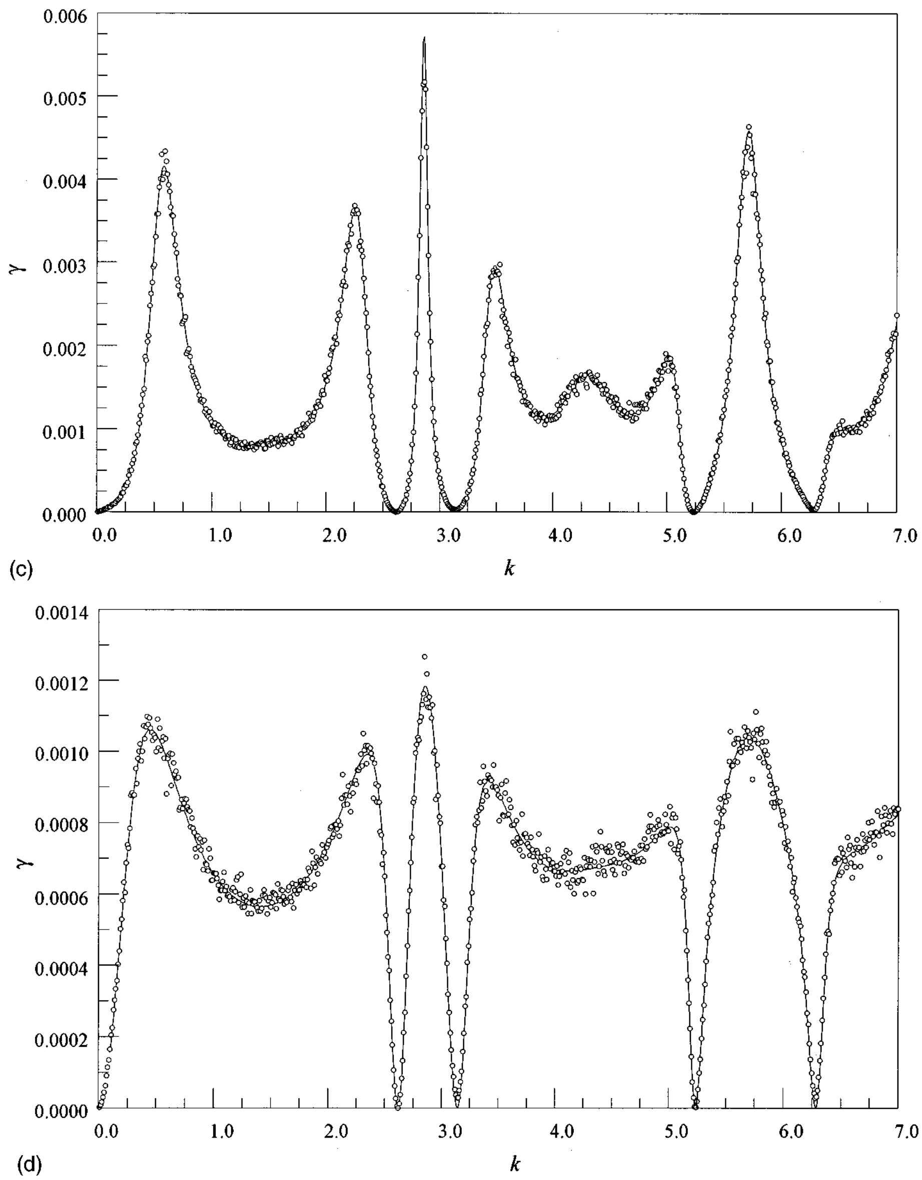


FIG. 3 (*Continued*).

such as a mean value and mean-root-square fluctuations [5]. Another approach exploited in Ref. [8] uses the generalized Lyapunov exponent.

