*Journal of Archaeological Method and Theory, Vol. 12, No. 1, March 2005 (© 2005)*
DOI: 10.1007/s10816-005-2396-6

# The Broken Past: Fractals in Archaeology

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*Many archaeological patterns are fractal. Fractal analysis, therefore, has much to contribute to archaeology. This article offers an introduction to fractal analysis for archaeologists. We explain what fractals are, describe the essential methods of fractal analysis, and present archaeological examples. Some examples have been published previously, while others are presented here for the first time. We also explain the connection between fractal geometry and nonlinear dynamical systems. Fractals are the geometry of complex nonlinear systems. Therefore, fractal analysis is an indispensable method in our efforts to understand nonlinearities in past cultural dynamics.*

**KEY WORDS:** fractals; nonlinear dynamics; chaos; self-organized criticality.

## INTRODUCTION

All archaeologists, theoreticians and shovelbums alike, search for patterns in the archaeological record. Indeed, archaeological practice consists very largely of detecting, describing, and interpreting patterns in the archaeological record. Many archaeological patterns are very complex and irregular and defy simple description. These kinds of patterns are usually *fractals*, but they have not been recognized as such. In this article, we tackle the subject of fractals in archaeology: where they are, how to find them, and what they mean.

Our purpose in this article is to show working professional archaeologists why they should use fractal analysis in their work. Fractals abound in archaeology, but

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typically they have not been seen for what they are. Here we will demonstrate (1) that fractals are ubiquitous in archaeological datasets; (2) that describing fractals properly yields important results; and (3) that fractal geometry has significant theoretical implications. As an added bonus, readers will be pleasantly surprised to find that fractal analysis is often easy, although the underlying ideas may at first seem strange.

Archaeologists continually face the problem of selecting useful and appropriate statistics to describe and analyze their data. Because the body of statistical literature is large, technical, and constantly evolving, we often find it difficult to make appropriate choices. Choosing our statistics is often a major decision. Inappropriate statistics will yield weak answers when stronger ones are available, or they may lead to errors of interpretation. Moreover, because one's statistical approach can and should determine what data are collected in the field and in the laboratory, it is essential to choose one's statistics wisely, or risk wasted time and effort. We hope that this article will clarify the role of fractal statistics in archaeological practice. To achieve that goal, we have selected examples of the widest possible applicability, so that archaeologists of all stripes can see the relevance to their work.

Many readers will be understandably reluctant to learn yet another statistic. Personally, we want to learn a new statistic as much as we want to learn the rules for cricket. But fractals are too important to ignore. We all need to understand what they are, learn how to find them, and recognize how they affect interpretation.

First, fractal analysis is not just a single statistic. It is a large suite of quantitative techniques for describing and analyzing complex and irregular phenomena. Some of the techniques will be new to most archaeologists, but many will be familiar. We promise that the basic and common techniques will be easily accessible to anyone who remembers their high school math or their one class in statistics.

Second, many archaeological phenomena are fractal. This is to be expected because fractal patterns have been shown to be common in other kinds of natural, cultural, and social data. Although fractal analysis is only beginning in archaeology, we will demonstrate in this essay that fractal patterns are commonplace in our data. Fractal geometry offers the most parsimonious description of these phenomena and, in some cases, it offers the *only correct description*. The *first* task of all science is description and measurement, without which interpretation and understanding are impossible. It is all very well to insist that description without explanation is inadequate, but the description first of all has to be right. In many cases in other scientific fields, it has been shown that patterns and processes that had long been thought to obey Gaussian or Poisson distributions are actually fractal.[5] The difference is significant.

[5]Gaussian and Poisson distributions are approximations to the Binomial distribution, the former when the average is moderate and the latter when it is very small. They are the best known of the "bell curve" distributions that are taught, almost exclusively, in all statistics courses.

Third, fractals are not only descriptive—they provide clues to the underlying dynamics that created the fractal patterns. It is, of course, a dictum of modern archaeology that the archaeological record is the static picture of past cultural dynamics (e.g., Binford, 1981). Fractals can help us infer the underlying dynamics of prehistoric social systems. Since fractals are strongly nonlinear patterns, they help us infer the properties of strongly nonlinear systems.

Fractals have generated controversy in some fields. For example, the idea that the spatial distribution of matter in the universe may be fractal has caused intense debate in astronomy because the Standard Cosmological Model assumes that matter is distributed as a homogenous Gaussian variable (Baryshev *et al.*, 1998; Sylos Labini *et al.*, 1998). Fractal analysis has also sparked debates in fields as diverse as human physiology and geology. We do not expect that the analyses discussed below will be tremendously controversial, but we do hope they will spark greater interest in fractals among archaeologists. As the reader will see, fractals are part of the large and rapidly advancing field of nonlinear science. This is such a wide-open and exciting field of study that, if we were graduate students, we would choose to work in it.

In the following pages, then, we outline the ways in which fractal geometry is relevant to archaeologists by reviewing some fractal patterns and processes known to exist in archaeology. We first provide a basic explanation of fractals and a brief introduction to fractal analysis. Then, we describe several fractal phenomena that are common in archaeology. We draw our examples from two inimitably archaeological problems: artifact analysis and spatial analysis. Along the way, we suggest several possible archaeological applications of fractal analysis that have yet to be tested. As we catalog below, fractal analysis can be applied to many essential archaeological problems and datasets. Finally, we discuss the relation between fractal geometry and dynamical systems theory.

## WHAT ARE FRACTALS?

Fractal analysis interests scientists because it yields elegant and parsimonious descriptions of immensely complex patterns. Fractal theory also unites disparate ideas from set theory, topology, cosmology, hydrology, geomorphology, linguistics, geography, and many other fields. The ability of fractals to unify similar ideas from very different fields is a measure of its power and evidence of the universality of fractal patterns in nature. Obviously, fractals have been a subject of intense interest in the physical and biological sciences. Hundreds of articles and books have been written about fractals in these fields. Some of the social sciences, particularly economics and geography, have developed modest literatures on fractals. Fractals have been a somewhat neglected topic in archaeology. The only general article on fractals in archaeology was Ezra Zubrow's prescient article in written in 1985. Since then, a corpus of information on fractal phenomena in archaeology and prehistory has accumulated slowly.

Fractal geometry is the study of the form and structure of complex, rough, and irregular phenomena. In the past, many fractal patterns were mistakenly treated as if they were non-fractal. In such cases, the patterns have typically been analyzed using conventional statistics, which often assume that the variation in the pattern is caused by normally distributed (Gaussian) effects. When the patterns are really fractal, classical statistical modeling yields faulty results that do not properly characterize the data. Not only are the estimates or predictions made using conventional parametric statistics relatively inaccurate, but worse, they are wrong more often than the errors associated with their parameter estimates would indicate (Brunk, 2002).

Benoit Mandelbrot is the father of modern fractal analysis (1967, 1983). He coined the term "fractal," and more importantly he recognized that many diverse natural and cultural phenomena are fractal. He had the insight that fractals all belonged to a single type of universal phenomenon that was being misunderstood and incorrectly analyzed. The basic premise of fractal analysis is that many complex and irregular patterns traditionally believed to be random, bizarre, or too complex to describe, are in fact strongly patterned and can be described by fairly simple algorithms that embody the principles of *self-similarity*.

Mandelbrot's technical definition of a fractal[6] would mean nothing to most archaeologists, so we shall substitute a transparent and accessible one. A fractal is a *set* with *self-similar geometry* and *fractional dimension*. This probably still seems cryptic, but each individual element is easy to understand. Let us look at the three parts of this definition.

By "*set*," we mean a mathematical set. Any kind of dataset can be a fractal: points, lines, surfaces, multi-dimensional data, and time series.

A pattern is "*self-similar*" if it is composed of smaller-scale copies of itself. Here, the term "similar" carries the mathematical denotation of objects that have the same shape but differ in size. One should envision an infinite regression of smaller and smaller images that constitute a whole that is similar to its parts. Think of a fern: it is composed of branches that look like little ferns; those branches in turn are made of smaller but structurally identical elements. Because of self-similarity, fractals are also "scale invariant." Scale invariance means that fractals appear (mathematically, if not visually) to be the same at all scales of observation. Why does one have to include a scale in a photograph of a rock? Rocks appear the same at all scales of observation. Looking at a photograph, the observer cannot know what the scale really is unless there is an object of known size in the picture. This phenomenon occurs because rocks are natural fractals.

[6]"*A fractal is by definition a set for which the Hausdorff Besicovitch dimension strictly exceeds the topological dimension*. Every set with a noninteger D is a fractal" (1983:15, emphasis in original).

