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Microwave transmission through a periodic three-dimensional metal-wire network containing random scatterers

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We have measured the transmission of microwave radiation through a nearly cubic metal-wire network with a lattice constant of 1 cm and a length of 8 cm on each side. We observe a sharp drop in the transmission below a cutoff frequency $\nu_c = 9.33$ GHz when the structure is empty. The gap persists when the network is filled with randomly positioned Teflon spheres, but ν_c is reduced by the ratio of the phase velocities in air and in a random medium of Teflon spheres and air. When aluminum spheres are substituted for some of the Teflon spheres, transmission peaks appear within the gap. The average transmission in an ensemble of random configurations shows a broadening of the band edge associated with precursors to localized states in this small sample. This indicates that the statistics of the localization transition can be studied in larger samples.
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Band gaps exist in the electromagnetic spectrum in a variety of periodic structures in analogy with the electronic band gaps in crystals.[1–3] In the photonic band gap (PBG), electromagnetic waves are evanescent. When disorder is introduced in a PBG structure, modes can be created in the gap but they are localized and again propagation is inhibited. Such localized states could be associated with a single defect in the photonic crystal which are analogous to localized electron states on isolated impurities in semiconductors[4] or to localized vibrational modes associated with defects in crystals.[5,6] They may also be associated with scattering from a statistically homogeneous random distribution of scatterers. John predicted the existence of a mobility edge separating localized from propagating electromagnetic waves within the pseudogap for perturbed periodic systems.[2] He proposed that introducing disorder would lead to Anderson localization for photons.[7]

In the present paper, we measure transmission through a nearly periodic copper-wire network with a structure close to simple cubic. A PBG is found in this structure both when it is empty and when it is filled with Teflon spheres producing a Teflon-air medium. By substituting aluminum spheres for some of the Teflon spheres, we create random scatterer configuations inside the structure. The transmission is measured in an ensemble of random scatterer configurations for various degrees of disorder introduced within the periodic metal matrix.

The band structure for various three-dimensional (3D) dielectric[3,8–10] and metallic[11–13] periodic structures possessing band gaps have been calculated and dielectric[3,14–16] and metallic[17] systems possessing band gaps have been constructed. Localized states have been created by adding to, removing, or displacing part of the structure in dielectric PBG materials and by cutting selected wires in metal systems.[15,17] These defects are introduced in a controlled manner so that it is possible to associate specific modes with particular defects. Our interest here is to create an ensemble of equivalent random configurations of scatterers within the PBG structure. We introduce disorder in a nearly periodic metal-wire network by filling it with mixtures of Teflon and aluminum spheres with diameters of 0.47 cm. This is approximately half the lattice constant of the wire network of $a = 1$ cm and the spheres readily fill the interior of the wire structure. The Teflon sphere medium has a scattering length which is much greater than the dimensions of the metal structure. Consequently, the Teflon spheres serve as an ideal support matrix into which scatterers can be substituted.

Sigalas *et al.* calculated the band structures for 3D periodic metal systems consisting of isolated metal scatterers and of metal wires which form a continuous network.[13] Periodic systems composed of isolated metal scatterers show behavior similar to that of dielectric PBG materials and do not exhibit a cutoff frequency below which transmission is sharply reduced. Calculations for metal networks with simple cubic and diamond-type structures predict a PBG in both systems. The cubic structure considered is infinite along the x and y directions and has a thickness along the z axis of $L=4a$ with a lattice constant a of 1.27 μm. The volume fraction occupied by the metal wires is 0.03. A band gap is found below a cutoff frequency $\nu_c=\gamma c/a$, where c is the speed of light in vacuum and $\gamma=0.4445$. At $\nu_c/2$, the attenuation is approximately 15 dB per unit cell.