Our analytical calculations utilize the approach developed in Ref. [5]. Nishiguchi, Tamura, and Nori established a useful relationship between the backscattering rate of waves $\ell(L,\omega)$ and the structure factor of a superlattice $I_s(\omega)$:

$$\ell(L,\omega)=\frac{2R^2}{\langle D_A\rangle+\langle D_B\rangle}I_s(L,\omega). \tag{7}$$

The structure factor in an infinite system is given by the expression

$$I_s(\omega)=\mathrm{Re}\left[\frac{(1-\varepsilon_A)(1-\varepsilon_B)}{1-\varepsilon_A\varepsilon_B}\right], \tag{8}$$

where $\varepsilon_A=\langle\exp(-2idk_An_A)\rangle$, $\varepsilon_B=\langle\exp(-2idk_Bn_B)\rangle$ and averaging is carried out over the distribution $P(n)$ of thicknesses of corresponding blocks given by Eq. (2). It is straightforward to show that for this distribution

$$\varepsilon_j=\exp(-2idk_j)\frac{1-p+(p-q)\exp(-2idk_j)}{1-q\exp(-2idk_j)}, \tag{9}$$

where $j=A,B$. Frequencies for which $2dk_j=2\pi n$, $n=0,1,2,\ldots$, correspond to the resonant transmission with $T=1$ in a system of any size. These frequencies are present in a system with any type of statistical distribution of layers; therefore, we will call them fundamental resonances. The short-range correlations, which occur when $p\neq q$, bring about new characteristic frequencies associated with the term $\exp(-4idk_j)$. We will see later that these frequencies actually manifest themselves as some additional maxima on frequency dependences of the localization length and the transmissivity. The effect of this term is the most prominent for $p=1$. The expression for $I_s(\omega)$ in the general case is rather cumbersome, so we only show it for the special case $p=1$, $q=1/2$:

$$I_s=\frac{4(1-\cos 2k_Ad)(1-\cos 2k_Bd)(5+2\cos 2k_Ad+2\cos 2k_Bd)}{|4-2\exp(-2\imath k_Ad)-2\exp(-2\imath k_Bd)+\exp[-2\imath(k_A+k_B)d]-\exp[-4\imath(k_A+k_B)d]|^2}. \tag{10}$$

The backscattering length $\ell(\omega)$ was shown in Ref. [5] to determine the Lyapunov coefficient $\gamma(\omega)$:

$$\gamma=\frac{1}{2\ell(\omega)}. \tag{11}$$

With the localization length found one can calculate the average transmission rate, fluctuations of the transmission, and other relevant characteristics [4,5]. In Figs. 2 and 3 we present results of numerical and analytical calculations for the average transmission coefficient and the Lyapunov exponent for different kinds of superlattices. For simulations we used superlattices with 300 layers and the ratio between layer parameters $\epsilon_A/\epsilon_B=1.2$. We averaged over 500 different randomly chosen realizations of the system. Figures 2(a) and 2(b) show the average transmission for uncorrelated and HT superlattices, respectively. They reproduce results of Refs. [4,5,9]. Figure 2(c) presents the frequency dependence of the average transmission for our model with the rigid short-range correlation $p=1$ and with no exponential correlation $q=1/2$. One can see that the average transmission reacts sharply on the short-range order: New maxima appear between fundamental resonance frequencies. The TM Markov superlattice also results in some structure in the frequency dependence of the average transmission rate [9]. However, the magnitude of the transmission at these new maxima for TM case is negligible for the lattice compounded of 300 layers. Nishiguchi, Tamura, and Nori [9] used a system with only 64 layers in their calculations. Therefore, in order to compare effects of different kinds of short-range order we show in Fig. 2(d) results of calculations of the average transmission coefficient for the TM superlattice and our model with $p=1$, $q=1/2$ for the system with 64 layers. This drastic decrease in transmission rate for the TM model is obviously due to the sharp increase of scattering interfaces in it compared to our situation.

Figure 2(e) presents the average transmission for the case when both short-range and exponential correlations are present ($p=1$, $q=0.8$). Such correlations favor like blocks stacked together; therefore, we observe an overall increase of the average transmission in accord with results for the HT model [Fig. 2(b)]. At the same time these correlations affect the shape of the dependence differently for different values of frequency. For frequencies below the first fundamental resonances, the general shape of the maximum is not changed, while the split maxima between the first and the second resonance is replaced by a smooth single maximum. This difference reflects the fact that a correlation radius of exponential correlations becomes an additional length scale in the system. Because of this, the behavior of the transmission as well as other characteristics should be different for wavelengths greater and smaller than the correlation radius. For larger wavelengths, the inhomogeneities associated with the exponential correlations tend to be averaged out and do not affect the system considerably. For shorter wavelengths, these inhomogeneities become more important and wash out some features caused by short-range correlations.

