Defect-induced whispering-gallery-mode resonances in optical microdisk resonators

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In this Letter we present results of theoretical and experimental studies of whispering-gallery modes in optical microdisk resonators interacting with subwavelength dielectric particles. We predict theoretically and confirm by direct observations that, contrary to the generally accepted models, both peaks of the particle-induced doublet of resonances are redshifted with respect to the position of the initial resonance.

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Whispering-gallery modes (WGMs) are excitations of axially symmetrical optical resonators characterized by very narrow spectral lines. High (up to 10^6–10^9) quality (Q) factors of these resonances make them attractive for a variety of applications [1], particularly as sensors with a potential for single-molecule sensitivity [2]. As a result, a great deal of attention has been paid over recent years to the problem of interaction between WGMs and subwavelength scatterers [3–9]. When the Q factor of the resonator is high enough, this interaction results in splitting of the initial WGM resonance into two peaks [3,7–9]. In the case of lower Q factors, the two split peaks overlap and are observed as a single shifted peak [2,4,7]. Lately, the origin of the splitting and the relative positions of the split frequencies with respect to the initial resonance have been the object of many studies [3,8,9]. Using a tip of an optical-near-field microscope as a subwavelength scatterer, Mazzei *et al.* in Ref. [3] were able to demonstrate formation of the doublet of resonances in a spherical resonator and to study their properties as a function of the scatterer's position. These authors clearly demonstrated that the doublet consists of the original single sphere resonance *not affected by the particle* and a particle-induced peak sensitive to the position of the scatterer. In order to explain these results, Mazzei *et al.* proposed a phenomenological model, in which formation of the doublet was related to an interaction between clockwise (cw) and counterclockwise (ccw) WGMs, mediated by their direct scatterer-induced coupling to a common set of "radiative modes."

A rigorous *ab initio* analysis of the interaction between WGMs of spherical resonators and dipole-sized particles carried out in Ref. [5] yielded a different physical picture of this interaction and showed that while the phenomenological model of Ref. [3] reproduced correctly certain significant features of this phenomenon, it did not agree with the microscopic analysis in such details as polarization or frequency dependence of the WGM-particle coupling and spatial distribution of the resonator-particle field. Nevertheless, due to its simplicity, convenience, and apparent agreement with experiments, the model of Ref. [3] has been adopted as a universal "paradigmatic" approach to interaction between WGM modes and dipole-sized particles [8–11].

In this Letter we revisit the issue of the applicability and universality of the phenomenological approach to the WGM-particle interaction by focusing on resonances of a planar disk resonator interacting with a subwavelength, also planar, particle. While a similar problem was considered in Ref. [12], the results presented here have not been previously reported to our knowledge. In particular, we demonstrate that the response of this system to a TE-polarized excitation consists of two resonance peaks, both of which are shifted toward longer wavelengths from the initial single-disk resonance. This prediction, which is in contradiction with the phenomenological model, is confirmed by direct experimental observation of the peaks in the optical response of a silicon microdisk interacting with Si nanoparticles functionalized on its surface. Our results indicate that the assumption of direct coupling of the WGM of the resonator to the propagating modes of free space made in Ref. [3] is not sufficient to capture the essential physics of WGM-particle interaction. A more appropriate model requires instead consideration of interaction between WGMs and modes of the particle, which are naturally coupled to the propagating modes.

Modeling the resonator-particle system as two coplanar disks with radii R_1 and R_2, we study their response to an external excitation adopting the two-dimensional (2D) approximation, in which Maxwell equations are reduced to a 2D scalar equation for normal to the disks' component of magnetic (TE modes) or electric (TM modes) fields. While this formulation is exact only for infinitely long cylinders, it is routinely used to describe WGMs in disks with thickness smaller than the WGM's wavelength [13]. The three-dimensional (3D) nature of this system manifests itself through an effective refractive index, which depends on the thickness of the disk, and can be found in a self-consistent manner [13]. However, in many cases, including this work, it is considered as a phenomenological parameter to be determined from the experiment.

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Using a multidisk modal expansion approach (see, e.g., Ref. [14]), we present the magnetic (electric) component of the TE (TM) fields scattered by the resonator and the defect as linear combinations of the Bessel functions (see details in Ref. [14]) and describe them by respective expansion coefficients $b_m^{(1)}$ and $b_m^{(2)}$. These coefficients obey the system of equations $b_m^{(1,2)} = \alpha_m^{(1,2)}(a_m^{(1,2)} - i\sum_l b_l^{(2,1)} t_{m,l})$, where indices m and l are azimuthal mode numbers characterizing the angular dependence of the field in WGMs, $\alpha_m^{(1,2)}$ are single-disk scattering amplitudes for the WGMs of the resonator and defect, respectively, and $t_{m,l}$ is a coupling coefficient, which characterizes the interaction strength between the mth mode of the resonator and the lth mode of the defect. This coupling coefficient is expressed in terms of the Hankel function: $t_{m,l} = iH_{m-l}(kR_{1,2})$, where k is the vacuum wavenumber corresponding to frequency ω of the external excitation, and $R_{1,2}$ is the distance between the centers of the resonator and the scatterer. In the case of higher-order modes $m \gg 1$, the Hankel function is dominated by its imaginary part, so that the coupling coefficients $t_{m,l}$ are mostly real. Coefficients $a_m^{(1,2)}$ represent amplitudes of the mth azimuthal component of the incident field; it is assumed that this field excites only one $m = M$ mode of the resonator so that $a_m^{(1)} = a\delta_{m,M}$ and $a_m^{(2)} = 0$.