The metal structure used in our experiment is a network of 1-mm-diam copper wires. The system has a geometry close to simple cubic structure with a lattice constant of 1 cm. The sample has a length of 8 cm on each side. Square wire meshes are created by overlaying one set of parallel wires onto another set of parallel wires oriented in the perpendicular direction. Straight wires are extended through parallel 2D wire meshes in the longitudinal direction. Wires oriented in the three direction are connected by a light solder applied at the vertices of the unit cells. The lattice is not perfectly periodic because of slight bending of the wires and because of variations in the solder joints. The volume fraction of the copper wires is 0.06. The front and back of the structure are open and copper plates are placed on the sides.

The transmission is measured in the frequency range between 4.0 and 10.4 GHz using a Hewlett-Packard 8722C network analyzer. Antennas, consisting of 1.5 cm pins, coupled the radiation to the metal network and detected the transmitted radiation. The input antenna and the front surface of the network are enclosed in an absorbing chamber and the detector and the output surface of the sample are within another chamber with absorbing walls. The antennas were placed 0.5 cm from the input and output faces of the sample. They are placed close to the sample to enhance the dynamic range of the measurements. As a result, however, the incident and detected waves cannot be approximated by plane waves.

The results of the measurements in an empty metal network are shown in Fig. 1(a). There is a sharp reduction in the transmission coefficient at a cutoff frequency of 9.33 ± 0.05 GHz. This gives $\gamma=0.31$ for the constant of proportionality in the relation $\nu_c=\gamma c/a$. There is a difference of more than 30% between the value of γ observed and the value calculated theoretically[13] which may be due to the difference in the filling fractions of metal wires in the sample used in the experiment and in the theoretical model. For frequencies below ν_c, the transmission is significantly suppressed. The average value of the attenuation in the gap from 6 GHz to ν_c is approximately 30 dB. At frequencies around 5 GHz, however, a transmission peak occurs in which attenuation is reduced to approximately 10 dB. This peak in transmission may arise because of deviations of the structure from simple cubic symmetry or it may be associated with surface states.[18] The sharp peaks in the spectrum observed above the cutoff frequency are a result of the interference between partial waves propagating through the system and reaching the detecting antenna. When the antennas are far from the sample only modes with small transverse components of the wave vector are detected and the spectrum appears relatively flat above the gap.

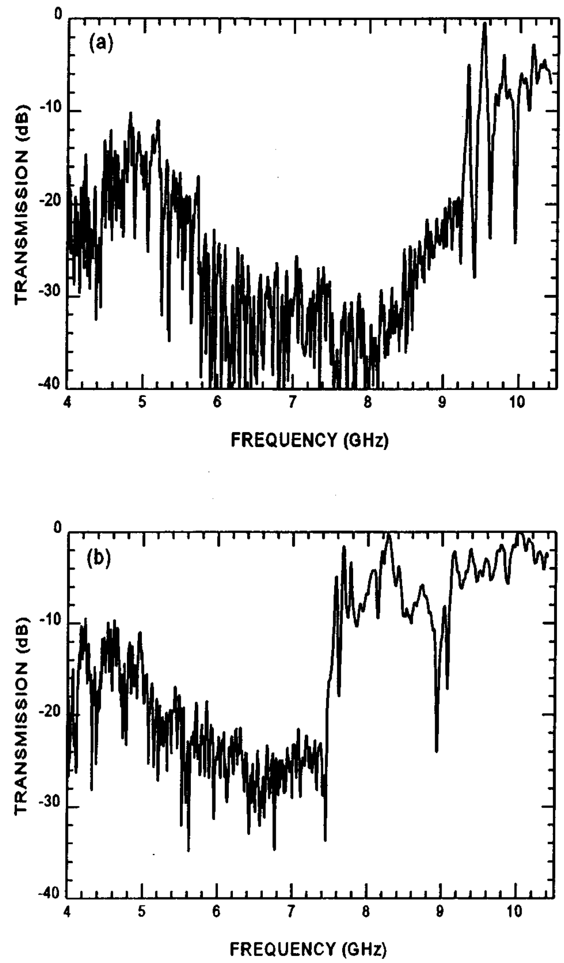

FIG. 1. Transmission of microwave radiation as a function of frequency through the metal network: (a) when it is empty and (b) when it is filled with Teflon spheres.

