

PACS numbers: 42.55.Sa; 42.60.Da; 73.21.–b; 78.67.–n
DOI: 10.1070/QE2015v045n03ABEH015627

Theory of a double-quantum-dot spaser*

E.S. Andrianov, A.A. Pukhov, A.V. Dorofeenko, A.P. Vinogradov, A.A. Lisyansky

Abstract. **We consider the influence of the number of quantum dots on spaser operation. It is shown that even in the presence of only two quantum dots, the spaser behaviour is qualitatively different from that of the previously studied spaser consisting of a nanoparticle and a single quantum dot. In particular, for nonzero detuning of resonant frequencies of a nanoparticle and quantum dots, an increase in the interaction constant between quantum dots first leads to a decrease in the spasing threshold and then to its growth and even the spasing breakdown.**

Keywords: *nanoplasmonics, spaser, nanolasers, suppression of lasing.*

1. Introduction

The use of active (gain) media to compensate for losses in artificial plasmonic metamaterials is currently of considerable interest [1–4]. Application of metamaterials usually assumes that the working range is a narrow region near the plasmon resonance of particles of which this metamaterial is made. Actually, this fact explains high losses. To compensate for these losses, Sarychev and Tartakovsky [2] proposed to introduce active inclusions into the matrix. When an active medium is embedded into a metamaterial at frequencies of the plasmon resonance, particles surrounded by this active medium turn into spasers [2–5]. Schematically [5–11], a spaser is a system of inversely excited two-level quantum dots (QDs) surrounding plasmonic nanoparticles (NPs). The principle of spaser operation is similar to that of a laser. Surface plasmons (SPs) localised on a NP [5, 9–11], which is a multimode resonator, play the role of photons. In other words, the spaser works as a near-field generator and amplifier of NPs (plasmons). NP amplification occurs due to nonradiative energy transfer from a QD to a NP. Since the probability of nonradiative excitation of a plasmon is $(kr)^{-3}$ times greater than the probability of radiative de-excitation of a photon [12] (r is the distance between QD and NP centres, $k = 2\pi/\lambda$), the interaction of a QD with a plasmonic NP can be described in the dipole–dipole approximation (or any other near-field approximation [13]).

Excitation of plasmon modes on a NP by near fields of a QD can lead to further stimulated emission of a QD, surrounding this NP, in the same plasmon mode and, finally, to the development of plasmon generation, i.e., to the appearance of a spaser. Previously, the authors of papers [5, 8, 14, 15] considered a system consisting of a NP and a QD. However, a single-quantum-dot spaser scheme is far from possible experimental realisation. Thus, in the experiment from paper [6], the number of atoms of the active medium is significantly greater than unity. Note that the behaviour of the spaser containing a large number of QDs [16, 17] is qualitatively different from the simple model of a spaser consisting of a NP and a QD [5, 8, 14, 15]. Below we consider collective phenomena in the simplest spaser model containing two QDs.

2. Statement of the problem, basic equations

Consider the interaction of a NP with two two-level QDs in the simplest case when QDs are pumped so that their dipole moments have the same directions (Fig. 1). In this case, the Hamiltonian of the system can be written as

$$\hat{H} = \hbar\omega_{\rm NP}\hat{a}^\dagger\hat{a} + \hbar\omega_{\rm TLS1}\hat{\sigma}_1^\dagger\hat{\sigma}_1 + \hbar\omega_{\rm TLS2}\hat{\sigma}_2^\dagger\hat{\sigma}_2 + \hbar\Omega_{\rm R1}(\hat{a}^\dagger\hat{\sigma}_1 + \hat{a}\hat{\sigma}_1^\dagger)$$
$$+\hbar\Omega_{\rm R2}(\hat{a}^\dagger\hat{\sigma}_2 + \hat{a}\hat{\sigma}_2^\dagger) + \hbar\Omega_3(\hat{\sigma}_1\hat{\sigma}_2^\dagger + \hat{\sigma}_1^\dagger\hat{\sigma}_2). \quad (1)$$

Here, $\omega_{\rm NP}$, $\omega_{\rm TLS1}$, $\omega_{\rm TLS2}$ are the frequencies of a SP and two QDs, respectively; and $\Omega_{\rm R1}$, $\Omega_{\rm R2}$, Ω_3 are the Rabi frequencies characterising the interaction of two QDs and NPs, as well as QDs with each other, respectively. The operators $\hat{a}$ and $\hat{a}^\dagger$ describe the SP creation and annihilation ($[\hat{a},\hat{a}^\dagger] = 1$), and the operators $\hat{\sigma}_1, \hat{\sigma}_1^\dagger, \hat{\sigma}_2, \hat{\sigma}_2^\dagger$ – the transition between the ground and excited levels of the first and second QD, respectively; in this case, $[\hat{\sigma}_1^\dagger,\hat{\sigma}_1] = \hat{D}_1$ and $[\hat{\sigma}_2^\dagger,\hat{\sigma}_2] = \hat{D}_2$, where $\hat{D}_1, \hat{D}_2$ are the population inversions in quantum dots. Note that the more general case of a four-level system does not lead to a qualitatively new behaviour of the system [18].