Figures 3(a)–3(d) present the frequency dependence of the Lyapunov coefficient for different situations shown in Fig. 2. One can see that a strong increase of the average transmission reflects the corrisponding increase of the localization length γ^{-1}. It was shown in Ref. [5] that there exists a universal critical value of the average transmission $\langle T\rangle_{\rm cr}=0.26$, which separates localized states from expanded states in systems with a finite length. For states with $\langle T\rangle<\langle T\rangle_{\rm cr}$ the localization length is less than the length of the system and the corresponding states are localized. In the reverse situation states are extended. One can see from the results presented that short-range correlations strongly influence localization properties of states in finite disordered systems. Correlations of the TM type do not support delocalization, while the structure with $p=1$, $q=1/2$ allows delocalized states at frequencies inside forbidden bands of the structure with no correlations. These new localized states are different, of course, from states at fundamental resonance frequencies because they do not survive in infinite systems. At the same time these states contribute considerably to transport properties of finite yet macroscopic systems.

IV. FLUCTUATION PROPERTIES OF THE TRANSMISSION RATE AND LYAPUNOV EXPONENT

In this section we consider the effect of correlations on fluctuation properties of the transmission rate and the Lyapunov exponent. Scaling properties of the distribution of the transmission rate were studied in Ref. [5]. These properties are known to be universal in a sense that their dependence upon the scaling parameter $t=\gamma L$, where L is the length of the system, remains the same for any kind of structure of a superlattice. The distribution function of the transmission rate $W(z,t)$, where $z=1/T$, is determined as [10]

$$W(z,t)=\frac{2}{\sqrt{\pi t^3}}\int_{x_0}^{\infty}\frac{x}{\sqrt{\cosh^2 x-z}}\exp[-(t/4+x^2/t)]dx.$$

For well-localized states with $t\gg 1$ this distribution reduces to the normal distribution for $\ln T^{-1}$ with a mean value equal to the Lyapunov exponent and a standard deviation equal to $2\sqrt{t}$ [10]. The transition between extended and localized states was investigated in Ref. [5]. Nishiguchi, Tamura, and Nori [5] suggested that $t=2$ is the boundary between the extended and localized regimes since at this point the average localization length becomes equal to the size of the superlattice. It can be shown, however, that the mean-square fluctuation of the localization length at this point is also equal to the size of the system. Therefore, the fluctuations of localization length wash out a distinctive boundary between these two regimes at $t=2$. At the same time, one can notice that relative fluctuations of the transmission coefficient show