When the network is filled with Teflon spheres at a volume filling fraction of 0.57, which is measured as a fraction of the total volume free of metal, the cutoff frequency shifts to 7.58 ± 0.05 GHz [Fig. 1(b)]. The frequency of the transmission peak in the gap undergoes the same fractional shift. This determines that the peak is intrinsic to the metal structure. Since $\nu_c\propto v_{\mathrm{ph}}/a$, where v_{ph} is the phase velocity in the medium, we expect that the fractional shift of the cutoff frequency is proportional to the ratio of the phase velocities in air and in the medium of random Teflon spheres and air.

In order to determine the phase velocity in the medium of Teflon spheres and air, we measured the field transmitted through a sample of Teflon spheres contained in a 7.5-cm-diam copper tube with a length $L=10$ cm. In these measurements, the filling fraction of the Teflon spheres is 0.60, which is slightly higher than it is in the metal network. We first determine the index of refraction n for this sample at a frequency $\nu_0=6.45$ GHz to be $n=1.26$, by measuring the frequency shift of a resonant cavity mode in a medium of Teflon spheres relative to that in air. The absolute phase of

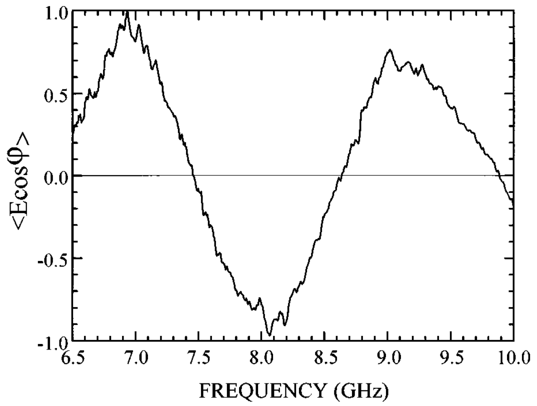

FIG. 2. Average of the real part of the field transmitted through samples of Teflon spheres contained in a copper tube. The field amplitude is normalized to unity.

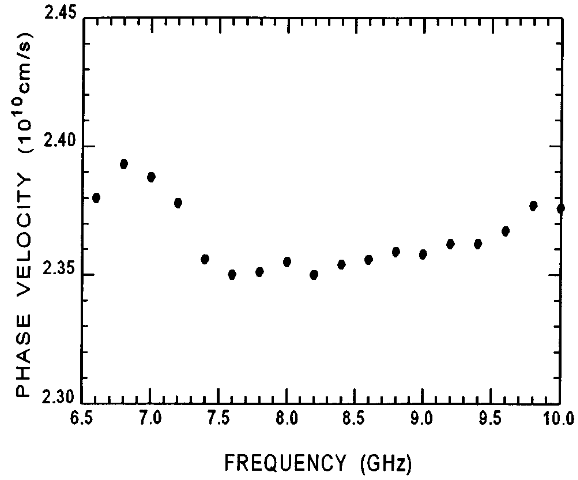

FIG. 3. Phase velocity in samples of Teflon spheres with filling fraction $f=0.60$.

