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Lev Deych, Carlos Meriles, and Vinod Menon

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Resonant enhancement of magneto-optical polarization conversion in microdisk resonators

Lev Deych,[1,a)] Carlos Meriles,[2] and Vinod Menon[1]
[1]Physics Department, Queens College of CUNY, Flushing, New York 11367, USA
[2]Physics Department of City College of CUNY, New York 10031, USA

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We theoretically investigate the effect of a periodically modulated magnetic field on the polarization properties of whispering-gallery-modes (WGM) in microdisk resonators. We show that by matching the modulation frequency to the frequency offset between WGMs with two mutually perpendicular polarizations one can achieve an enhancement of the magneto-optical polarization effect by a factor of $M^{1/3}Q_M^{(av)}$, where M is the modal order of the WGM and $Q_M^{(av)}$ is the geometric average of the quality factors of the TE and TM polarized modes. © *2011 American Institute of Physics*. [doi:10.1063/1.3670354]

Optical whispering gallery mode (WGM) resonators are dielectric structures with axial symmetry, in which light, propagating almost tangentially with respect to their circumference, is confined by total internal reflection. Examples of such structures are spherical, toroidal, disk, or ring resonators. WGM resonators have attracted a great deal of attention because their modes are characterized by very small volume and high (up to 10^9) quality (Q) factors. The correspondingly large power build-up inside these resonators has already made them a platform for a broad variety of studies in ultra-low threshold lasing,[1,2] low power optical non-linearities and cavity quantum electrodynamics.[3–6] These structures also appear as promising candidates for sensing applications because the near-field-mediated interaction with the surrounding medium introduces detectable changes in the resonator frequencies.[7–11]

A question of practical interest is whether the sensitivity of WGM resonators to small changes in the index of refraction of the surrounding medium can also be exploited for magnetic field detection. For example, it is known[12–15] that by confining light in planar cavities one can enhance the magneto-optical Faraday polarization rotation (FPR) proportionally to the cavity quality factor. A similar effect in WGM resonators, which have much larger Q-factors, would be of great interest for multiple applications. However, the magneto-optical properties of WGM resonators are more complicated than those of planar cavities, which can be seen from the following simple arguments. FPR (along with other polarization magneto-optical effects) depends on the existence of frequency-degenerate, mutually perpendicular polarizations of light, which can be combined to form circular (or, more generally, elliptical) polarization states. Fabry-Perot resonators preserve this degeneracy and are thus inherently compatible with FPR. The situation is quite different in WGM resonators where modes with orthogonal polarizations (usually TE and TM modes) are non-degenerate, and, therefore do not allow for standard polarization effects such as FPR.

a)Author to whom correspondence should be addressed. Electronic mail: lev.deych@qc.cuny.edu.

Despite significant efforts devoted to understanding the optical properties of WGM resonators, the study of their magneto-optical response remains virtually unexplored (the investigation of magnetic-field-induced polarization switching in Ref. 16 is possibly the only exception). In this letter, we make an initial foray into this area by theoretically considering the polarization properties of a dielectric disk in an axially symmetric external magnetic field. This particular configuration has the advantage of allowing for a relatively simple analytical treatment while being readily reproducible in an experiment. More importantly, it provides insights into more general features of the magneto-optical properties of WGMs and their possible applications.

We consider a dielectric disk characterized by a radius R and refractive index n_d surrounded by a medium with refractive index n_0. If the thickness of the disk is much smaller than any relevant wavelengths in the spectral range under consideration, the high-Q WGM of the disk can be described using a two-dimensional approximation with a thickness-dependent effective refractive index determined self-consistently as described in Ref. 17. Applicability of the two-dimensional approximation to disks with subwavelength thickness might appear counterintuitive because this approximation becomes exact in the opposite limit of infinity long cylinders. It can be understood, however, by noting that the field of WGM is strongly confined within the disk with only evanescent tails protruding outside of the disk in both in-plane and perpendicular to the plane of the disk directions. In this case, and taking into account the subwavelength thickness of the disk, one can introduce the two-dimensional approximation as an averaging of the field over the thickness of the disk. The validity of this approach has been verified with rigorous numerical calculations by several authors, see, for instance, Ref. 18.