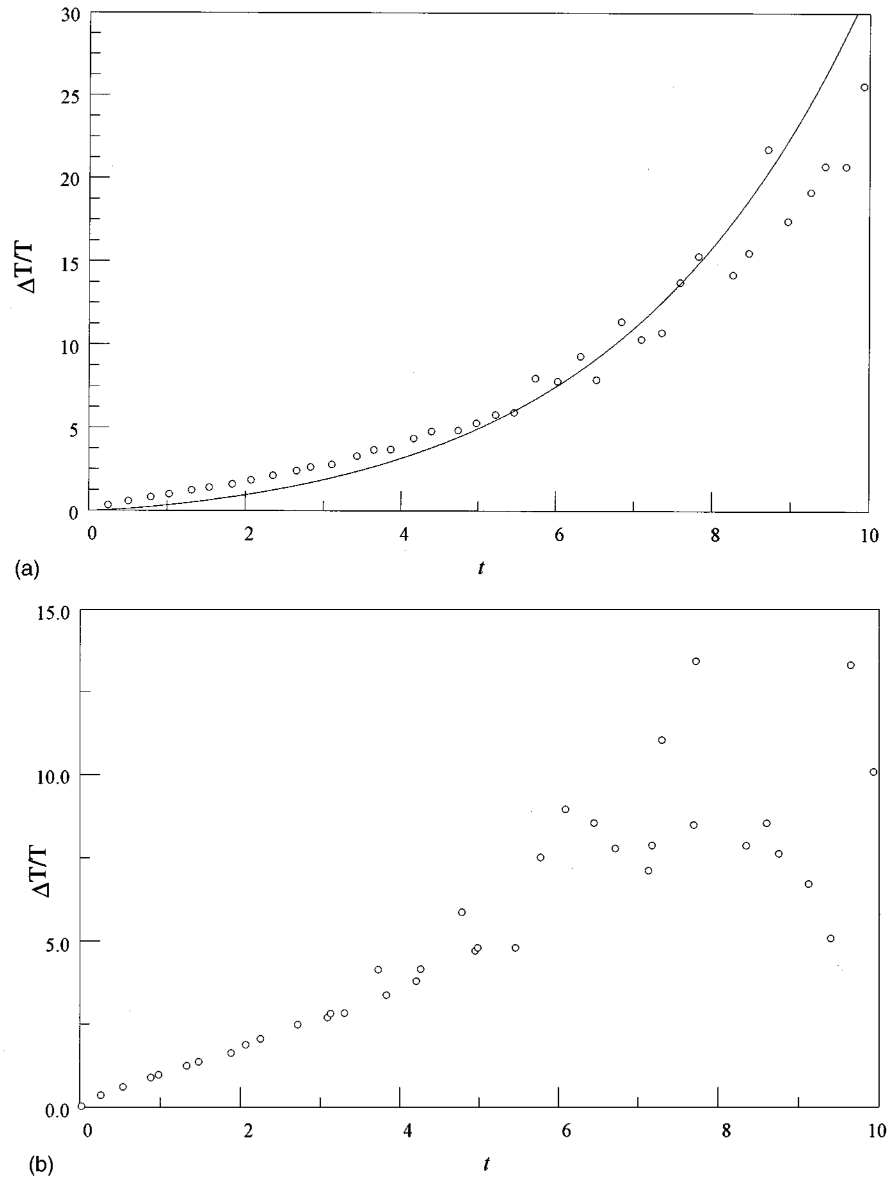


FIG. 4. Relative fluctuations of the transmission rate versus the scaling parameter t. Circles present the results of computing and the solid line shows the theoretical results. The numerical data were obtain from averaging over (a) 2000 realizations and (b) 200 realizations.

a sharp increase when the average transmission becomes approximately half of its value at $t=2$ [Fig. 4(b) in Refs. [5]]. Based upon this observation, we find that it is interesting to consider the scaling behavior of this parameter. Its dependence upon the scaling parameter t obtained by simulations along with the results of the corresponding theoretical calculations is shown in Fig. 4(a). We would like to point out a sharp increase in relative fluctuations of the transmission at $t \approx 5-6$. It can be seen as an increase in the slope of the averaged curve, but also as a drastic increase of scattering of points in the numerical experiment. Actually, in order to obtain a more or less smooth line in the region $t>5$ we had to increase the number of realizations for averaging from 200 for the region $t<5$ to 2000 for $t>5$. Figure 4(b) presents the same dependence with a smaller number of averaging equal to 200. At $t>2$ the fluctuations of localization length become smaller than the system's size and localized states begin to contribute more distinctively to such characteristics as the relative fluctuations of the transmission rate. One can conclude, therefore, that a sharp change in the behavior of relative fluctuations of transmission at $t \approx 5$ can be attributed to the transition between extended and localized regimes in a finite sample.