The equations for the expansion coefficients show that the M mode of the resonator couples to different m modes of the defect with a coupling strength determined by respective scattering amplitudes $\alpha_m^{(2)}$. In the limit $n_d kR_2 \ll 1$, where n_d is the refractive index of the scatterer, the TE amplitudes with $m > 0$ are proportional to $(kR_2)^{2|m|}$, while $\alpha_0^{(2)} \propto (kR_2)^4$ so that the largest contribution to the scattering of TE modes comes from interaction with $m = \pm 1$ modes. In the case of TM modes, the $(kR_2)^2$ contribution to the $|m| = 1$ scattering amplitude vanishes, while the amplitude of the s-scattering ($m = 0$) $\alpha_0^{(2)}$ becomes of the order of $(kR_2)^2$, making this channel dominating for this polarization. This result contradicts the assumption of Ref. [15] that the scattering of the WGM of both TM and TE polarizations due to a point defect is dominated by the s channel. Accordingly, one needs to take into account the coupling of the $\pm M$ modes of the resonator with $m = \pm 1$ modes of the scatterer for TE polarization and with the $m = 0$ particle's mode for TM polarization. Neglecting coupling to all other resonators' modes, one can demonstrate that the response of the resonator-particle system is best described in terms of symmetric and antisymmetric modes defined as $b_M^{(\pm)} = b_M^{(1)} \pm (-1)^M b_{-M}^{(1)}$:

$$b_M^{(\pm)} = \frac{a_M}{[\alpha_M^{(1)}]^{-1} - \alpha_1^{(2)}(t_{M,-1} \pm t_{M,1})^2}, \quad \text{(TE)}, \qquad (1)$$

$$b_M^{(+)} = \frac{a_M}{[\alpha_M^{(1)}]^{-1} - 2\alpha_0^{(2)} t_{M,0}^2}, \quad b_M^{(-)} = a_M \alpha_M^{(1)}, \quad \text{(TM)}. \qquad (2)$$

Here the particle-scattering amplitudes for both $|m| = 1$ and $m = 0$ scattering channels can be approximated as $\alpha_{1,0}^{(2)} \approx -1/(1 + ip_{1,0}^{-1})$, where both p_1 and p_0 are proportional to $(kR_2)^2$: $p_{1,0} = \kappa_{1,0}(kR_2)^2$, but with different coefficients: $\kappa_1 = [\pi(n_d^2 - 1)]/[4(n_d^2 + 1)]$, $\kappa_0 = [\pi(n_d^2 - 1)]/4$. The scattering amplitude describing the response of a single-disk resonator in the vicinity of a respective WGM with $m = M$ characterized by frequency ω_M and resonance width γ_M can be presented as $\alpha_M = -i\gamma_M/(\omega - \omega_M + i\gamma_M)$.

Equations (1) and (2) clearly demonstrate the difference between the coupling of TM and TE WGMs to the subwavelength particle. In the former case, only the frequency of the $a_M^{(+)}$ mode of the disk-particle system is affected by the particle, while $a_M^{(-)}$ resonates at the same frequency as a single disk. At the same time, the symmetric and antisymmetric modes of TE polarization are characterized by resonance frequencies, both of which differ from ω_M. Considering the poles of expression in Eqs. (1) and (2) and neglecting the small imaginary part of the coupling coefficient $t_{M,l}$, one can derive the following approximate expressions for the particle-induced resonances. For TE polarized modes one has two new frequencies,

$$\omega_{M,TE}^{(\pm)} \approx \omega_M - \gamma_M p_1 (t_{M,-1} \pm t_{M,1})^2 - i\gamma_M[1 + p_1^2 (t_{M,-1} \pm t_{M,1})^2], \qquad (3)$$

both shifted downward from the single-disk resonance in contradiction to Ref. [3]. The behavior of TM modes is consistent with phenomenological theory and features a single particle-induced frequency split off the single-disk resonance:

$$\omega_{M,TM}^{(+)} \approx \omega_M - 2\gamma_M p_0 t_{M,0}^2 - i\gamma_M[1 + 2p_0^2 t_{M,0}^2]. \qquad (4)$$