The third element of our definition says that fractals must have "*fractional dimension*," by which we mean that when it is measured the fractal dimension should be a fraction, not an integer. For a thing to be fractal, therefore, it is not enough for it to be self-similar: the power-law exponent that describes the relation among the copies of different sizes must be a fraction. A power law is a function of the form $f(x) = Cx^b$, where $C$ is a constant and the exponent $b$ is the basic parameter that describes the behavior of the distribution.[7] This exponent $b$ captures essential information about the patterns, and it can help us understand the processes that produced them. In some fractals, this exponent is the fractal dimension, $D$. In other fractals, the exponent is a simple function of the fractal dimension.

The idea of a dimension that is a fraction is contrary to the Euclidean concept of dimension. Euclidean dimensions are integers: 0 for a point, 1 for a line, 2 for a plane, and so forth. Modern mathematicians, however, have developed a number of other ways of measuring dimension that can produce fractions, and are, therefore, strictly speaking, non-Euclidean. These methods include the correlation dimension, the information dimension, capacity dimension, and others, all of which are mathematically related. Here, we will discuss the "fractal" or "self-similarity" dimension. This dimension is described by the following relation:

$$a = \frac{1}{s^D} \tag{1}$$

where $a$ is the number of self-similar "pieces," $s$ is the linear scaling factor of the pieces to the whole, and $D$ is the dimension that we want calculate.

To make tangible the concept represented these abstract mathematical symbols, let us visualize a stalk of broccoli. As the size $s$ of the florets shrinks, their number, $a$, grows. The fractal dimension, $D$, tells us how many new florets we will find as the size of the florets gets smaller (Fig. 1).

Re-arranging the elements of the equation, one can solve for $D$ (Mandelbrot, 1983, p. 37):

$$D = -\left(\frac{\log a}{\log s}\right) \tag{2}$$

For most fractals, $D$ is not an integer.[8] $D$ measures the complexity of the set and expresses the power law that relates the self-similar parts to the whole. For example, in a plane a fractal curve will have a fractal dimension between

[7] People often confuse power-laws with exponential functions. A power-law function is different from an exponential function because in the latter the variable of the function occurs in the exponent, thus: $f(x) = b^x$, whereas in a power law the variable, $x$, is in the base.

[8] In fact, some fractals can have an integer dimension (Mandelbrot, 1983), but they are rare, particularly with empirical datasets.

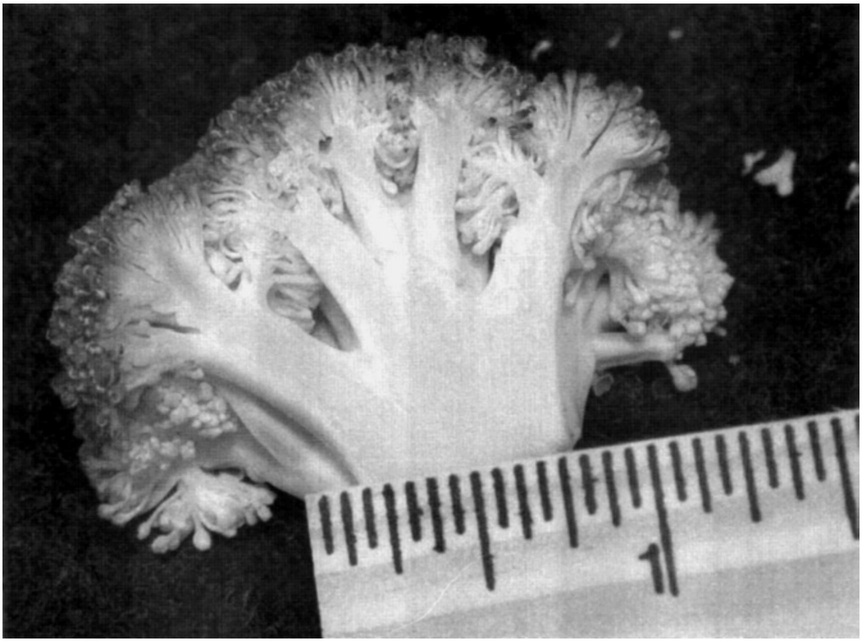


**Fig. 1.** Photograph of a cross-section of a stalk of Broccoli showing fractal branching in florets.

1 (the Euclidean dimension for a line) and 2 (the Euclidean dimension for a plane). The more complex the curve, the closer the dimension will be to 2. The theoretical maximum dimension for a fractal curve is 2, when it becomes so complex it fills the plane. This apparently counter-intuitive, non-Euclidean, conceptualization of dimension is a fundamental characteristic of fractals. It relates to their irregularity and complexity and is the source of their apparent "naturalness."

The same properties that make the fractal dimension the appropriate parameter to describe a fractal object often make the mean and variance unstable measures of the characteristics of the phenomenon. When we sample a population, we normally expect that the sample mean will be an estimate of the population mean, and we expect that that estimate will improve as the size of the sample grows. Similarly, we expect the variance of the sample to quantify the spread or dispersion of the data in the population. With some kinds of fractals, this is not the case: the mean and variance are sometimes not stable—they can increase or decrease with sample size without converging on a finite "true" value (Liebovitch, 1998, pp. 74–105; Liebovitch and Scheurle, 2000; Liebovitch and Todorov, 1996). A statistics textbook would say, "The sample mean and variance are not consistent estimators of the population parameters." Another way to state this is to say that because Eq. (1) is a highly skewed function, with no resemblance to a normal curve, the mean and variance have little value in describing it. That is precisely why one calculates the fractal dimension instead: it *is* the stable, consistent estimator of the population parameter.

Is the fractal dimension just a number for numbers' sake? What does it mean? A distribution of numbers that is Gaussian, which is also called a "normal" distribution or a "bell curve," has many numbers near a certain value, along with some smaller and some larger numbers. The mean is a good way to summarize and characterize those numbers. But a fractal distribution has values that extend over a much larger range. Here the mean is not a good descriptor of the numbers. In fact, as described in the previous paragraph, the mean does not converge to any one value as the sample size grows. The mean depends on the size of the sample or the resolution used to measure it. Thus, the mean, the single most common statistical measure, perhaps the one we are most comfortable with, fails us badly as a descriptor of the data. But all is not lost; we can use a new fractal measure to characterize the data.

This new measure is the fractal dimension $D$ that summarizes and characterizes how the mean depends on the size of the sample or the resolution used to measure it. What does it represent? The larger the dimension, the more a fractal object in space fills up the space around it. For example, a tree with fractal dimension 2.1 in our common three-dimensional space is a very sparse and spare tree, a few thin lines very open to the sun and wind. An object with fractal dimension 2.5 is much denser, with many more branches at each joint. An object with fractal dimension 2.9 is thick with many branches blocking the sun and wind. Just as the mean describes Gaussian data, the fractal dimension is the single most important descriptor of fractal objects, processes, and data.

Thus, the dimension $D$ of a fractal object tells us something important about the character of the phenomenon. So, for example, the fractal dimension of a surface measures how rough it is. The fractal dimension is also a guide to the type of nonlinear process that generated the pattern. As we examine empirical fractal patterns, we will consider the nonlinear processes that generate patterns with comparable fractal dimensions. Most of this paper involves estimating the dimensions of empirical archaeological fractal patterns.

Let us look at a couple of classic examples of fractals to see what these ideas mean in practice.

The Cantor set is historically and mathematically the primordial fractal. It was conceptualized by Cantor (1845–1918), a German mathematician who carried out fundamental work on the foundations of mathematics in the field that today we call "set theory" (Peitgen *et al.*, 1992, p. 67). The Cantor set is constructed by starting with a line segment representing the interval [0, 1]. The square braces indicate that the endpoints are included. Remove the middle third of the line. Two line segments are left: [0, 1/3] and [2/3, 1]. Next, remove the middle third of these two remaining line segments. Then, remove the middle third of the four remaining segments. Continue to iterate the same rule, removing the middle thirds

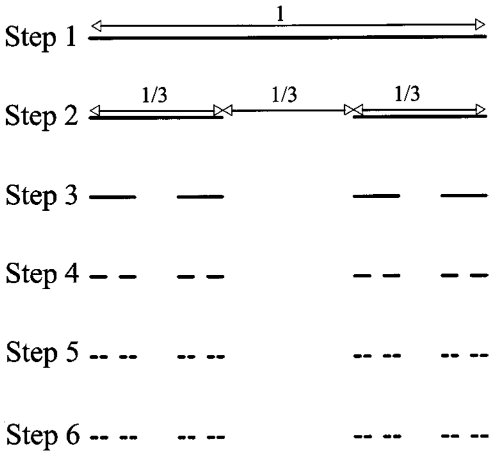


**Fig. 2.** First six iterations of the middle third Cantor set.

of the remaining line segments. Figure 2 illustrates the first six iterations of this process. The Cantor set is the "dust" remaining in the limit as the number of iterations approaches infinity. Thus, like other fractals, the Cantor set is formed by an iterative process involving a reduction in scale (or a "contraction"). The Cantor set is self-similar: smaller parts of the set are identical to the whole except for the difference in scale. The self-similarity also creates scale invariance: if one could really see the Cantor set, it would look the same at all scales, at any magnification.