the field transmitted through a sample with length L is $\varphi=kL=2\pi\nu L/v'_{\mathrm{ph}}$. Here v'_{ph} is the phase velocity in a copper tube containing the Teflon spheres and is given by $v'_{\mathrm{ph}}=v_{\mathrm{ph}}[1-(\nu_c^{\mathrm{wg}}/\nu)^2]^{-1/2}$,[19] where $v_{\mathrm{ph}}=c/n$ is the phase velocity in an unbounded medium of Teflon spheres and ν_c^{wg} is the cutoff frequency of the cylindrical waveguide. The value of $\nu_c^{\mathrm{wg}}=2.06$ GHz in this case was determined experimentally from measurements of the transmission through the copper tube filled with Teflon spheres from 1 to 4 GHz. Using these relations and the index of refraction measured at ν_0, we determine the absolute phase $\varphi_0=2\pi\nu_0 L/v'_{\mathrm{ph}}$. Then, starting from this frequency, we measure the frequency dependence of the field transmission through the sample of Teflon spheres contained in a copper tube. Transmission spectra were obtained for 1000 different sample configurations. The small variation between configurations of less than 1% of the field amplitude indicates that the scattering due to the Teflon spheres is extremely weak. This explains the persistence of the band gap when the network is filled with Teflon spheres. The real part of the ensemble average complex field $\langle E\cos\varphi\rangle$ is shown in Fig. 2. The well-defined sinusoidal frequency dependence of $\langle E\cos\varphi\rangle$ allows us to follow the phase roll-up with frequency. By following the increase in the phase of the transmitted field when the frequency is changed in small increments of 2.5 MHz, we are able to obtain the phase $\Delta\varphi$ accumulated between 6.45 and 10.0 GHz. By adding the accumulated phase $\Delta\varphi$ to the absolute phase φ_0 we determine the absolute phase in this frequency range. From this phase we calculate the phase velocity v_{ph} in a medium of Teflon spheres and air contained in a copper tube. The frequency dependence of v_{ph} is shown in Fig. 3. To determine the phase velocity in the Teflon-air medium within the metal network, small corrections are made using Maxwell-Garnet approximation.[20] At a frequency equal to the cutoff frequency for the metal structure filled with Teflon spheres, the phase velocity is 2.39×10^{10} cm/s. This gives $v_{\mathrm{ph}}/c=0.80$, which is close to the ratio of the cutoff frequencies of the network in Teflon and in air of 0.81.

When some of the Teflon spheres are replaced by aluminum spheres, transmission peaks appear below the band edge. Measurements were made for two filling fractions of aluminum spheres $f_{\mathrm{al}}=0.05$ and $f_{\mathrm{al}}=0.10$. A typical transmission spectrum at a filling fraction of the aluminum spheres of 0.05 is shown in Fig. 4. The peaks below the cutoff frequency are reminiscent of modes associated with ''donor'' defects, which have been observed in 3D dielectric PBG materials.[15] However, the radius of the scattering spheres in the present case is at least an order of magnitude smaller than the wavelength. Thus, these transmission peaks are more likely associated with random configurations of many metal spheres than with individual scatterers.

We characterize the strength of the disorder of the scattering medium at an aluminum filling fraction of 0.10 by measuring the scattering length l_s of the Teflon-aluminum mixture apart from the metal lattice. The scattering length is determined by measuring the intensity associated with the average field $I_{\mathrm{coh}}=|\langle E\rangle|^2$ as a function of the sample length L. In a sample tube with a diameter which is larger than the

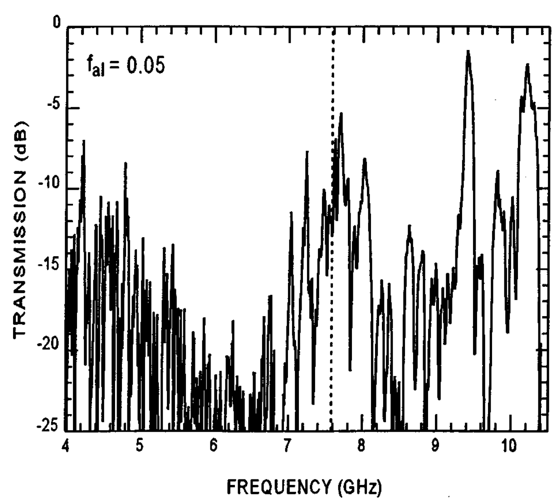

FIG. 4. Transmission of microwave radiation as a function of frequency for a single sample configuration for mixtures of Teflon-aluminum spheres at a filling fraction of aluminum spheres $f_{\mathrm{al}}=0.05$. The vertical dashed line indicates the band-gap edge for the system filled with Teflon spheres.