The universal relations described above do not imply, however, that localization properties of individual states at different frequencies are also universal. Below we present results of our study of fluctuation properties of localization lengths at some characteristics frequencies of the system. We are primarily interested in a dependence of these properties upon the correlation characteristics of the system. In order to study this problem, we first fix the probability $q=1/2$ and consider the dependence of the Lyapunov parameter γ upon the probability p. This choice of parameters allows one to study the influence of the short-range structure in which exponential correlations are absent. The value $p=0$ leads to periodic ordering of the layers with the period equal to $2d$, $p=1/2$ describes the system without correlations, and $p=1$ leads to the structure opposite to the TM model, as was ex-

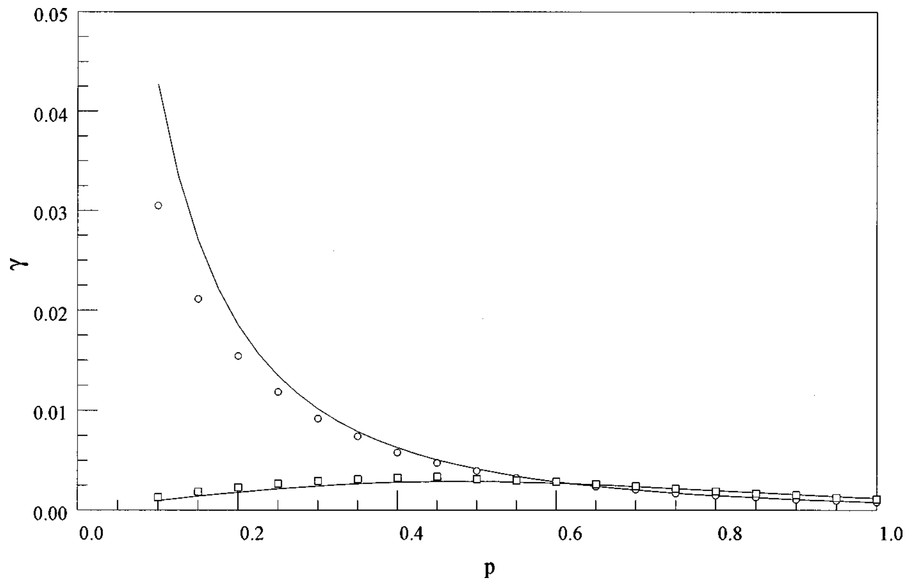


FIG. 5. Dependence of the Lyapunov coefficient versus the probability parameter p for $q=0.5$. Circles and squares show results for $k=1.45k_0$ and for $k=3.9k_0$, respectively; solid lines present corresponding theoretical data.

plained in Sec. III. As reference frequencies we consider $k=1.45k_0$ and $k=3.9k_0$, where k_0 corresponds to the vacuum. In the system without correlations, these frequencies are positioned in the middle of forbidden bands, becoming resonance frequencies at $p=1$ (see Fig. 2).

Figure 5 present results of computer simulations of the Lyapunov exponent versus the probability parameter p along with theoretical curves based upon Eq. (9). It is seen that the Lyapunov exponent at these frequencies demonstrates *qualitatively* different behavior. The Lyapunov exponent at $k=1.45k_0$ shows a monotonic decrease with an increase of the parameter p, while at $k=3.9k_0$ it exhibits a nonmonotonic behavior with the minimum value at approximately $p=1/2$. The difference in behavior between these frequencies can be understood if one recalls that $p=0$ corresponds to the periodic structure with a period of $2d$. The frequency $k=3.9k_0$ falls into a transmission band of this periodic structure; therefore, it demonstrates a small Lyapunov exponent when p approaches 0. At the same time the frequency $k=1.45k_0$ falls in a forbidden band for the periodic structure arising at $p=0$ and hence their Lyapunov coefficient sharply increases at $p\rightarrow 0$. When p approaches 1 both frequencies belong to resonance regions associated with the resonance transmission from blocks with doubled thickness of individual layers. Though the structure with $p=1$ does not lead to exact doubling of all layers, it does favor such a situation causing a decrease of scattering boundaries and consequently maxima of transmission at these frequencies. Therefore, the Lyapunov exponent at all frequencies considered decreases when p approaches 1.