The fractal dimension of the Cantor set can be calculated easily using Eq. (2), which relates the fractal dimension $D$ to the size $s$ and number $a$ of the pieces. In each iteration, the line segments scale down by 1/3 (the variable $s$ in Eqs. (1) and (2)) and yield two new pieces (the variable $a$ in Eqs. (1) and (2)). So,

$$D = -\frac{\log 2}{\log 1/3} = 0.6309$$

Cantor's original purpose in proposing this strange set was to illustrate how one could have a set with an uncountably infinite number of line segments but with zero length. This weirdness in measuring fractals is typical of the species. They often seem to have counterintuitive properties, at least until one gets used to them. Mandelbrot's famous article "How Long is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension" (1967) offers a wonderful illustration of these ideas.

The Cantor set, as we have seen, is a perfect, mathematical fractal. Real, empirical fractals usually exhibit their fractality statistically, within finite bounds. An example of this kind of empirical fractal is the coast of Britain, first analyzed by Richardson (1961) and made famous in this context by Mandelbrot (1967). Imagine a very large and detailed map of a coastline. Measure the total length of the coast with a pair of dividers set to some large setting, say, 100 km. Walk the dividers down the irregular line formed by the coastline. Much of the detail of the coastline, small bays and inlets and narrow promontories, will be skipped over by the path of the dividers. The total length of the coastline measured in this way will be equal to the number of "steps" taken by the dividers, multiplied by their setting (i.e., 100 km). Then reset the dividers to a smaller setting, say 10 km, and repeat the measuring process. The result will be very different: the coastline will appear much longer, in fact very much longer, because much more detail will have been measured with the smaller divider setting. Repeat the measuring procedure with the dividers set to 1 km, 100 m, 10 m, and so forth, down to 1 mm. The measured length of the coast approaches infinity as the unit of measurement, (i.e., the divider setting) approaches zero. What is the "real" length of the coast? This is indefinable. In an important sense, one cannot speak of the "true" length of a river or a coastline; one can only specify how irregular it is. The fractal dimension quantifies how irregular and complex the curve is, and it is easily calculated from the data collected with the dividers.

The measurements of the coastline taken with the divider describe a simple, power-law relation. The total length of the curve $L(G)$ is related to the length of the unit of measurement $G$ according to the following function:

$$L(G) = MG^b \tag{3}$$

where $M$ is a positive constant of proportionality. In this equation, the exponent $b = 1 - D$, where $D$ is the fractal dimension. The exact mathematical relationship between $b$ and $D$ depends on the nature of the fractal (Liebovitch, 1998, pp. 58–59). It is this parameter one wishes to calculate. To calculate $b$, take the logarithms of the step length (divider setting) and plot them against the logarithms of the total length of the coastline as measured at each setting. The base of the logarithms is unimportant, provided they are the same, because the result will be the ratio of two logarithms (and their ratio will be the same regardless of the base). The function represented in the scatterplot will be linear if the coastline curve is fractal. If the relation in the scatterplot is not linear, the curve that was measured is not fractal. If it is linear, then use least-squares regression to draw a best-fit line through the data points. The slope of the line is an empirical estimate of $b$, while the $Y$-intercept of the regression line is a measure of $M$.

The dimension $D = 1 - b$ tells us how irregular and complex the curve of the coastline is. For a straight coastline it will be equal to 1 and it will increase with the complexity of the coastline until, at $D = 2$, the line actually fills the plane. Mandelbrot (1967) suggested that most coastlines could be modeled by the von Koch curve, a deterministic fractal of known dimension, $D = 1.26$. Much empirical data supports this argument (Turcotte and Huang, 1995, pp. 14–17). The same concepts also apply to all fractal curves, which include most coastlines, lakeshores, topographic contour lines, river plan views, and city outlines.

As one would hope, this method will not yield fractal results for Euclidean figures; they will yield integer, not fractional, dimensions. If one attempts to use the divider method to measure the fractal dimension of a circle, the total perimeter measured quickly converges to a limit representing the real length and the log–log plot approaches a horizontal line. This means that $b = 0$ and so $D = 1$, an integer, not a fraction. So, this method of analysis also gives us the correct dimension for Euclidean figures.

Thus, the mathematics of fractals gives us a more general way to think of dimension, namely, fractional dimensions, which also include our intuitive, Euclidean, integer dimensions as special cases. The fractal method can tell us when a figure is fractal and has a fractional dimension, and when it is not fractal and has an integer dimension.

The divider method is not appropriate for all kinds of fractals. It cannot be applied to overlapping or discontinuous lines, or to phenomena of more than two Euclidean dimensions. A wide array of other methods of measuring fractal dimension exists that are appropriate to other datasets.

There are hundreds of scientific articles and scores of books about fractals, so we will not belabor their description any further. Some excellent books on fractals have been written for non-technical college classes or even high school students (Devaney, 1990; Liebovitch, 1998; Peitgen *et al.*, 1992).

## ARCHAEOLOGICAL FRACTALS

Do any of these complicated ideas have practical application to archaeology as most of us practice it? The answer is unequivocally "yes."

Fractal analysis applies to a wide range of archaeological problems, such as:

- Archaeological fragmentation. This is a fractal process whose results can be described using fractal size–frequency distributions.
- The discovery and description of patterns in the archaeological record. Fractals are patterns and fractal analysis is devoted to the identification and description of those patterns. Much archaeological research is a search for pattern in the archaeological record. For example, archaeological remote sensing, settlement pattern analysis, and various kinds of artifact analysis all focus on describing different kinds of patterns; many of those patterns have fractal characteristics.
- The discovery of scale-free phenomena in archaeology. There has been much commentary devoted to the issue of scale in archaeology (e.g., Ebert, 1992; Stein and Linse, 1993). The discussion of scale in archaeology is

> incomplete without a consideration of scale-free or scale-invariant phenomena, which turn out to be common, and which are, by definition, fractals.

In the text below, we offer two sets of examples, one involving artifacts, the other, archaeological spatial patterns. We have recapitulated some published examples, we have described some new examples published here for the first time, and we have offered some suggestions for future research. We have picked cases that illustrate the ubiquity of fractal patterns in the archaeological record. We hope thereby to persuade the reader that fractal analysis is indispensable, and that it is not some weird, arcane, or recondite method that is merely a curiosity for specialists.

## Artifact Analysis

The following examples include lithic analysis, ceramic analysis, and a little glass, both in the form of a glass jar and in the guise of obsidian.

### *Lithic Analysis*

Stone tools and debitage are possibly the most common types of archaeological artifacts. They are also fruitful sources of fractal patterns. This is not surprising because rocks have many fractal properties (Turcotte, 1997).

Kennedy and Lin (1988) showed that the outlines of stone tools are fractals, like coastlines. They used the divider method to measure the fractal dimension of the tools (Kennedy and Lin, 1986). They found (1988) that the outlines of bifaces could be described as *bifractals*, that is, could best be characterized by the use of two fractal dimensions. One value of $D$ reflects the overall form of the outline (e.g., stemmed, notched, shouldered, triangular), while the other quantified the irregularity of the pattern of flake scars along the edge.

This is an area in which fractal analysis is underutilized. Analyzing entire points, as Kennedy and Lin did, may be of limited use. The fractal analysis of edge form or flake patterns would seem to be more useful, but this has not been done. The fractal dimension would usefully measure the roughness, fineness, and straightness of a lithic tool edge. Similarly, the fractal dimension could usefully characterize the pattern of flaking in a formal tool, although it would not, of course, be a complete description. It would, however, provide a precise and quantitative measure of flaking patterns and retouch to replace current qualitative descriptions.