For dissipation to be consistently taken into account, we should bear in mind that a spaser is an open quantum system

*Presented at the 6th Workshop on Fibre Lasers, Novosibirsk, April 2014.

E.S. Andrianov All-Russia Research Institute of Automatics, Sushchevskaya ul. 22, 127055 Moscow, Russia; Moscow Institute of Physics and Technology (State University), Institutskii per. 9, 141700 Dolgoprudnyi, Moscow region, Russia;
e-mail: p314159262006@yandex.ru;
A.A. Pukhov, A.V. Dorofeenko, A.P. Vinogradov All-Russia Research Institute of Automatics, Sushchevskaya ul. 22, 127055 Moscow, Russia; Moscow Institute of Physics and Technology (State University), Institutskii per. 9, 141700 Dolgoprudnyi, Moscow region, Russia; Institute for Theoretical and Applied Electrodynamics, Russian Academy of Sciences, Izhorskaya ul. 13, 125412 Moscow, Russia; e-mail: a-vinogr@yandex.ru, asdf00@yandex.ru, pukhov@mail.ru;
A.A. Lisyansky Department of Physics, Queens College of the City University of New York, Flushing, New York 11367, USA

Received 16 July 2014; revision received 6 October 2014
Kvantovaya Elektronika **45** (3) 245–249 (2015)
Translated by I.A. Ulitkin

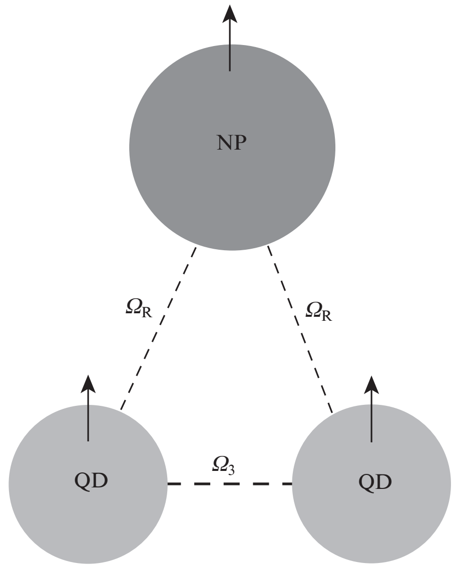

Figure 1. Geometry of the problem. Arrows indicate the directions of the dipole moments.

[19]. To this end, we should introduce into consideration the spaser environment with which NPs and QDs interact [20–22]. Without loss of generality, we can assume that these are the reservoirs, representing a continuum of boson field modes in interactions with which NPs and QDs relax. Depending on the dominant relaxation mechanism [23] such bosons can be phonons, polaritons, surface plasmons, etc. [24]. Then, under the assumption that the correlation time of the variables of the reservoir is much smaller than the characteristic time of change in the system (Markov approximation), the Heisenberg equations of motion for the slowly varying amplitudes and operators, $\hat a$, $\hat\sigma_1$, $\hat\sigma_1$, $\hat D_1$, $\hat D_2$ can be written in the form:

$$\begin{aligned}
\dot{\hat a} &= \left(\mathrm{i}\Delta - \frac{1}{\tau_a}\right)\hat a - \mathrm{i}\Omega_{\mathrm{R}1}\hat\sigma_1 - \mathrm{i}\Omega_{\mathrm{R}2}\hat\sigma_2,\\
\dot{\hat\sigma}_1 &= \left(\mathrm{i}\delta_1 - \frac{1}{\tau_{\sigma1}}\right)\hat\sigma_1 + \mathrm{i}\Omega_{\mathrm{R}1}\hat a\hat D_1 + \mathrm{i}\Omega_3\hat\sigma_2\hat D_1,\\
\dot{\hat D}_1 &= 2\mathrm{i}\Omega_{\mathrm{R}1}\left(\hat a^\dagger\hat\sigma_1 - \hat a\hat\sigma_1^\dagger\right) + 2\mathrm{i}\Omega_3\left(\hat\sigma_2^\dagger\hat\sigma_1 - \hat\sigma_1^\dagger\hat\sigma_2\right) - \frac{\hat D_1 - \hat D_{01}}{\tau_{D1}},\\
\dot{\hat\sigma}_2 &= \left(\mathrm{i}\delta_2 - \frac{1}{\tau_{\sigma2}}\right)\hat\sigma_2 + \mathrm{i}\Omega_{\mathrm{R}2}\hat a\hat D_2 + \mathrm{i}\Omega_3\hat\sigma_1\hat D_2,\\
\dot{\hat D}_2 &= 2\mathrm{i}\Omega_{\mathrm{R}2}\left(\hat a^\dagger\hat\sigma_2 - \hat a\hat\sigma_2^\dagger\right) + 2\mathrm{i}\Omega_3\left(\hat\sigma_1^\dagger\hat\sigma_2 - \hat\sigma_2^\dagger\hat\sigma_1\right) - \frac{\hat D_2 - \hat D_{02}}{\tau_{D2}}.
\end{aligned} \tag{2}$$