Without the external magnetic field, the normal modes of the disk resonator in this approximation are described by a two-dimensional wave equation of the form

$$\Delta_\perp F(\mathbf{r},t) - \frac{n^2(\mathbf{r})}{c^2}\frac{\partial^2 F(\mathbf{r},t)}{\partial t^2} = 0, \qquad (1)$$

where $\Delta_\perp$ is the two dimensional Laplacian taken with respect to coordinates in the plane of the disk, $n(\mathbf{r}) = n_d$

inside the disk and $n(\mathbf{r}) = n_0$ outside, and the field function $F(\mathbf{r}, t)$ can represent either TM or TE polarized field. In the case of TM polarized modes, F is identified with the component of the electric field E normal to the disk, while for TE modes this function represents the analogous component of the magnetic field B.

Equation (1) is conveniently solved by transforming to the spectral domain, and by expressing the temporal Fourier transform of the field $F(\mathbf{r}, \omega)$ in the form of a linear combination of Bessel (J_m) and Hankel (H_m) functions. The field scattered by the disk in this case takes the form $F_{sc}(\mathbf{r}, \omega) = \sum_m b_m H_m(k\rho)\exp(im\varphi)$, where ω is the spectral variable, $k = \omega/c$ with c being the speed of light in vacuum, while ρ and φ are, respectively, the radial and angular coordinates of the point $\mathbf{r}$ in the polar system with origin at the center of the disk. Maxwell's boundary conditions relate the coefficients b_m to corresponding coefficients $a_m(\omega)$ of the incident field via scattering amplitudes $\alpha_m^{(TE,TM)}(\omega)$ according to

$$b_m^{(TE,TM)}(\omega) = \alpha_m^{(TE,TM)}(\omega) a_m^{(TE,TM)}(\omega). \tag{2}$$

If the incident field is monochromatic, the spectral parameter in Eq. (2) can be identified with the incident frequency. WGMs appear in this formulation as scattering resonances determined by the poles of the respective scattering amplitudes, whose real and imaginary parts give the frequency and the width of these resonances. For each mode number m and polarization, there are multiple poles distinguished by their radial number $s \geq 1$ specifying the number of oscillations of the field in the radial direction. Resonances with $s = 1$ are characterized by the highest Q-factors and smallest mode volumes.

The effect of an external magnetic field $\mathbf{H}$ on the optical properties of the WGMs can be studied with the help of a phenomenological constitutive relation between the electric displacement vector $\mathbf{D}$ and the electric field $\mathbf{E}$ of the form[19] $\mathbf{D}(t) = \varepsilon\mathbf{E}(t) + ig\mathbf{E}(t) \times \mathbf{H}(t)$, where g is a magneto-optical constant. Unlike more traditional models, we allow for the external magnetic field to be time-dependent and assume that this dependence has the form of $\mathbf{H}(t) = \mathbf{H}^0\cos\Omega t$, where the modulation frequency Ω is assumed to be much smaller than the TE and TM resonance frequencies under consideration. Then, in the spectral domain, the constitutive relation takes the form

$$D(\omega) = \varepsilon E(\omega) - ig\mathbf{H}^0 \times [\mathbf{E}(\omega - \Omega) + \mathbf{E}(\omega + \Omega)]. \tag{3}$$

To first order in $\mathbf{H}$, Maxwell's equations with the constitutive relation, Eq. (3), can again be written in terms of the normal components of the optical electric, E, and magnetic, B, fields

$$\begin{aligned} \Delta_\perp E + x^2 n_d^2 E &= \frac{gx}{n_d^2 \rho} H_\varphi^0 \frac{\partial U^{(TM)}}{\partial \varphi}, \\ \Delta_\perp B + x^2 n_d^2 B &= -\frac{gx}{\rho} H_\varphi^0 \frac{\partial U^{(TE)}}{\partial \varphi}. \end{aligned} \tag{4}$$

Here, we assumed that $\mathbf{H}^0$ is oriented in the plane of the disk and is characterized by zero radial and φ-independent azimuthal components. Such a field can be created, for instance, by a straight current passing through the center of the disk, or by an alternating uniform electric field perpendicular to the disk. We also introduced in Eq. (4), the dimensionless spectral variable $x = kR$ and used the normalized radial coordinate $\rho/R \to \rho$. Functions $U^{(TM)}(\omega, \Omega)$ and $U^{(TE)}(\omega, \Omega)$ on the right hand side of Eq. (4) are defined as

$$\begin{aligned} U^{(TM)} &\equiv B(\omega - \Omega) + B(\omega + \Omega); \\ U^{(TE)} &\equiv E(\omega - \Omega) + E(\omega + \Omega). \end{aligned} \tag{5}$$