Beauchamp and Purdy (1986) showed that heat treatment of chert decreased its toughness ($K_{Ic}$). Subsequently, Mecholsky and Mackin (1988), using the same samples of chert, demonstrated that the fracture surfaces of the chert were fractal and could be measured using fractal mathematics. The roughness and fractal dimension of the fracture surfaces correlated with toughness of the chert. The fracture surfaces became smoother with reduced toughness and greater heat treatment. The authors conclude that "fracture in Ocala [Florida] chert can be modelled as a fractal process" (Mecholsky and Mackin, 1988, p. 1147).

Many chert and obsidian debitage size–frequency distributions are fractal (Brown, 1999; Brown, 2001). That is, the number of pieces of debitage of different sizes constitutes a fractal frequency distribution. This is of a piece with the general fractality of fragmentation. Fractal fragmentation has been explained by physical models (Barton, 1995; Borodich, 1997; Coutinho *et al.*, 1993; McDowell *et al.*, 1996; Redner, 1990; Sammis and Biegel, 1989; Sammis and Steacy, 1995; Steacy and Sammis, 1991; Turcotte, 1997, pp. 42–50).

In geology, Turcotte (1986, 1997; Turcotte and Huang, 1995) has demonstrated that rock fragmentation creates a size–frequency distribution of fragments that obeys the fractal (power-law) relation

$$N(>r) = \frac{1}{r^D} \quad (4)$$

where $N(>r)$ is the number of fragments with a characteristic linear dimension greater than $r$, and $D$ is the fractal dimension (Turcotte, 1986, p. 1921, cf. 1997, p. 42). (Equation (4) is a simple re-writing of Eq. (1).) The exponent $D$ characterizes a specific distribution. It is a measure of the relative abundance of objects of different sizes. Because there are fragments of so many different sizes, no single number, such as the mean from classical statistics, can give us a good description of the frequency distribution. The classical statistical methods, the statistics we grew up with, and maybe struggled with, are not capable of properly describing this distribution that extends over so many different scales.

This kind of power-law relation is considered to be fractal because it has no natural scale; it is scale-free or scale invariant. As mentioned earlier, scale invariance is diagnostic of fractal sets. "The wide applicability of scale invariance provides a rational basis for fractal statistics just as the central limit theorem provides a rational basis for Gaussian statistics" (Turcotte, 1997, p. 39).

Although ideal mathematical fractals display this scale-free property over all possible scales, typical real-world fractals are scale-free only over a finite range of scales. How large such a range needs to be so that we can appropriately call an object a fractal is a matter of debate. We are certainly pleased if this range covers two orders of magnitude. But it is not the absolute size of this range that is important; more significant is whether the realization that the data is fractal tells us something useful about the data and, or, the processes that produced it.

It is important to understand how the relationship between size, or magnitude, and frequency can take a power-law or fractal form. Such power-law distributions are an endless source of fractal patterns (Schroeder, 1991, pp. 103–120), and we will see them repeatedly in this article. We illustrate the calculation below using a small dataset from the knapping of obsidian flakes from a core. This will serve

**Table I.** Screen Sizes for Fragmentation Analysis

| Nominal edge length of screen opening, in inches | Actual edge length of square opening in screen, in mm | Computed diagonal length of square screen opening ($r$) |
|---|---|---|
| 4 | 100.0 | 141.421 |
| 2 | 50.0 | 70.711 |
| 1 | 25.0 | 35.355 |
| 0.5 | 12.5 | 17.678 |
| 0.25 | 6.30 | 8.91 |
| 0.11 | 2.80 | 3.96 |
| 0.0555 | 1.40 | 1.98 |

as an example of how to determine whether any frequency distribution is fractal. It should also persuade the reader of how simple fractal analysis can be.

After collecting the debitage from knapping a few obsidian flakes on a tarp, we passed the debitage through a series of brass U.S. standard graduated geology sieves (certified W. S. Tyler brand ASTM E-11 specification, ISO 656 3310-1, BS 410). We measured the apertures of the mesh of the sieves to verify the sizes of the apertures. Since the fractal analysis depends on logarithms, it is best when the screen sizes decrease in a logarithmic (that is a geometric) rather a linear way. Therefore, we chose sieve sizes such that at each successive screen the aperture-width would be approximately halved. In Table I we list the screen sizes.

We show the calculation in Table II. First, one counts the fragments retained in each screen. Then, one has to calculate $N\,(>r)$, which is the cumulative frequency of fragments with a linear dimension greater than $r$. One sums the raw frequencies in the second column to form the cumulative frequency shown in the third column. Then, one calculates the logarithms of screen aperture ($r$) and of the cumulative frequency ($N > r$). One should use the real aperture as $r$ because the fundamental form of the fractal relation considers the proportion of fragments *larger than* a given linear size to the total number of fragments. The size of the screen aperture for sieve data should represent the smallest size debitage in the group.

The next step is to plot the logarithm of the flake size $r$ against the logarithm of the cumulative frequency $N\,(>r)$. The plot of these data is shown in Fig. 3. For

**Table II.** Calculation of Fractal Dimension for Knapping Obsidian Flakes

| Screen aperture in mm ($r$) | Frequency | Cumulative Frequency ($N > r$) | ln ($r$) | ln ($N > r$) |
|---|---|---|---|---|
| 1.4 | 257 | 354 | 0.3365 | 5.8693 |
| 2.8 | 65 | 97 | 1.0296 | 4.5747 |
| 6.3 | 16 | 32 | 1.8405 | 3.4657 |
| 12.5 | 10 | 16 | 2.5257 | 2.7726 |
| 25 | 6 | 6 | 3.2189 | 1.7918 |

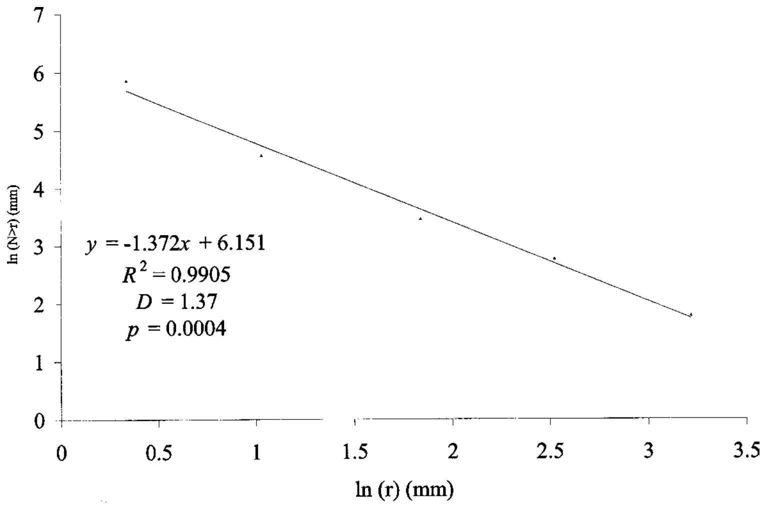


**Fig. 3.** Fractal size-frequency plot for knapping obsidian flakes.

the relation between the two variables to be fractal, this plot must be linear. If it is, the slope of the least squares regression line provides an estimate of $-D$. Thus, we have $D = 1.37$, which is merely the negative of the slope. The coefficient of determination, $R^2$, which measures the proportion of the variation in the data explained by the regression, is a nearly perfect .99, or 99%. This demonstrates that the relation is highly linear, because it is almost perfectly modeled by the linear regression. The significance of the regression taken from the analysis of variance table is $p = .0004$, which tells us that the probability of this pattern occurring by chance alone is almost nil.

Many, although not all, datasets from experimental reduction of stone tools exhibit power-law frequency distributions. Their fractal dimensions range from 1.2 to 3.3 (Brown, 2001, Table II). Archaeological assemblages of debitage also exhibit fractal size–frequency distributions (Brown, 1999; Brown, 2001). For such fractal datasets, the fractal method is, by far, the simplest and the mathematically most correct way of evaluating the frequencies of various sizes of debitage.

It is important to know to whether an assemblage of debitage has a fractal size–frequency distribution: knowledge of the shape of the frequency distribution is a key to sampling and to inferring the population parameters from one's sample. If one knows that an assemblage has a fractal frequency distribution, one can sample much more efficiently and judiciously.

The fractal dimension captures important information about the process of tool production. The fractal dimension of an assemblage of debitage is a measure of stage of reduction (Brown, 2001). This makes sense because the fractal dimension quantifies the proportion of small to large flakes, and the proportion of small flakes normally increases with stage of reduction, so it is logical that the fractal dimension should correlate with stage of reduction. Thus, fractals offer a powerful new approach that carries great inferential potential.