The case of several independent scatterers can be approximately described by the same Eqs. (3) and (4) by replacing the polarization parameters $p_{1,0}$ with $Np_{1,0}$, where N is the number of scatterers. This approach is justified if, on the one hand, the number of particles is small enough to neglect multiple-particle scattering, but, on the other hand it is large enough to average out the effects of fluctuations in the size and position of individual particles [16]. The predicted behavior of TE WGMs has been verified by direct experimental observations of particle-induced spectral shifts of WGM resonances in a microdisk resonator. The experimental structure consists of an on-chip fabricated silicon microdisk resonator integrated with two bus waveguides for light injection and signal extraction; see Fig. 1. A wavelength-tunable laser around 1.55 μm placed at the input port of the first

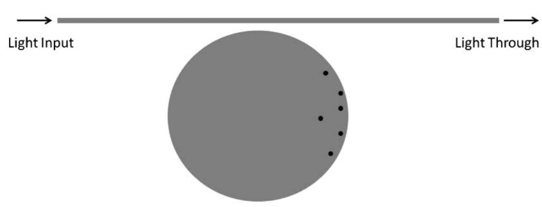

Fig. 1. Schematic of a structure used in measurements. Disk is of radius $R_1 = 4\,\mu$m, and thickness 160 nm. Silicon nanoparticles have average radius of 100 nm.

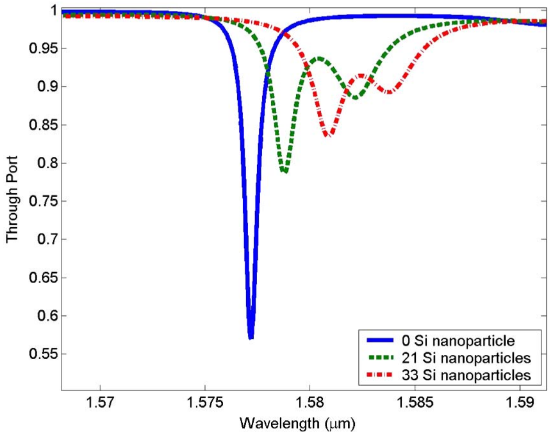

Fig. 2. (Color online) Spectra of a signal in a through port of the bus waveguide (solid line, zero particle; dashed line, 21 particles; dashed-dotted line, 33 particles).

bus waveguide is used to launch the waveguide mode; the photodetector is placed at the through port of the same waveguide behind the region where the waveguide couples to the microdisk. The silicon-on-isolator wafer was used to fabricate the disk on SiO_2 as the bottom cladding material, and the 100 nm gap between the bus waveguide and the resonator was fabricated using electron beam lithography. Silicon nanoparticles were functionalized on the surface of the disk; while particles were distributed throughout the entire disk, only those on the rim of the disk were counted using portable scanning electron microscopy.

The results of the measurements are presented in Fig. 2, where one can clearly see the emergence of a doublet of particle-induced resonances between 1.5 μm and 1.6 μm, both of which are shifted with respect to the single-disk resonance at around 1.550 μm. The solid line is the bus waveguide through port signal without any Si nanoparticles showing original resonance at 1.5772 μm. With an increasing number of Si nanoparticles (21 and 33) attached to the resonator, a doublet resonance modes appear at 1.5787 μm and 1.5821 μm for 21 nanoparticles (dashed line), and at 1.5808 μm and 1.5837 μm for 33 nanoparticles (dashed-dotted line).

Even though the structure used in these experiments does not correspond exactly to the theoretical model due to the 3D nature of the scatterers and their positions in the interior of the resonator, the experimental results are in qualitative agreement with the theory. This can be understood by noting that the main effect of the three-dimensionality of the scatterers consists in coupling between the TE and TM modes of the resonator. While this coupling might result in the shift of the previously unaffected TM resonance, it will not change the qualitative behavior of the TE mode. The positioning of the scatterers in the interior of the disk also would not change the main qualitative predictions of the model: TE (TM) modes will still couple to $m = \pm 1$ ($m = 0$) modes of the particle with the same consequences as in the exterior case. Despite these differences, the model produces an order-of-magnitude agreement with experimental results. Indeed, using an effective refractive index of the resonator $n_d \approx 2.3$ and estimating the mode azimuthal index to be $M \approx 36$, one finds for $N = 21$ particles that $(\omega_- - \omega_+)c/R_1 \approx 4.3 \times 10^{-3}$, which is smaller than the experimental value $\approx 5.5 \times 10^{-3}$. The interior position of the particles, where they feel a stronger field, can partially account for this difference.

In summary, we demonstrated, theoretically and experimentally, that the spectrum of WGM resonators interacting with a subwavelength particle depends on the geometry of the resonator and cannot be described by a single universal "one size fits all" model. Unlike the case of a spherical resonator, the spectral response of the disks-particle system to an external excitation of TE polarization is characterized by a doublet of peaks, which are both shifted from the original single-disk resonance.

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