They introduce coupling not only between the TE and TM polarized fields but also between their different spectral components. When deriving Eq. (4), we neglected frequency Ω as compared to ω everywhere except in the terms appearing in Eq. (5). It will be seen below that this approximation is justified by the resonant nature of the frequency dependence of $U^{(TM,TE)}$.

To solve Eq. (4) to first order in H_φ^0, we write $E = E^{(0)} + E^{(1)}$ and $B = B^{(0)} + B^{(1)}$, where $E^{(0)}$ and $B^{(0)}$ are TM and TE polarized solutions of Eq. (1) in the absence of the magnetic field. If the incident field is of TE (TM) polarization one has $E^{(0)} = 0$ ($B^{(0)} = 0$). In this case, the magnetic field-induced correction $E^{(1)}$ ($B^{(1)}$) satisfies Eq. (4), where the right-hand side is found using $E^{(0)}$ ($B^{(0)}$), and the boundary conditions do not contain any incident field. Then, it can be shown that $E^{(1)}$ ($B^{(1)}$) outside of the disk can be presented as a combination of Hankel functions ($E^{(1)}$,$B^{(1)}$) $= \sum_m b_m^{(TM,TE)} H_m(n_0 k\rho)\exp(im\varphi)$ with expansion coefficients given by

$$b_m^{(TM)} = \frac{\alpha_m^{(TM)}}{2\pi x p_m^{(TM)}} S_m^{(TM)}; \quad b_m^{(TE)} = \frac{n_0 \alpha_m^{(TE)}}{2\pi x n_d p_m^{(TE)}} S_m^{(TE)}, \tag{6}$$

where $S_m^{(TM,TE)} = \mp(gx/n_d^{\lambda_{TM,TE}})\int_0^1\int_0^{2\pi} H_\varphi^0(\rho) J_m(n_d x\rho)(\partial U_0^{(TM,TE)}/\partial\varphi)\exp(im\varphi)d\rho d\varphi$ with $\lambda_{TM} = 2$ for TM coefficients and $\lambda_{TE} = 0$ in the TE case. The subscript 0 in $U_0^{(TM,TE)}$ indicates that these quantities are calculated with the zero order internal fields $B^{(0)}$ or $E^{(0)}$, respectively. Parameters $p_m^{(TM,TE)}$ in Eq. (6) are defined as

$$\begin{aligned} p_m^{(TE)}(x) &= n_d J'_m(n_0 x) J_m(n_d x) - n_0 J'_m(n_d x) J_m(n_0 x) \\ p_m^{(TM)}(x) &= n_0 J'_m(n_0 x) J_m(n_d x) - n_d J'_m(n_d x) J_m(n_0 x). \end{aligned} \tag{7}$$

Also, note that in the vicinity of a particular WGM resonance, the scattering amplitudes, $\alpha_m^{(TM,TE)}(x)$, can be written as

$$\alpha_m^{(TM,TE)}(x) = -\frac{i\gamma_m^{(TM,TE)}}{x - x_m^{(TM,TE)} + i\gamma_m^{(TM,TE)}}, \tag{8}$$

where $x_m^{(TM,TE)}$ and $\gamma_m^{(TM,TE)}$ are the dimensionless resonance frequency and width, respectively. Unlike the scattering amplitudes, $p_m^{(TM,TE)}(x)$, are slowly changing functions of x and can be replaced by their values at $x = x_m^{(TM,TE)}$.

Using well-known solutions of the single disk scattering problem, one finds

$$b_m^{(TM,TE)} = -\frac{2gm\alpha_m^{(TM,TE)}(x)\alpha_m^{(TM,TE)}(x)\alpha_m^{(TE,TM)}(x\pm X)a_0}{\pi x n_d n_0^{\lambda_{(TM,TE)}-1} p_m^{(TM)}(x)p_m^{(TE)}(x)} \times \int_0^1 H_\varphi^{(0)}(\rho)[J_m(n_d x\rho)]^2 d\rho, \qquad (9)$$

where $X = \Omega R/c$ is the dimensionless version of the modulation frequency of the external magnetic field. When deriving Eq. (9) we neglected X in the arguments of $p_m^{(TM,TE)}(x)$ and the Bessel functions as well as in all other slowly changing functions of frequency.