Equally interesting are those archaeological collections of debitage that do not exhibit fractal frequency distributions (Brown, 1999, 2001). These seem to cases of re-deposition of debitage through a disposal process that left lag deposits of small flakes near their original locations as primary refuse while a disproportionate number of larger flakes were removed and transported to become secondary refuse. It is well-documented that cleaning and re-disposal of knapping debris tends to leave small flakes behind in primary contexts as a lag deposit (e.g., Behm, 1983; Clark, 1991a, 1991b; Healan, 1995). Thus, the fractal approach can also provide a clue to the character of the assemblage.

So, the fractality of size distributions of debitage relates to taphonomy and their fractal dimensions can relate to stage of reduction. Are there other possible implications for fractal analysis? Turcotte (1986, p. 1925) has suggested that the fractal dimension of fragmented material may relate to the strength of the raw material, with more fragile materials yielding a lower fractal dimension. The effect of material strength on fractal dimension is not clear as far as we can tell from published data, but then there is almost no relevant data on the strength, toughness, or fragility of archaeological lithic materials. This lack of data could be remedied, and it certainly merits additional study.

In addition, all other things being equal, different reduction sequences and reduction techniques likely produce different fractal dimensions in the resulting debitage assemblages. Any fragmentation process that creates significantly different proportions of small or large pieces of debitage will generate a different size–frequency distribution. So, for example, pressure flaking, which characteristically produces relatively small flakes, will tend to generate an assemblage of debitage with a larger fractal dimension than, say, percussion flaking, because the former would produce proportionately more small fragments than large ones. Similarly, one can predict that different reduction trajectories should create assemblages with varying fractal dimensions to the degree that they generate debitage with different size–frequency characteristics.

Thus, the fractal size–frequency relations of debitage should be further investigated through the systematic characterization of different raw materials, reduction techniques, and reduction sequences. We suggest that the best approach would be experimentation followed by confirmation with real archaeological data.

It would also be interesting to see if other archaeological rock-size distributions, such as fire-cracked rock from stone middens in Texas, are fractal. As we shall see below, fragmentation of many materials is fractal, and this method can therefore be extended to other classes of artifacts.

In sum, the edges of chipped stone tools are fractals, their manufacture is a fractal process, their debris exhibits a fractal frequency distribution, and analyzing the process and the resulting patterns using fractal methods advances our understanding of both.

### *Ceramic Analysis*

These facts provide a starting point for the study of ceramic fragmentation. The fractal fracture and fragmentation of rock is directly applicable to the understanding of ceramic fracture and fragmentation because stone and ceramic share a number of important physical characteristics. They are both brittle and inhomogeneous materials, and they are sometimes made of the same clay minerals. As the material of archaeological stone tools grades from chert into obsidian, so do ceramic materials grade from coarse wares into porcelain and glass.

To examine the possibility that ceramic fragmentation might be a fractal process, we fragmented six ceramic and glass vessels (Witschey and Brown, 2003). The specimens were chosen to provide a range of paste textures and probable firing temperatures. We also chose specimens that were similar to some archaeological specimens, so that the experiment would provide a basis for archaeological inference.

We processed the fragments through the same screens described in Table I. The size distributions of the fragments indeed present fractal relations. The fractal dimensions range from 0.86 to 1.47. The coefficients of determination ($R^2$) range from 89 to 99%, which, again, indicate a linear, and therefore, fractal relation. The probabilities associated with these relations range from $p = 0.0044$ to 0.0002, indicating that these patterns are statistically significant. We can conclude that ceramic fragmentation appears to be a process with a fractal outcome. It is important to note that other types of functions, such as stretched exponential or log-normal distributions, can be similar to power-law distributions, especially over limited ranges in scale. When there is some scatter in the data, these other forms may come equally close to fitting the data, and so they cannot necessarily be excluded as possible models for this phenomenon. It is also true that multiplicative processes, other than true fractal growth, can produce power-law-like distributions (Laherrère and Sornette, 1998). Thus, the power-law distribution found here is indicative of a fractal process, but it does not necessarily prove that it was only a fractal process.

The magnitude of $D$ may respond to paste texture, but seems to be influenced most strongly by the method and extent of breakage, which are themselves related to disposal and taphonomic processes.

Fractal fragmentation of ceramics carries several important implications for any archaeologist who works with ceramics. Different ceramic types, particularly if they have different pastes or are subject to differing fragmentation processes, may have different fractal fragmentation dimensions. We know from ethnoarchaeological research that different types may fragment in different ways (e.g., David, 1972; Orton, 2000, pp. 51–53), resulting in different patterns of sherd sizes. Different fractal dimensions imply that the proportions of small to large fragments are different; the differences in those proportions affect the size sampling that takes place when we screen archaeological sediments through our hardware cloth. So, different type frequencies can be the result of size sampling of types with different fractal fragmentation dimensions. Knowing the fractal dimensions of the different types, derived from the samples collected, will allow one to infer the real, original proportions of the types.

Just as with lithic debitage, one can use this approach to identify cases in which natural or cultural processes have resulted in biased size-sorting of archaeological sherd assemblages. If the size–frequency relation has become distorted because of natural geomorphic processes, such as those that result in the size-sorting of sediments, or cultural processes, such as disposal (perhaps by sweeping), fractal analysis would probably highlight such biases.

Obviously, fragmentation is both an important fractal process and a major aspect of the archaeological record. We do not, however, wish to leave the impression that fragmentation is the only arena in which fractal analysis can contribute to ceramic analysis. For example, in a highly original analysis, Bentley and Maschner (2001, 2003) have argued that the fractal statistics of ceramic type frequencies demonstrate the existence of self-organized critical dynamics in the evolution of style. The strength of this argument derives from the fact that self-organized critical systems generate fractal statistics in time and space (Bak, 1996; Bak *et al.*, 1987, 1988).

Another important application of fractal analysis is the characterization and description of patterns in art and design. Fractals are, after all, patterns, and fractal analysis consists of methods of statistically estimating the parameters of the patterns—that is, mathematically describing the patterns. This approach has already been applied to a variety of subjects. For example, Richard Taylor has shown that Jackson Pollock's drip paintings are fractal patterns and that their fractal dimension evolved over time, reflecting the evolution of his style (Taylor *et al.*, 1999, 2002). Taylor has also studied the underlying dynamics of Pollock's painting style: the chaotic dynamics created the fractal patterns. Others have examined the fractality of the Nazca lines, and they have found that the fractal dimension of the designs varies, although their assumption that the complexity of the designs must have increased with time cannot be defended (Castrejón-Pita *et al.*, 2003). Eglash (1999) has identified numerous examples of fractal patterns in African art.

Evidence is accumulating that human visual perception is particularly attuned to the detection and appreciation of fractal patterns. Much of nature is fractal: landscapes, plants, clouds, and so forth are all fractal patterns. It would be surprising indeed if our visual faculties were not attuned to their perception because they are, in a very real sense, the patterns and geometry of nature. Cognitive and experimental psychologists are exploring these issues (e.g., Aks and Sprott, 1996; Mitina and Abraham, n.d., Spehar *et al.*, 2003).

The rigorous description of form and pattern is a fundamental task of archaeology. It is an elementary part of the analysis of artifact style. The documentation of the stylistic and morphological similarities and differences among artifact assemblages is a basic part of what archaeologists do when they describe and classify material culture. For archaeologists, it is always important to identify, describe, and quantify variation in material culture. Of course, artifact style, its psychological basis, and its cultural meaning are major topics in archaeology (e.g., Hodder, 1979, 1982; Miller, 1985; Sackett, 1990; Wiessner, 1983, 1984; Wobst, 1977).

Notwithstanding all the discussion of style in archaeology, Washburn (e.g., 1977, 1990, 1994; Washburn and Crowe, 1988; Washburn and Matson, 1985) is one of only a small coterie of investigators who have made a detailed and successful effort to develop a system to describe archaeological artifact designs. Her approach is called "symmetry analysis." Its historical origins lie in the geometry of crystallography. The method consists of identifying and recording "symmetrical" patterns of translation, reflection, and rotation in artifact designs. She has developed a systematic procedure to identify, and a nomenclature to describe the patterns. Most of her work has addressed patterns on ceramics, baskets, and fabrics. Washburn is to be credited with a thoughtful and effective approach to this fundamental problem. Her methods have been adopted by other archaeologists as well as investigators in other fields.