Here, $\delta_1 = \omega_{\mathrm{S}} - \omega_{\mathrm{TLS1}}$; $\delta_2 = \omega_{\mathrm{S}} - \omega_{\mathrm{TLS2}}$; $\Delta = \omega_{\mathrm{S}} - \omega_{\mathrm{NP}}$; ω_{S} is the oscillation frequency, which will be determined below; $\hat D_{01}$ and $\hat D_{02}$ determine the pump in the first and second QDs, respectively; and τ_{D1} and τ_{D2} determine the pump rate. The terms containing the relaxation times τ_a (NP) and $\tau_{\sigma1}$, $\tau_{\sigma2}$ (first and second QDs) are obtained, as we have already pointed out, in the Markov approximation (see, for example, [21, 22]).

Below, we make the following simplifying assumptions. First, we assume that QDs are similar and, therefore, $\omega_{\mathrm{TLS1}} = \omega_{\mathrm{TLS2}}$, $\delta_1 = \delta_2$, $\tau_{\sigma1} = \tau_{\sigma2}$, $\tau_{D1} = \tau_{D2}$, $D_{01} = D_{02}$.

Second, we assume that both QDs are located at the same distance from the NP and, therefore, $\Omega_{\mathrm{R}1} = \Omega_{\mathrm{R}2} = \Omega_{\mathrm{R}}$.

Taking the above into account, system (2) takes the form:

$$\begin{aligned}
\dot{\hat a} &= \left(\mathrm{i}\Delta - \frac{1}{\tau_a}\right)\hat a - \mathrm{i}\Omega_{\mathrm{R}}\hat\sigma_1 - \mathrm{i}\Omega_{\mathrm{R}}\hat\sigma_2,\\
\dot{\hat\sigma}_1 &= \left(\mathrm{i}\delta - \frac{1}{\tau_\sigma}\right)\hat\sigma_1 + \mathrm{i}\Omega_{\mathrm{R}}\hat a\hat D_1 + \mathrm{i}\Omega_3\hat\sigma_2\hat D_1,\\
\dot{\hat D}_1 &= 2\mathrm{i}\Omega_{\mathrm{R}}\left(\hat a^\dagger\hat\sigma_1 - \hat a\hat\sigma_1^\dagger\right) + 2\mathrm{i}\Omega_3\left(\hat\sigma_2^\dagger\hat\sigma_1 - \hat\sigma_1^\dagger\hat\sigma_2\right) - \frac{\hat D_1 - \hat D_0}{\tau_D},\\
\dot{\hat\sigma}_2 &= \left(\mathrm{i}\delta - \frac{1}{\tau_\sigma}\right)\hat\sigma_2 + \mathrm{i}\Omega_{\mathrm{R}}\hat a\hat D_2 + \mathrm{i}\Omega_3\hat\sigma_1\hat D_2,\\
\dot{\hat D}_2 &= 2\mathrm{i}\Omega_{\mathrm{R}}\left(\hat a^\dagger\hat\sigma_2 - \hat a\hat\sigma_2^\dagger\right) + 2\mathrm{i}\Omega_3\left(\hat\sigma_1^\dagger\hat\sigma_2 - \hat\sigma_2^\dagger\hat\sigma_1\right) - \frac{\hat D_2 - \hat D_0}{\tau_D}.
\end{aligned} \tag{3}$$

3. Steady-state solution, generation conditions

Let us find the steady-state solution to system (3). To do this, we set the time derivatives equal to zero. As a result, the system takes the form