More detailed information about states corresponding to the selected frequencies can be obtained from Figs. 6(a) and 6(b), which present relative fluctuations of the Lyapunov exponent, $\Delta\gamma/\gamma$ and relative fluctuations of the transmission rate $\Delta T/T$ versus the probability parameter p. Small $\Delta\gamma/\gamma$ and big $\Delta T/T$ for $k=1.45k_0$ at small values of p reflect strong localization of the corresponding states. This is exactly what one would expect for the states arising in a forbidden gap of a nearly periodic structure. It is interesting to note, however, that an increase of the degree of a disorder associated with the increase of p does not enhance localization of the states. One can see from Figs. 6(a) and 6(b) that the state at $k=1.45k_0$ becomes ''less'' localized with increasing p. The reason for this behavior is that an increase of p destroys the periodicity of the structure washing out its forbidden gaps and weakening opportunities for localization. States at other frequencies show almost delocalized behavior for small p since they belong to a passband of the periodic structure and become more localized when traces of periodicity of the structure gradually disappear as p approaches 1/2. For $p>1/2$, both frequencies behave in approximately the same way since a memory about their different origin is lost in this situation.

It is interesting to note that results qualitatively similar to those presented in Fig. 5 were found in Ref. [7], though that paper dealt with a quite different model. Crisanti, Paladin, and Vulpiani [7] studied the effect of ''long-range'' exponential correlations on localization properties of the nearest-neighbor tight-binding model with the two-state Markov-type distribution of site energies (the HT model). It was found that at the states far enough from the band edge and band center of the pure system the Lyapunov exponent exhibits behavior similar to the curve presented by squares in Fig. 5 and states at the center of the band behave similarly to the second line in this figure. This similarity can be understood if one considers these two models in their extreme realizations. We have already discussed that the state at $k=1.45_0$ in our model falls into the forbidden band of the periodic structure arising at $p=0$. The same is valid for the states in the center of the band in Ref. [7] in the case of extreme anticorrelation between adjacent values of the site energies. This similar origin causes similar behavior when the structures change. The second type of behavior is associated with states that belong to passbands of the respective models; therefore, they also demonstrate similar properties. The third type of behavior of the Lyapunov exponent found in Ref. [7], in which the Lyapunov exponent monotonically increases along with the Markov transition probability, does