Washburn's approach is applicable to a wide range of designs, which she refers to as "symmetrical" (1977, pp. 17–22). These designs are not fractal because they do not include those for which the transformations include a reduction in scale. "All symmetry operations involve congruence. During each transformation the fundamental part is shifted a constant distance so that only its position relative to the point axis and/or line axis is altered. The *size of the fundamental part* and its relation to the axis *remain constant*" (1977, p. 12; emphasis added). All fractals, however, include a reduction in scale, called a "contraction mapping" (Peitgen *et al.*, 1992, pp. 228–296). Fractals typically also include translations and rotations, but they must include a scaling, which creates a contraction, or otherwise they are not fractals. The contraction mapping is what creates the self-similarity and scale invariance in fractal patterns.

So, symmetrical patterns (*sensu* Washburn) are not fractal and neither are fractal patterns symmetrical (in the same sense). Indeed, fractal analysis and symmetry analysis are complementary. If an artifact displays an ideal and perfect fractal design, one can decipher the rules of construction of the pattern (initiator and generator) and even figure out the entire mapping formula (i.e., translation, rotation, and scaling factors). This is approximately the equivalent to Washburn's

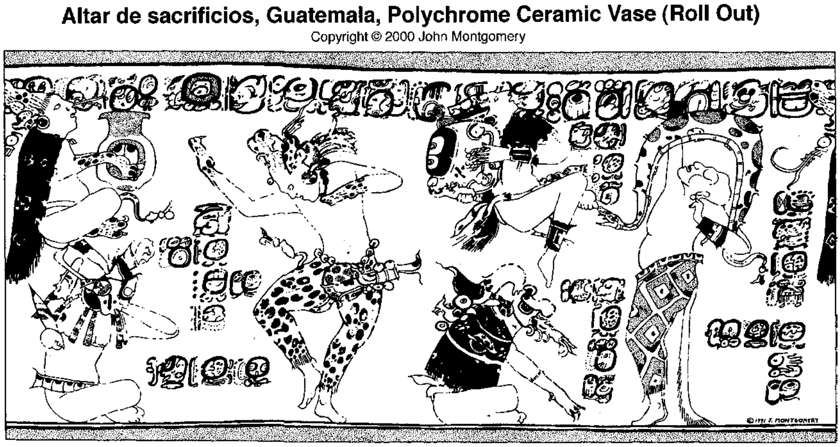


**Fig. 4.** Roll out drawing of the "Altar Vase" by John Montgomery (courtesy of John Montgomery).

method applied to fractal patterns. On the other hand, if the fractal is a "statistical" one, rather than an ideal and perfectly geometrical one, then one can measure its fractal dimension, which indicates the scaling in the design.

As an example, we will describe the fractal analysis of the "Altar Vase." This vessel, Number 58–104 from Burial 96, was excavated at Altar de Sacrificios, Guatemala, by the Harvard Peabody Museum expedition led by Gordon Willey. The pot is a cylindrical Maya funerary vase painted with a narrative, figural scene of apparently historical content (Fig. 4), because the hieroglyphic captions include the emblem glyphs of Yaxchilan and Tikal, as well as possibly the name of a personage known from the former (Adams, 1971, 1977). The painted scene includes six human figures, two dancing and four sitting. There are six hieroglyphic captions in the scene in addition to a primary band of larger glyphs around the rim of the vessel.

Obviously, in such a case, the self-similarity of the design must be statistical rather than ideal. The possibility of statistical self-similarity is suggested by clusters of detail at many different scales. The artist did not clutter the painting but used his small canvas judiciously, balancing empty space and painted content. The figures provide much of the detail in the drawing: their clothing and regalia contains much fine detail, as do the various objects and animals they hold. This approach to composition creates a statistical clustering of painted detail at varying scales.

Because of the structure of pattern, one cannot analyze it using any of the methods described so far. The appropriate method is called the "box-counting method." The box-counting method is probably the most commonly used method to analyze images in fractal analysis, and we would be remiss if we did not explain it in this article.

The idea is this: one overlays a grid of squares on the design to be measured, and one counts the number of boxes containing part of the design. The number of squares, $N$, required to cover the design will depend on the size of the squares, $s$, so $N$ is a function of $s$, or we can write $N(s)$. Now one reduces the size of the grid repeatedly, recording the two variables, $N$ and $s$. One plots the log of $N(s)$ versus the log of $s$. If the relation between the two is linear, then the pattern is fractal. If we call the slope of the best-fit line $b$, then the fractal dimension is $D = -b$. The procedure is illustrated in Fig. 5.

This procedure is difficult to perform with precision by hand, but it is easily automated. One program worth noting is FD3, written by DiFalco and Sarraille (Sarraille and Myers, 1994), which is based on an algorithm devised by Liebovitch and Toth (1989). This program takes as input an ASCII file containing a series of coordinates for points, with one set of coordinates for one point on each line. Then the program calculates the difference between the maximum and minimum values in the dataset and uses this figure to determine the box sizes. It begins with a single box, the sides of which are the length of the difference between the maximum

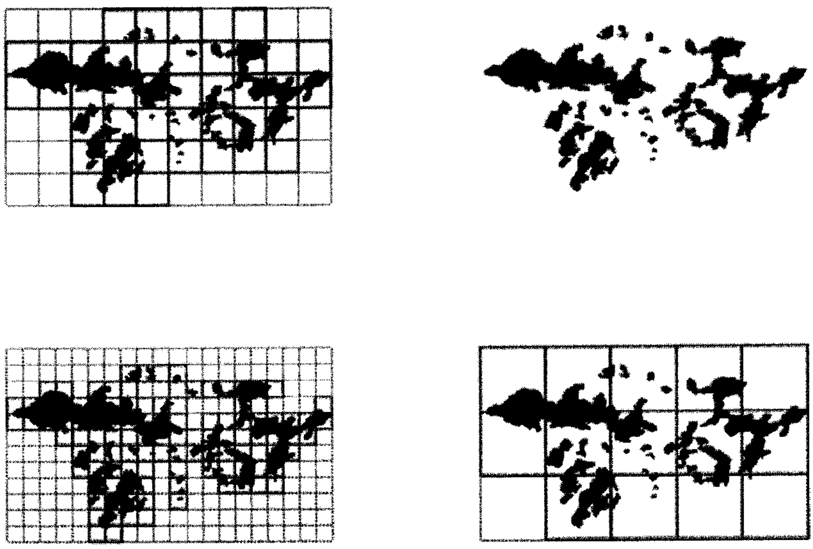

**Fig. 5.** The box counting method can be used to determine the fractal dimension $D$. Different grids with boxes of size $s$ are here laid over a map of the lakes around Jyväskylä, Finland. For each grid the number of boxes $N$ that contain any part of any lake is counted. Then, the slope, $b$, of log of $N(s)$ versus the log of $s$ is determined. The fractal dimension, $D$, of this set of lakes is given by $D = -b$. For these lakes, $D = 1.45$. This figure is reproduced, with permission, from the CD-ROM, *The Mathematics and Science of Fractals* (copyright 2004 L. S. Liebovitch). North is to the right.

and minimum values. This box is then divided into four cells by bisecting each side of the original box. The next division is into 16 cells by subdividing each side into four segments. The next division is into 64 boxes by a linear division of each side into eight segments, and so on. To accelerate computation, the program shifts and rescales the dataset. Logarithms of base 2 are used in the calculation of dimension. Versions of the program for different operating systems are available free on Professor Sarraille's web site.

In the case of the Altar Vase, we used version 1.3 of a commercial computer program entitled "Benoit," published by TruSoft International, Inc. (reviewed in *Science* by Steffens [1999]). This program implements the standard box counting algorithm.[9] It operates on an input file in the form of a black and white bitmap file for which it calculates the fractal dimension of the white pixels. We used Benoit to take advantage of Montgomery's beautiful rollout drawing[10] of the Altar vase posted on the web site of the Foundation for the Advancement of Mesoamerican Studies (http://www.famsi.org), which we converted to a bitmap. We cropped the drawing to remove the borders, leaving the band of glyphs around the rim. We also trimmed the drawing laterally a little so that there was no horizontal overlap at the ends of the rollout. We made a negative because Benoit measures the fractal dimension of the white pixels. We used Adobe PhotoDeluxe Business Edition for all the image manipulation. Using the box-counting command in Benoit, we calculated that the fractal dimension of the Altar Vase drawing is $D = 1.67$ (SD = 0.016).

So, we can assert that at least some Classic Maya art is fractal in structure. We predict that further research will reveal systematic spatial and temporal variability in the fractal dimension of Maya art. One obvious suggestion is that the fractal dimension increases through time, from the relative simplicity of Late Formative reliefs to the extreme complexity of Late Postclassic designs related to the Mixteca-Puebla horizon.