$$\begin{aligned}
&\left(\mathrm{i}\Delta - \frac{1}{\tau_a}\right)\hat a - \mathrm{i}\Omega_{\mathrm{R}}\hat\sigma_1 - \mathrm{i}\Omega_{\mathrm{R}}\hat\sigma_2 = 0,\\
&\left(\mathrm{i}\delta - \frac{1}{\tau_\sigma}\right)\hat\sigma_1 + \mathrm{i}\Omega_{\mathrm{R}}\hat a\hat D_1 + \mathrm{i}\Omega_3\hat\sigma_2\hat D_1 = 0,\\
&2\mathrm{i}\Omega_{\mathrm{R}}\left(\hat a^\dagger\hat\sigma_1 - \hat a\hat\sigma_1^\dagger\right) + 2\mathrm{i}\Omega_3\left(\hat\sigma_2^\dagger\hat\sigma_1 - \hat\sigma_1^\dagger\hat\sigma_2\right) - \frac{\hat D_1 - \hat D_0}{\tau_D} = 0,\\
&\left(\mathrm{i}\delta - \frac{1}{\tau_\sigma}\right)\hat\sigma_2 + \mathrm{i}\Omega_{\mathrm{R}}\hat a\hat D_2 + \mathrm{i}\Omega_3\hat\sigma_1\hat D_2 = 0,\\
&2\mathrm{i}\Omega_{\mathrm{R}2}\left(\hat a^\dagger\hat\sigma_2 - \hat a\hat\sigma_2^\dagger\right) + 2\mathrm{i}\Omega_3\left(\hat\sigma_1^\dagger\hat\sigma_2 - \hat\sigma_2^\dagger\hat\sigma_1\right) - \frac{\hat D_2 - \hat D_0}{\tau_D} = 0.
\end{aligned} \tag{4}$$

Adding the second and fourth, as well as the third and fifth equations of system (4), we obtain

$$\begin{aligned}
&\left(\mathrm{i}\delta - \frac{1}{\tau_\sigma}\right)(\hat\sigma_1 + \hat\sigma_2) + \mathrm{i}\Omega_{\mathrm{R}}\hat a(\hat D_1 + \hat D_2) + \mathrm{i}\Omega_3(\hat\sigma_2\hat D_1 + \hat\sigma_1\hat D_2) = 0,\\
&2\mathrm{i}\Omega_{\mathrm{R}}[\hat a^\dagger(\hat\sigma_1 + \hat\sigma_2) - \hat a(\hat\sigma_1^\dagger + \hat\sigma_2^\dagger)] - \frac{(\hat D_1 + \hat D_2) - 2\hat D_0}{\tau_D} = 0.
\end{aligned} \tag{5}$$

Now we will use the equalities $\hat\sigma_1\hat D_1 = \hat\sigma_1$, $\hat\sigma_2\hat D_2 = \hat\sigma_2$ and transform the first equation (5) to the form

$$\begin{aligned}
&\left(\mathrm{i}\delta - \frac{1}{\tau_\sigma}\right)(\hat\sigma_1 + \hat\sigma_2) + \mathrm{i}\Omega_{\mathrm{R}}\hat a(\hat D_1 + \hat D_2)\\
&\quad + \mathrm{i}\Omega_3[(\hat\sigma_2 + \hat\sigma_1)(\hat D_1 + \hat D_2) - (\hat\sigma_2 + \hat\sigma_1)] = 0.
\end{aligned} \tag{6}$$

As a result, we obtain the system of equations determining the steady-state solution:

$$\begin{aligned}
&\left(\mathrm{i}\Delta - \frac{1}{\tau_a}\right)\hat a - \mathrm{i}\Omega_{\mathrm{R}}(\hat\sigma_1 + \hat\sigma_2) = 0,\\
&2\mathrm{i}\Omega_{\mathrm{R}}[\hat a^\dagger(\hat\sigma_1 + \hat\sigma_2) - \hat a(\hat\sigma_1^\dagger + \hat\sigma_2^\dagger)] - \frac{(\hat D_1 + \hat D_2) - 2\hat D_0}{\tau_D} = 0,
\end{aligned}$$

$$\left(\mathrm{i}\delta - \frac{1}{\tau_\sigma}\right)(\hat{\sigma}_1 + \hat{\sigma}_2) + \mathrm{i}\Omega_{\mathrm{R}}\hat{a}(\hat{D}_1 + \hat{D}_2) + \mathrm{i}\Omega_3[(\hat{\sigma}_2 + \hat{\sigma}_1)(\hat{D}_1 + \hat{D}_2) - (\hat{\sigma}_2 + \hat{\sigma}_1)] = 0. \quad (7)$$

One can see that system (4) has been reduced from five equations to a system of three equations (7), depending on the variables $\hat{a}, (\hat{\sigma}_1 + \hat{\sigma}_2)$ и $(\hat{D}_1 + \hat{D}_2)$.

We now turn to the corresponding c-number equations, as is usually done in the study of the dynamics of lasers [8, 14, 21, 25]. System (7) takes the form

$$\left(\mathrm{i}\Delta - \frac{1}{\tau_a}\right)a - \mathrm{i}\Omega_{\mathrm{R}}(\sigma_1 + \sigma_2) = 0,$$

$$2\mathrm{i}\Omega_{\mathrm{R}}[a^*(\sigma_1 + \sigma_2) - a(\sigma_1 + \sigma_2)^*] - \frac{(D_1 + D_2) - 2D_0}{\tau_D} = 0,$$

$$\left(\mathrm{i}\delta - \frac{1}{\tau_\sigma}\right)(\sigma_1 + \sigma_2) + \mathrm{i}\Omega_{\mathrm{R}}a(D_1 + D_2) + \mathrm{i}\Omega_3[(\sigma_2 + \sigma_1)(D_1 + D_2) - (\sigma_2 + \sigma_1)] = 0. \quad (8)$$