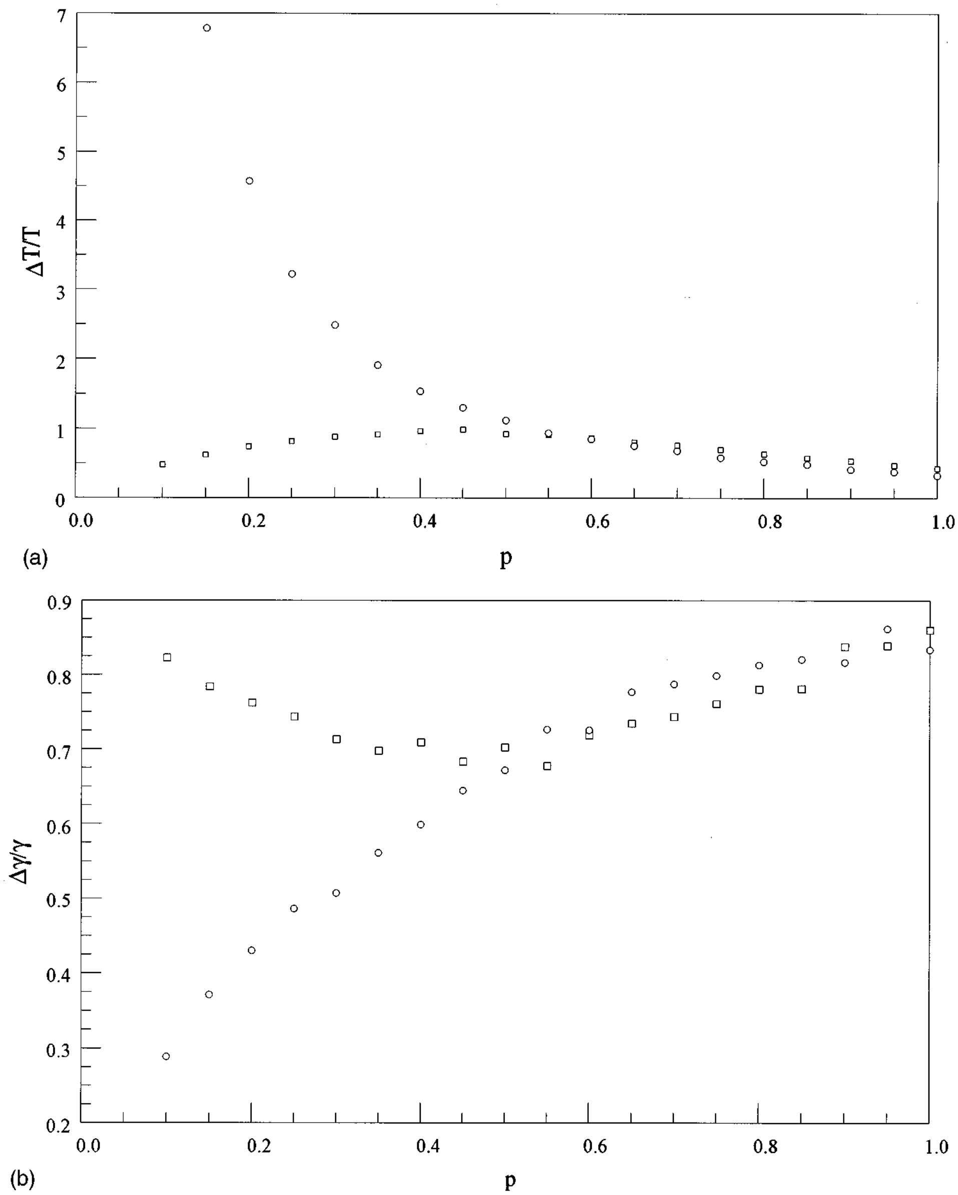


FIG. 6. Relative fluctuations of (a) the transmission rate and (b) the Lyapunov coefficient (b) versus the probability parameter p. All notations are the same as in Fig. 5.

not exist in our model with the parameter q set equal to 1/2. The reason for this is that the second extreme structure of Ref. [7] corresponds to an almost homogeneous structure, a situation, that cannot be realized in our model with $q=1/2$.

V. CONCLUSION

In this paper we carried out a detailed analysis of the effects of correlations on localization properties of classical waves in random superlattices. The correlations between different layers of the superlattice were introduced within the framework of the generalized random Thue-Morse model. The statistical properties of the model are controlled by two parameters p and q. By changing the values of these parameters we were able to consider different kinds of random structures including the classical random Thue-Morse model and the Hendricks-Teller model introduced in Ref. [9] and structures with weak random deviations from periodicity. We found that correlations between the constituent layers strongly affect localization properties of superlattices and can lead to a great variety of transmission patterns. This property can allow one to create superlattices with controlled rates of transmission in different frequency regions.

We pointed out that relative fluctuations of the transmission rate increase sharply for a value of the scaling parameter of $t\approx 5$. This point can be considered as a more exact threshold between localized and extended states in finite systems instead of the $t=2$ suggested in Ref. [5].

We also considered the dependence of localization properties of our model upon the type of short-range structure associated in the model with the probability parameter p. Since knowing the value of the Lyapunov exponent itself is not enough to determine whether the state considered is localized or extended, we also considered relative fluctuations of this parameter along with relative fluctuations of the transmission rate. These quantities are size independent and therefore are convenient for discussing localization properties. We found that there exist two kinds of states exhibiting different behavior when p changes from 0 to 1. The behavior of the states is mainly determined by their position in the spectrum of the deterministic periodic structure arising at $p=0$. The states from passbands of this structure show a decrease of their localization length with an increase of p, while states from stop bands depend upon p in nonmonotonic way. For small values of p, the localization length increases when p increases and reaches its maximum value for $p=1/2$; for $p>1/2$ its dependence upon p is similar to that of other states of the system. Comparing these results with those obtained in Ref. [7], where the tight-binding model with correlations of the Hendricks-Teller type were considered, shows a surprising similarity between them. The general conclusion that one can draw from this comparison is that the localization properties of states in 1D systems depend strongly upon properties of deterministic systems, which are opposite extremes of the random systems considered, and upon the position of the states in the spectra of these deterministic systems and the localization properties are less sensitive to details of the structure of a random system itself.

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