Fractals with the same fractal dimension do not necessarily look similar. Therefore, Mandelbrot (1983; Gefen *et al.*, 1984) proposed a measure called "lacunarity" to specify the "clumpiness" of fractals and thereby to describe them statistically in greater detail. A number of approaches to measuring lacunarity have been developed (Allain and Cloitre, 1991; Lin and Yang, 1986; Turcotte, 1997, pp. 109–112). A detailed discussion of lacunarity is beyond the scope of this paper, so we will only note that in combination with the fractal dimension,

[9] Benoit adds a refinement to the preceding description of the box-counting method: it rotates the grid through 90° in specified increments to minimize the number of occupied boxes at each box size. Minimizing the number of boxes required to cover the design improves the accuracy of the estimate of the fractal dimension.

[10] We experimented with digitized rollout photographs of cylindrical Maya funerary vases, but we found that the combination of archaeological weathering, the burnish or polish of the vessels, and the color variation of the paints tended to create spurious patterns of white pixels, i.e., ones unrelated to artistic composition of the scene on the vessel. Consequently, we chose to analyze a drawing.

lacunarity can be used to describe further the fractal patterns on ceramics and other types of artifacts.

In summary, we have shown that most common archaeological artifacts have a range of fractal characteristics. We have demonstrated that fragmentation is a fractal process that applies to such common archaeological materials as ceramics, lithics, and glass. In addition, a number of investigators have noted fractal patterns in art and design. Fractal analysis provides appropriate methods for analyzing and describing these patterns, which exhibit statistically similar levels of detail at many different scales. It can be applied to painted, incised, or impressed patterns. Fractal analysis complements symmetry analysis of artifact patterns.

## Spatial Patterns

In this section we will discuss archaeological spatial patterns including human settlement patterns and horizontal distributions of artifacts. We will divide the settlement patterns into two sub-categories, those associated with sedentary settlement systems and those derived from mobile foragers and collectors. We realize, of course, that sedentism versus mobility is a false dichotomy, that the reality is that different cultures fall along a spectrum of mobility.

### *Settlement Patterns*

A wide range of human settlement patterns are known to be fractal; others look suspiciously self-similar but have yet to be analyzed. We discuss both kinds below.

*Geomorphology and Settlement.* Geomorphology is integral to understanding archaeological settlement, taphonomy, and stratigraphy. Settlement is commonly correlated with landforms, river networks, water sources, coasts, and soil types. Archaeological taphonomy and stratigraphy are largely determined by soils, geomorphology, and geomorphic processes. Fractal geometry has brought fundamental changes in the understanding of geomorphology, particularly in the study of topography, river networks, and coastlines (Baas, 2002; Dodds and Rothman, 2000). Several of Mandelbrot's early and seminal articles on fractals were about geomorphology (e.g., 1967, 1975). The literature on fractal analysis in geomorphology is now large. We cannot include here even a minimal review, but can only touch upon a few major topics. Topography is a self-affine fractal; the use of fractal analysis to describe topography is related to fractals as a measure of surface roughness (Burrough, 1981; Clarke, 1986; Klinkenberg, 1992; Mandelbrot, 1975; Turcotte, 1997, pp. 162–178). River plan views, river networks, and drainage systems are all fractal (Rodríguez-Iturbe and Rinaldo, 1997; Snow, 1989; Stølum, 1996; Turcotte, 1997, pp. 183–207).

Hydrology, erosion, and stratigraphy are all dynamically linked by nonlinear systems theory. The formation of fractal erosional topography and the development of fractal sedimentary stratigraphy are explained by a model of self-organized criticality, which is strongly supported by simulation (Bak, 1996; Bak *et al.*, 1987, 1988), observation (Gomez *et al.*, 2002; Turcotte, 1997, p. 321), and experimentation (Roering *et al.*, 2001). Self-organized critical systems generate fractal statistics in time and space.

These ideas are directly relevant to archaeology because some archaeological settlement patterns depend closely on topography, river network structure, or coastal morphology. For example, it is widely believed in the archaeology of eastern North America that archaeological sites occur preferentially at stream junctions and that larger sites occur at the junctions of higher order streams. For that reason alone some human settlement patterns are related to fractal geometry.

*Fractal Patterns in Urban Settlement.* In addition to and independent from the fundamentally fractal nature of topography and geographic space, many aspects of human settlement are known to be fractal. The body of literature in modern geography on the fractal characteristics of human settlement is significant and growing (Batty, 1991; Batty *et al.*, 1993, 1989; Batty and Kim, 1992; Batty and Longley, 1986, 1994; Batty and Xie, 1996; Bovill, 1996, pp. 144–149; Brown and Witschey, 2003; Carvalho and Pen, 2003; Deadman *et al.*, 1993; Eglash, 1999, pp. 20–38; Eglash *et al.*, 1994; Longley *et al.*, 1991; Makse *et al.*, 1995; White and Engelen, 1993). Several different kinds of modern settlements have been shown to be fractal in form. A number of investigators have studied the boundaries of modern cities and concluded that they are fractal curves that can be modeled by a process called diffusion limited aggregation (e.g., Batty, 1991; Batty and Longley, 1994; Batty *et al.*, 1989). Others have discovered fractal patterns in the complex, maze-like streets of Tokyo (Rodin and Rodina, 2000). Central Place lattices are ideal fractals (Arlinghaus, 1985, 1993; Batty and Longley, 1994, pp. 48–56).

A number of studies of the fractal characteristics of archaeological settlement patterns have appeared. One excellent example is given by Maschner and Bentley (2003). They demonstrate the existence of scale invariant frequency distributions in archaeological house floor areas from southern Alaska (Maschner and Bentley, 2003). Ancient Maya intrasite residential settlement patterns are also fractal—that is, the complex, nested spatial pattern of buildings within the site forms a fractal pattern (Brown and Witschey, 2003) (Fig. 6). More generally, the size−frequency relation for sites in many settlement patterns is a fractal (power-law) relation (Brown, 1999; Brown and Witschey, 2003; De Cola and Lam, 1993, pp. 17–19). In addition, the segmentary internal structure of some traditional settlements is also fractal (Bovill, 1996, pp. 144–149; Eglash, 1999, pp. 20–38; Eglash *et al.*, 1994).

Not all settlement patterns are fractal. For example, the orthogonal grid pattern of an archetypal Roman city tends to be Euclidean rather than fractal, although