The first equation of system (8) determines the steady-state value of the dipole moment of the NP, the second – the common dipole moment of two QDs, and the third – the steady-state value of the total inversion in the QD. It is worth emphasising that the reduction of five operator equations (4) to a system of three equations (8) for c-numbers is not a trivial replacement of operators by c-numbers, because in system (7) unknown are collective operators $\hat{\sigma}_1 + \hat{\sigma}_2$ and $\hat{D}_1 + \hat{D}_2$. This transition involves the use of operator equalities $\hat{\sigma}_1\hat{D}_1 = \hat{\sigma}_1$ and $\hat{\sigma}_2\hat{D}_2 = \hat{\sigma}_2$, which is invalid in c-numbers. Thus, although the operator system (4) is equivalent to the operator system (8), c-number systems are not equivalent. The transition to c-numbers in (4) means decoupling of all its correlators; however, the transition to c-numbers in system (8) allows one to avoid the 'decoupling' procedure of the majority of them, by making a controlled process out of this procedure, because system (8) is equivalent to the system of a single-quantum-dot spaser (accuracy of the transition to c-numbers is investigated in detail in [19, 26]).

Before examining the general case, we note that if QDs do not interact with each other ($\Omega_3 = 0$), the mathematical system (8) is almost equivalent to the problem of finding the steady-state solutions in a system with a single QD and NP (see [2, 5, 8, 14]), with the only difference that now $2D_0$ instead of D_0 is used in the equations.

Next, we introduce the notations $\Omega_a = -(\mathrm{i}\Delta - 1/\tau_a)$, $\Omega_\sigma = -(\mathrm{i}\delta - 1/\tau_\sigma)$. We find an expression for a from the first equation (8):

$$a = \frac{-\mathrm{i}\Omega_{\mathrm{R}}(\sigma_1 + \sigma_2)}{\Omega_a}, \quad (9)$$

and then substitute it into the second equation:

$$2\mathrm{i}\Omega_{\mathrm{R}}\left[\frac{\mathrm{i}\Omega_{\mathrm{R}}(\sigma_1 + \sigma_2)^*(\sigma_1 + \sigma_2)}{\Omega_a^*} + \frac{\mathrm{i}\Omega_{\mathrm{R}}(\sigma_1 + \sigma_2)^*(\sigma_1 + \sigma_2)}{\Omega_a}\right] - \frac{(D_1 + D_2) - 2D_0}{\tau_D} = 0. \quad (10)$$

Then,

$$D_1 + D_2 = -2\Omega_{\mathrm{R}}^2\tau_D|\sigma_1 + \sigma_2|^2\left(\frac{1}{\Omega_a} + \frac{1}{\Omega_a^*}\right) + 2D_0. \quad (11)$$

Substituting (11) and (9) into the third equation of system (8) we obtain

$$(\sigma_1 + \sigma_2)\left\{\mathrm{i}\Omega_{\mathrm{R}}\frac{-\mathrm{i}\Omega_{\mathrm{R}}}{\Omega_a}\left[-2\Omega_{\mathrm{R}}^2\tau_D|\sigma_1 + \sigma_2|^2\left(\frac{1}{\Omega_a} + \frac{1}{\Omega_a^*}\right) + 2D_0\right] + \mathrm{i}\Omega_3\left[-2\Omega_{\mathrm{R}}^2\tau_D|\sigma_1 + \sigma_2|^2\left(\frac{1}{\Omega_a} + \frac{1}{\Omega_a^*}\right) + 2D_0 - 1\right]\right\} - \Omega_\sigma = 0. \quad (12)$$

For equation (12), there are two types of solutions. Firstly, it is a trivial solution (in the absence of spasing) when $\sigma_1 + \sigma_2 = 0$, $a = 0$, $D_1 + D_2 = 2D_0$. However, there is also a nontrivial solution, determined from equality of the expression in the curly brackets to zero, from which one can find

$$|\sigma_1 + \sigma_2|^2 = \frac{2D_0 - (\Omega_\sigma + \mathrm{i}\Omega_3)(\Omega_{\mathrm{R}}^2/\Omega_a + \mathrm{i}\Omega_3)^{-1}}{2(1/\Omega_a + 1/\Omega_a^*)\Omega_{\mathrm{R}}^2\tau_D}. \quad (13)$$

One can see that the left-hand side of (13) is a positive real number. Therefore, from the condition of equality of the imaginary part of the expression on the right-hand side of (13) to zero, we obtain the equation for the oscillation frequency, which, generally speaking, can have several solutions.