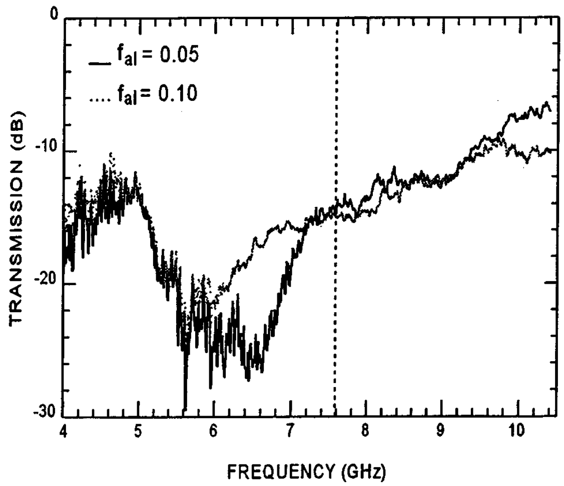

FIG. 5. Average transmission spectra obtained from 200 sample configurations of Teflon-aluminum mixtures at filling fractions of aluminum spheres of 0.05 and 0.10.

wavelength in the medium, the coherent intensity falls exponentially with an attenuation length which is close to the scattering length, once the sample is long enough that a single waveguide mode predominates. In this case, $I_{\mathrm{coh}} \sim I_0 \exp(-L/l_s)$. The coherent part of the field is obtained by averaging the field in 2000 sample configurations in the frequency range from 4 to 10 GHz. From measurements with sample length from 7.5 to 25 cm we find that $l_s \gtrsim 20$ cm at 4 GHz and that it decreases to $l_s = 4.5 \pm 0.5$ cm at 10 GHz. Since $l_s \geqslant l$, where l is the transport mean free path, the measurements show that kl is greater than 10 over the entire frequency range. Since localization in samples without long-range order is expected when $kl \sim 1$, this indicates that microwave radiation in the mixtures of Teflon and aluminum spheres apart from the metal network is far from the localization threshold. The values of k used in the estimate are calculated using the phase velocity in a random medium of Teflon spheres.

In Fig. 5 we present the average transmission spectra for 200 sample configurations for two concentrations of aluminum spheres. In these measurements, different sample configurations were created by shaking the sample by hand. In the average spectrum the band edge is broadened relative to that of the metal network in air and with Teflon spheres inside. The band gap becomes less pronounced as the density of metal scatterers increases. The frequency of the peak in the transmission at low frequencies, however, is not affected. This indicates that the phase velocity is not changed substantially by the presence of metal spheres into the network. The smearing of the band edge is not, therefore, a result of fluctuations in the phase velocity due to the presence of metal scatterers.

In conclusion, we have observed a band of attenuated microwave transmission through a 3D periodic metal network. We have shown that filling the metallic network with dielectric scatterers, which are small compared to the electromagnetic wavelength, shifts the band edge to a value proportional to the phase velocity in the medium. This demonstrates that a well-defined band gap can be produced in a structure which deviates from perfect periodicity and suggests that useful photonic devices can be produced using structures which are not perfectly periodic. The extent of disorder that can be introduced before the band gap is significantly altered can be studied by introducing a variety of defects into the periodic lattice. In the present study, we have measured transmission through a periodic metal-wire structure with various concentrations of randomly positioned metallic spheres. We observe a broadening of the band edge which is associated with the presence of localized states in the gap. Because the systems studied are small, the transmission peaks observed may not be associated with sharply defined localized states which could only be seen in larger structures. Studies in larger systems would make it possible to investigate the localization transition in this system. For such samples the ability to observe sharp transmission peaks associated with individual localized modes would be limited by the absorption of the sample.

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