Assuming that the incident field excites only a single TM (TE) WGM with $m = M$ and is monochromatic with frequency x_0 close to the respective resonance $a_m(x) = a_0\delta(x - x_0)$, the spectral response of the system, according to Eqs. (2) and (9), can be described as follows. The initial TM (TE) component oscillates at frequency $x = x_0$ with its amplitude determined by Eq. (2), which reaches its maximum value when $x_0 = x_M^{(TM,TE)}$. At the same time, the induced TE (TM) component oscillates at frequencies $x = x_0 \pm X$ with amplitudes determined by the products $\alpha_m^{(TM)}(x_0)\alpha_m^{(TE)}(x_0 \pm X)\alpha_M^{(TE)}(x_0 + X)\alpha_M^{(TM)}(x_0)$ (or $\alpha_m^{(TE)}(x_0)\alpha_m^{(TM)}(x_0 \pm X)$, respectively). In order for these amplitudes to have an appreciable value one has to simultaneously require $x_0 = x_M^{TM(TE)}$ and $x_0 \pm X = x_M^{TE(TM)}$ so that both initial and induced polarization components oscillate at their own resonance frequencies. This is a frequency matching condition reflecting conservation of energy in any nonlinear frequency conversion process. Since in disk resonators $x_M^{TE} > x_M^{TM}$, this condition can only be fulfilled for the $x_0 + X$ amplitude in the case of TM excitation (or for the $x_0 - X$ amplitude if the TE mode is originally excited) as expressed in Eq. (9), provided that $X = x_M^{TE} - x_M^{TM}$. Taking into account that according to Eq. (8) both TE and TM scattering amplitudes are equal to -1 at exact resonances, we have for the on-resonance amplitude of the induced polarization components

$$b_M^{(TM,TE)} = -\frac{(n_d^2 - n_0^2)gM^{4/3}\Lambda_M\sqrt{Q_M^{(TM)}Q_M^{(TE)}}}{2^{4/3}\pi^2 n_0^{\lambda_{(TM,TE)}+1}\upsilon^2}a_0, \qquad (10)$$

where $\Lambda_M = \int_0^1 H_\varphi^{(0)}(\rho)[J_M(n_d x\rho)]^2 d\rho$ characterizes the strength of the external magnetic field, $Q_M^{(TM,TE)} = x_M^{(TM,TE)}/\gamma_M^{(TM,TE)}$ are the Q-factor of the respective mode and $\upsilon \approx 0.7$ is the derivative of the Airy function taken at its first zero. When deriving Eq. (10), we took into account that for $M \gg 1$, $n_d x_M^{(TM,TE)} \approx M$ and functions $p_M^{(TM,TE)}$ at the respective resonances can be estimated as[20]

$$\left|p_M^{(TM,TE)}\right| \approx \left(\frac{2}{M}\right)^{2/3}\upsilon n_0\sqrt{\frac{2\pi}{Q_M^{(TM,TE)}(n_d^2 - n_0^2)}}. \qquad (11)$$

In the case of the magnetic field of a straight wire $H_\varphi^0 = h/\rho$, one finds, $\Lambda_M \approx 2h/M$ for $x \gg 1$. This result can actually be extended to an arbitrary radial dependence of the magnetic field by taking into account that for $M \gg 1$ and x in the vicinity of the WGM resonance, the Bessel function in the definition of Λ_M is different from zero only in a small interval $M/nx < \rho < 1$. Therefore, unless the external magnetic field changes fast over this interval, its actual radial dependence should not affect the dependence of the integral upon M.

Equation (10) demonstrates that by using a periodically modulated external magnetic field to "bridge" the frequency difference between TE and TM polarized WGMs one can significantly enhance the magneto-optical polarization response of a WGM cavity. The main enhancements in Eq. (10) is due to the geometric average of the quality factors of the TE and TM resonances, which is further amplified by an additional $M^{1/3}$ factor, which might provide an extra order of magnitude enhancement for resonances with significantly large M.