Thus, the equation for the oscillation frequency is given by

$$-\frac{\Omega_3}{\tau_\sigma}\Delta^2 - \frac{\Omega_{\mathrm{R}}^2}{\tau_\sigma}\Delta - \frac{\Omega_{\mathrm{R}}^2}{\tau_a}\delta + \frac{\Omega_3\Omega_{\mathrm{R}}^2}{\tau_a} - \frac{\Omega_3}{\tau_a^2\tau_\sigma} = 0. \quad (14)$$

Note that the characteristic experimental values of the damping constants $\tau_a \sim 10^{-14}$ s, $\tau_\sigma \sim 10^{-11}$ s and interaction frequencies $\Omega_{\mathrm{R}} \sim 10^{12}$ s^{-1}, $\Omega_3 \sim 10^{11}$ s^{-1} satisfy the condition $\tau_\sigma^{-1}, \Omega_3 \ll \Omega_{\mathrm{R}} \ll \tau_a^{-1}$. In this approximation, we obtain the solution to equation (14):

$$\omega_{\mathrm{S}} = \omega_{\mathrm{NP}} + \Omega_3\left(1 - \frac{1}{\Omega_{\mathrm{R}}^2\tau_a\tau_\sigma}\right). \quad (15)$$

In this case, the second root of equation (14) can be estimated as $\omega_{\mathrm{S2}} \approx \omega_{\mathrm{NP}} - \Omega_{\mathrm{R}}^2\tau_\sigma/\Omega_3\tau_a$. The quantity $\omega_{\mathrm{S2}} \sim 10^{16}$ s^{-1} and it is not related to the region of approximations used in the paper. The positivity condition of expression (13)

$$2D_0 \geqslant \mathrm{Re}\left(\frac{\Omega_\sigma + \mathrm{i}\Omega_3}{\Omega_{\mathrm{R}}^2/\Omega_a + \mathrm{i}\Omega_3}\right) \quad (16)$$

determines the lasing threshold (per one QD):

$$D_{\mathrm{th}} = \frac{1}{2}\frac{\Omega_\sigma + \mathrm{i}\Omega_3}{\Omega_{\mathrm{R}}^2/\Omega_a + \mathrm{i}\Omega_3} = \frac{1}{2}\times \frac{[(\tau_a\tau_\sigma)^{-1} + \Delta(\Omega_3 - \delta)](\Omega_{\mathrm{R}}^2 + \Omega_3\Delta) + \Omega_3\tau_a^{-1}[(\Omega_3 - \delta)/\tau_a - \Delta/\tau_\sigma]}{(\Omega_{\mathrm{R}}^2 + \Omega_3\Delta)^2 + (\Omega_3/\tau_a)^2}. \quad (17)$$

Note that, unlike the case of a single QD, the lasing threshold D_{th} in the presence of two QDs increases with increasing interaction constant between them (Fig. 2). This is due to the fact that when QDs interact with each other, their levels split, resulting in a detuning between the QD transition frequencies and NP-enhanced plasmon resonance frequency, which leads to an increase in the lasing threshold.

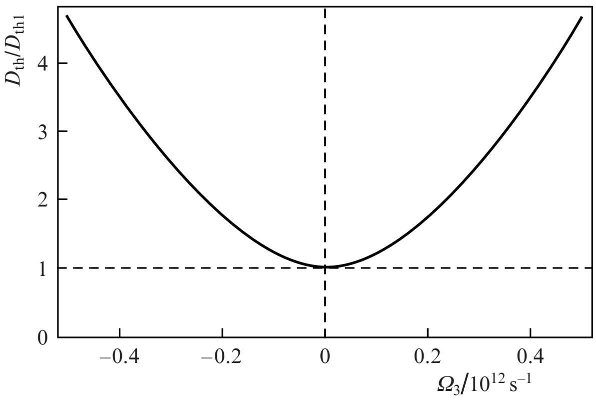

Figure 2. Dependence of the pump threshold D_{th} (in appropriate units for a single-quantum-dot spaser, D_{th1}) on the interaction constant Ω_3 between the QDs.

Thus, with increasing interaction constant between the QDs, the pump threshold increases. Consider the case when lasing is observed at $\Omega_3 = 0$, i.e., $D_0 > D_{th}(0)$. By increasing the interaction constants Ω_3, as has been stated above, the lasing threshold will increase and eventually exceed D_0, resulting in suppression of lasing of the NP dipole moment (Fig. 3).