In the standard experimental setup, when one records a signal in a tapered fiber transmitted beyond the point of contact with the resonator, the induced polarization component is identified as a transmission maximum when recorded as a function of the modulation frequency of the magnetic field. The spectral position of the resonant value of the modulating frequency is determined by the frequency splitting between TM and TE modes. The latter can be approximately estimated as $x_M^{(TE)} - x_M^{(TM)} \approx \sqrt{n_d^2 - n_0^2}/n_d$.[20] Thus, the resonance value of X is typically a fraction of the free spectral range of the resonator and scales inversely with its radius.

The dependence of the resonant value of the magnitude of the polarization-converted signal upon the quality factor and mode number predicted by Eq. (10) is illustrated in Figure 1, where we present the resonant value of the enhancement factor $\kappa_M(X_{res}) \equiv |b_M/(gha_0)|$ calculated directly from Eq. (9). Normalizing $\kappa_M(X_{res})$ by the Q-factors of the respective modes, determined numerically from the poles of the scattering coefficients $\alpha_M^{(TM,TE)}$, and plotting it as a function of M, we find that the calculated data follow a $M^{1/3}$ dependence in agreement with Eq. (10).

The polarization conversion in the case under discussion is accompanied by up- or down-conversion of frequency stimulated by the modulated magnetic field. Unlike more traditional frequency conversion processes utilizing electrical Kerr nonlinearity,[21] the effect considered here is based on magnetic field induced gyrotropy. Taking into account that for millimeter sized resonators, X is in the GHz spectral region, one can see that the effect under consideration

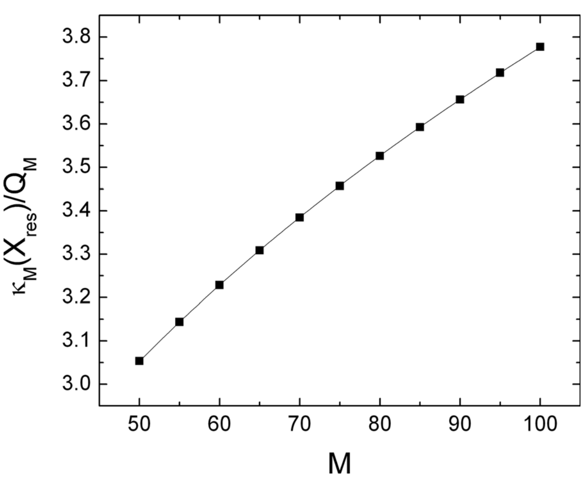

FIG. 1. Resonance value of the enhancement factor versus mode number. Squares—calculated values, line-fit with the $M^{1/3}$ dependence.

provides an additional modality for optical detection of microwave signals, in which frequency conversion is accompanied by polarization conversion as well. For numerical illustration, we consider a disk resonator with radius $R \approx 2000\,\mu$m, width $d \approx 1\,\mu$m and effective refractive index $n \approx 1.5$. Assuming $\lambda \approx 1.5\,\mu$m, $M \approx 1.2 \times 10^4$, and a Q-factor of order 10^9, Eq. (11) predicts an enhancement factor of the order of 10^9. Thus, taking into account that the magneto-optical parameter for silica is approximately $g \approx 4 \times 10^{-7} T^{-1}$,[22] we estimate the ratio of the power of the magnetic-field induced polarization component P_{ind} to the incident power P_0 to be of the order of $P_{inc}/P_0 \propto |b_M/a_0|^2 \approx 10^{-9} P_{mw}/A_d$, where P_{mw} is the power of the microwave radiation generating the modulated magnetic field, and A_d is the area through which this power flows, both in SI units. For the present geometry—where the microwave energy flows through the side walls of the disk with area $A_d = 2\pi R d \approx 1.3 \times 10^{-8}\,\mathrm{m}^2$—one can relate the powers of incident, polarization-converted and microwave signals as $P_{ind}/P_{mw} \approx 0.1 P_0$. It is also interesting to rewrite this estimate in terms of the strength of the magnetic field detectable with this effect. Assuming that one can measure the converted signal of at least one nW of power, we estimate that with a mW incident signal, this corresponds to detecting an external magnetic field of 0.1 G.

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