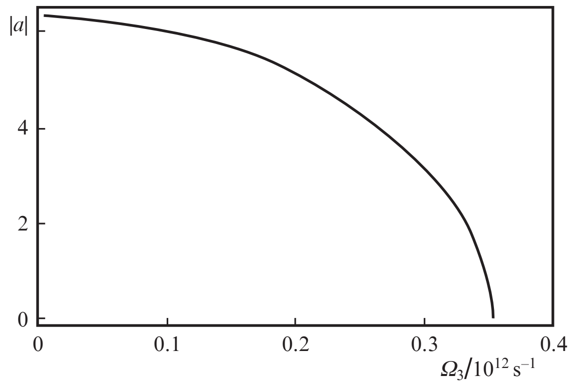

Figure 3. Dependence of the dipole moment of the nanoparticles a on the interaction constant Ω_3 between the QDs.

If there is a detuning between the NP and QD frequencies, the interaction between the quantum dots can lead to a decrease in the lasing threshold (Fig. 4). This is due to the fact that when QDs interact with each other, there occurs level splitting and an effective change in the QD transition frequencies, which can make the detuning between the NP and QD frequencies equal to zero. This leads to the minimisation of the lasing threshold at a certain interaction constant between the QDs. With a further increase in the interaction between the QDs the pump threshold increases (see Fig. 4). The dependence of the NP dipole moment on the interaction constants Ω_3 in the case of a detuning between the NP and QD frequencies is shown in Fig. 5. One can see that at a certain optimal interaction a maximum value of the dipole moment is reached, which corresponds to a minimum threshold value in Fig. 4. If the pump level is less than the threshold, lasing is suppressed and the NP dipole moment vanishes.

Note that the change in the interaction constant Ω_3 between the QDs inevitably leads to a change in the interaction constant Ω_R between the NP and QD. However, since

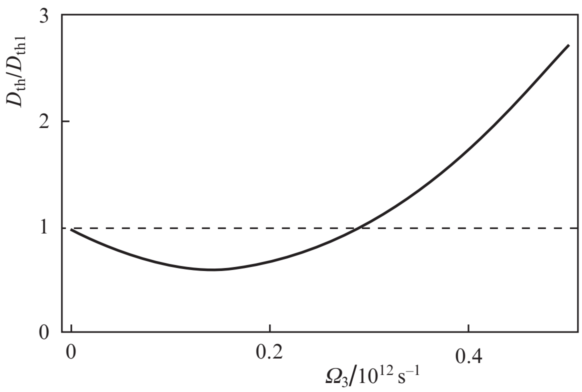

Figure 4. Dependence of the pump threshold D_{th} (in appropriate units for a single-quantum-dot spaser, D_{th1}) on Ω_3 at a frequency detuning between the nanoparticles and quantum dots, $\Delta = 3\times10^{13}$ s^{-1}.

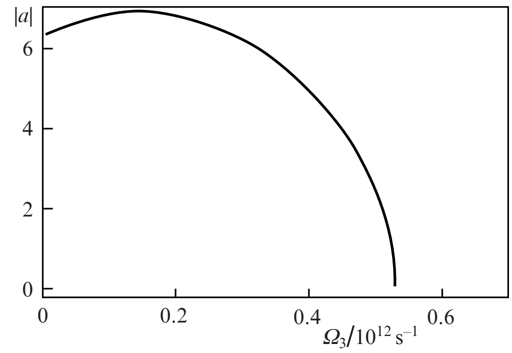

Figure 5. Dependence of the dipole moment of the nanoparticles a on Ω_3 at $\Delta = 3\times10^{13}$ s^{-1}.

$$\Omega_3 \sim r_{\mathrm{TLS-TLS}}^{-3} \text{ и } \Omega_R \sim \left(r_{\mathrm{TLS-TLS}}^{2} + r_{\mathrm{NP-TLS}}^{2}\right)^{-3/2},$$

where $r_{\mathrm{TLS-TLS}}$ is the distance between the QDs and $r_{\mathrm{NP-TLS}}$ is the distance between the NP and the middle line connecting QDs, changing $r_{\mathrm{TLS-TLS}}$ greatly affects Ω_3 and weakly Ω_R. Therefore, the change in the interaction constant between the QD and NP is neglected in this work.

It should be noted that in the near-field region the thermal losses in the nanoparticle substantially increase. Indeed, as shown in [27], the thermal effects play a significant role in the spaser dynamics. However, when using repetitively pulsed pumping and pulse duration of 10^{-9} s, a hundred plasmons can exist in a nanoparticle, which is one-to-two orders of magnitude larger than the number of plasmons we consider. Also it is worth noting that pumping can be performed by quantum wires which work in the ballistic rather than the diffusion regime [28]. In this situation, the pump rate can be quite large at small currents.

4. Conclusions

We have shown that the presence of two quantum dots can lead both to an increase and a decrease in the lasing threshold D_{th}. A decisive role is played by the interaction constant Ω_3 between the QDs. At a zero detuning ($\omega_{NP} = \omega_{TLS}$) the interaction between the QDs always leads to an increase in the lasing threshold. There exists a critical value of the interaction constant, at which lasing is suppressed, i.e., a single-quantum-dot spaser scheme may be preferable to a scheme with two

QDs. However, if there is a detuning between the QD transition frequency and the NP-enhanced plasmon resonance frequency, then by changing the interaction between the QDs one can reduce the lasing threshold and increase the steady-state value of the dipole moment.

Thus, the allowance for the collective interaction is important in calculating the spaser dynamics. Particular attention should be paid to the interaction of the elements of the active medium, which can be varied by changing the distance between the elements.

Acknowledgements. This work was partially supported by the Foundation for Advanced Studies (Contract No. 7/004/2013 – 2018 dated 23.12.2013), Russian Foundation for Basic Research (Grant Nos 12-02-01093, 13-02-00407, 13-02-92660), National Science Foundation (Grant No. DMR-1312707) and Dynasty Foundation.

References

1. Ramakrishna S.A., Pendry J.B. *Phys. Rev. B*, **67**, 201101 (2003).
2. Sarychev A.K., Tartakovsky G. *Phys. Rev. B*, **75**, 085436 (2007).
3. Sarychev A.K., Tartakovsky G. *Proc. SPIE Int. Soc. Opt. Eng.*, **6320**, 6320A (2006).
4. Lagar'kov A.N., Sarychev A.K., Kissel' V.N., Tartakovsky G. *Usp. Fiz. Nauk*, **179**, 1018 (2009).
5. Bergman D.J., Stockman M.I. *Phys. Rev. Lett.*, **90**, 027402 (2003).
6. Noginov M.A., Zhu G., Belgrave A.M., Bakker R., Shalaev V.M., Narimanov E.E., Stout S., Herz E., Suteewong T., Wiesner U. *Nature*, **460**, 1110 (2009).
7. Sarychev A.K., Pukhov A.A., Tartakovsky G. *PIERS Online*, **3**, 1264 (2007).
8. Protsenko I.E., Uskov A.V., Zaimidoroga O.A., Samoilov V.N., O'Reilly E.P. *Phys. Rev. A*, **71**, 063812 (2005).
9. Kuiru L., Xiangting L., Stockman M.I., Bergman D.J. *Phys. Rev. B*, **71**, 115409 (2005).
10. Stockman M.I. *Nature Photon.*, **2**, 327 (2008).
11. Stockman M.I. *J. Opt.*, **10**, 024004 (2010).
12. Klimov V.V. *Usp. Fiz. Nauk*, **178**, 305 (2008).
13. Gabitov I.R., Kennedy B., Maimistov A.I. *IEEE J. Sel. Top. Quantum Electron.*, **16**, 401 (2010).
14. Andrianov E.S., Pukhov A.A., Dorofeenko A.V., Vinogradov A.P., Lisyansky A.A. *Phys. Rev. B*, **85**, 035405 (2012).
15. Vinogradov A.P., Andrianov E.S., Pukhov A.A., Dorofeenko A.V., Lisyansky A.A. *Usp. Fiz. Nauk*, **182**, 1122 (2012).
16. Fyodorov I.A., Sarychev A.K. *arXiv:1303.7358v1* (2013).
17. Parfenyev V.M., Vergeles S.S. *Opt. Express*, **22**, 13671 (2014).
18. Parfenyev V.M., Vergeles S.S. *Phys. Rev. A*, **86**, 043824 (2012).
19. Rosenthal A.S., Ghannam T. *Phys. Rev. A*, **79**, 043824 (2009).
20. Andrianov E.S., Pukhov A.A., Dorofeenko A.V., Vinogradov A.P., Lisyansky A.A. *Zh. Eksp. Teor. Fiz.*, **144**, 112 (2013) [*JETP*, **117**, 205 (2013)].
21. Haken H. *Laser Light Dynamics (Light, Vol. 2)* (Amsterdam: North-Holland, 1985; Moscow: Mir, 1988).
22. Scully M.O., Zubairy M.S. *Quantum Optics* (Cambridge: Cambridge University press, 1997; Moscow: Fizmatlit, 2003).
23. Fedorov A.V., Rukhlenko I.D., Baranov A.V., Kruchinin S.Yu. *Opticheskie svoistva poluprovodnikovykh kvantovykh tochek* (Optical Properties of Semiconductor Quantum Dots) (St. Petersburg: Nauka, 2011).
24. Zuev V.S. Preprint FIAN No. 3 (Moscow, 2006).
25. Pantell R.H., Puthoff H.E. *Fundamentals of Quantum Electronics* (New York: Wiley, 1969; Moscow: Mir, 1972).
26. Protsenko I.E. *Usp. Fiz. Nauk*, **182**, 1116 (2012).
27. Fyodorov I.A., Parfenyev V.M., Tartakovsky G., Vergeles S.S., Sarychev A.K. *arXiv:1308.0811v1* (2013).
28. Li D., Stockman M.I. *Phys. Rev. Lett.*, **110**, 106803 (2013).