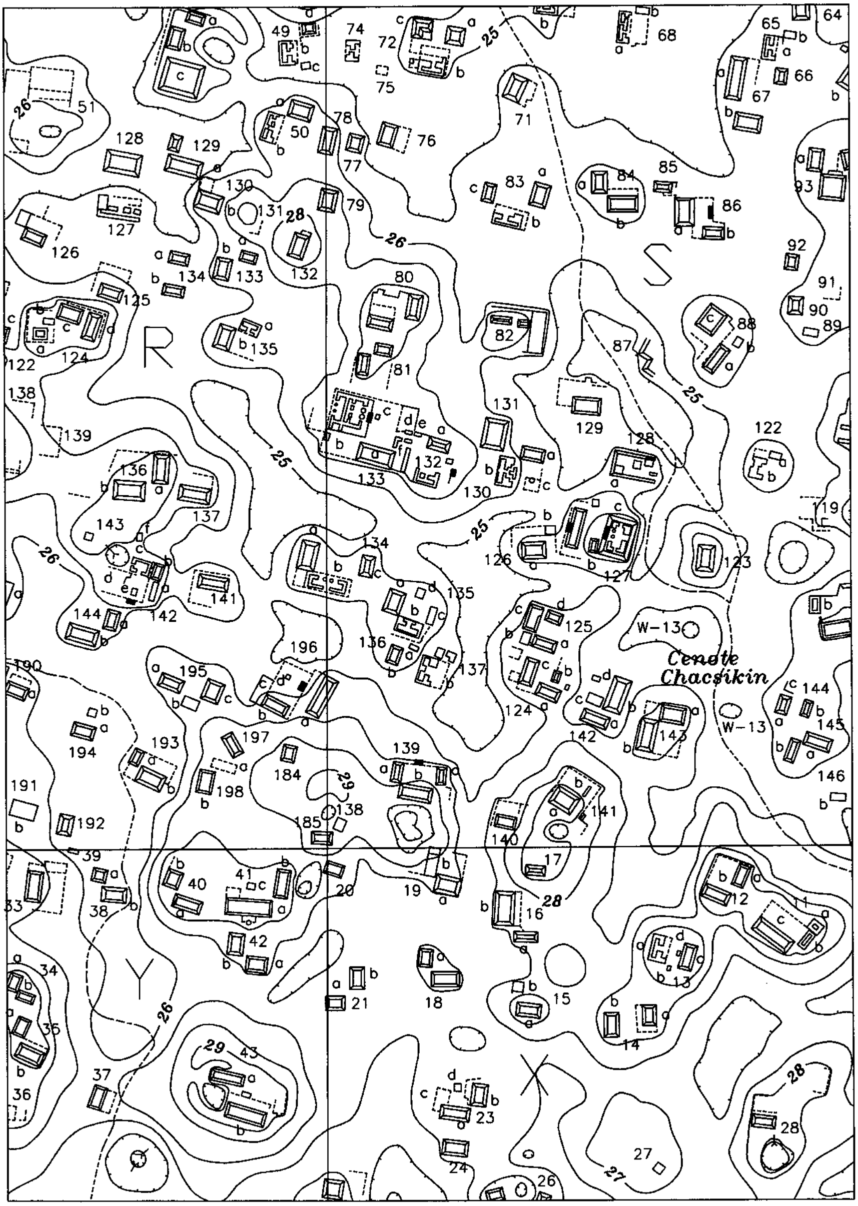


**Fig. 6.** Detail of the Central Part of the Carnegie Institution Map of Mayapán, Yucatán, México. Re-drafted by Lynn A. Berg based on data in Jones (1952).

its fractality depends on the details of the grid squares. Thus, although the grid is self-similar, it is not fractal because the dimension is an integer not a fraction. If, however, one were to leave out certain grid squares from the pattern, it could become a fractal similar to a "Sierpinski Carpet." So, for example, the internal grid layout of Teotihuacan, Mexico, might not be fractal (but see Oleschko *et al.*, 2000 for the fractality of the Ciudadela), while its irregular outline might well be fractal. In sum, the fractality of settlement patterns cannot be assumed. It must be demonstrated by argument and measurement.

For the sake of brevity, we will touch only upon a couple of examples of the fractality of human settlement. The first example focuses on the use of the rank-size rule to describe settlement hierarchies. This is a large topic with a robust literature, and we cannot do it justice here. We will only provide the briefest possible explanation. The rank-size rule is an empirical observation that expresses the relationship between settlement size (population) and settlement rank (its numerical position in the series created by ordering all the settlements in the system from large to small). The idea that settlement size and rank have a systematic relationship was popularized by Zipf (1949), who expressed it as:

$$P_r = \frac{P_1}{r^k}, \quad r = 1, 2, \ldots \tag{5}$$

where $P_r$ is the size of the settlement of $r$-th rank in the system and $k$ is a constant, which is typically of the magnitude of 1, in the ideal case described by Zipf. The exponent $k$ is calculated empirically by plotting the logarithm of rank against the logarithm of size: $k$ is the slope of the best-fit line.

The rank-size rule has been applied to archaeological data in various contexts (e.g., Adams, 1981; Cavanagh and Laxton, 1994; Hodder, 1979; Hodder and Orton, 1976, pp. 69–73; Johnson, 1980; Laxton and Cavanagh, 1995; Paynter, 1983). It is noteworthy that this is an empirical rule and does not depend on any sociological theory, like Zipf's "Law of the Least Effort." Rank-size has been used to analyze ancient settlement data from the Maya area and other parts of Mesoamerica (Adams, 1981; Hammond, 1974; Inomata and Aoyama, 1996; Kowalewski, 1990; Kowalewski *et al.*, 1983). There has also been study of the deviation of settlement systems from the expectations of the rank-size rule (Hodder, 1979; Johnson, 1980; Zipf, 1949, pp. 374–375, 416–444), such as, for example, primate settlement systems that may be related to colonialism. Consequently, use of the rule is not merely a mechanistic exercise, but an informative model.

It has recently been shown that the rank-size rule is a fractal relation (Cavanagh and Laxton, 1994; Laxton and Cavanagh, 1995). Therefore, settlement hierarchies that obey the rank-size rule are fractal, too. The fractal formulation of the rank-size rule provides an important theoretical advantage over the original. The inherent self-similarity of the fractal relation means that a regional sample can be extrapolated to a whole settlement system, which is particularly appropriate in archaeological cases because most archaeological surveys do not comprehend entire regions or settlement systems (Laxton and Cavanagh, 1995, p. 327; Cavanagh and Laxton, 1994). The fractal dimension is related to the rank-size rule by:

$$D = -\frac{1}{k} \tag{6}$$

(Cavanagh and Laxton, 1994, p. 62). Thus, for $k = 1$, Zipf's "classic" case, $D = 1$ also. Cavanagh and Laxton found that the fractal dimension of settlement systems in Laconia, the territory of ancient Sparta, varied through time (from about 0.7 to 1.0) and reflected important changes in the distribution of population among settlements of different size. This is a technique that can and should be applied to many other existing datasets.

*Foraging*. Another connection between archaeological settlement patterns and fractal theory involves Lévy flights and optimal foraging theory. Lévy flights, poetically named after the French mathematician Lévy, are a kind of random walk. Brownian motion, the best-known kind of random walk, is the special case of a walk in which the sizes of the steps are normally distributed. Lévy, in contrast, studied patterns in which the step lengths follow a Lévy probability density function, which is a family of distributions characterized by power-law tails. Lévy flights have many applications in physics and even in economics.

Lévy distributions have a number of interesting characteristics (ben-Avraham and Havlin, 2000; Shlesinger *et al.*, 1993). Their variance or second moment is infinite, in contrast to the finite variance of a normal distribution and Brownian motion. Lévy flights lead to what is called anomalous diffusion or superdiffusion. Not surprisingly, because of the power-law distribution of step lengths, Lévy flights produce fractal patterns in space (Mandelbrot, 1983).

Researchers have recently discovered that some insects (ants, bumble bees) and animals (albatrosses, deer), including non-human primates (spider monkeys), forage in patterns that are Lévy flights (Boyer *et al.*, 2003; Ramos-Fernández *et al.*, 2003; Viswanathan *et al.*, 1996; Viswanathan *et al.*, 1999). These foragers may be moving around in Lévy spatial patterns because the resources they are seeking, such as fruit trees, have fractal distributions: movement along a fractal lattice creates Lévy flights. A variety of evidence suggests that plants have complex, aggregated spatial distributions, and that many vegetation patches are fractal (Condit *et al.*, 2000; Hastings and Sugihara, 1993; Solé and Manrubia, 1995).

The Lévy flight patterns of foragers have assumed importance because it has been shown recently that Lévy flights with a negative squared exponent are the optimal search pattern for random foraging searches in certain environments (da Luz *et al.*, 2001; Viswanathan *et al.*, 2000, 2001, 1999, 2002). These findings have influenced optimal foraging theory in biology.

Optimal foraging theory is perennially popular among some archaeologists in modeling forager behavior (Bettinger, 1987; Kelly, 2000; Lake, 2000; Smith, 1983;

Smith and Winterhalder [eds.], 1992; Winterhalder and Smith, 2000). A recent volume of *World Archaeology* was devoted to this topic (Shennan, 2002). We must ask ourselves, therefore, whether human foragers might use Lévy flights patterns when foraging. We can show, using ethnological data from modern foraging groups, that at least some hunter-gatherers use Lévy patterns in their movements across the landscape (Brown and Liebovitch, 2004). Such foraging movements result in a fractal pattern of archaeological sites. It will be exciting to work out the implications of these findings for the study of Paleolithic and Neolithic foragers.

*Diffusion*. As mentioned above, Lévy flights in physics and chemistry create anomalous diffusion or superdiffusion. In archaeology, cultural diffusion has typically been modeled as "normal" diffusion by assuming a Gaussian distribution of migration distances (Ammerman and Cavalli-Sforza, 1979, p. 280), although Hodder and Orton (1976, pp. 126–154) considered negative exponential distributions as well as Brownian motion. Brownian motion is the special case of a random walk in which the step lengths have a Gaussian distribution. Archaeologists need to examine whether the conventional and statistically convenient assumption of normality is true and to test whether a model based on a Lévy distribution is not a better fit to the data. In fact, one group of researchers (Rodríguez Alcalde *et al.*, 1995) has already proposed that fractal percolation theory may offer a useful alternative to Ammerman and Cavalli-Sforza's model of demic diffusion for the spread of agriculture to the western Mediterranean region.

*Remote sensing*. Today, many archaeological projects use remote sensing data, from simple aerial photographs to the newest space-based sensors, to find ancient roads, walls, fortifications, cities, and other archaeological remains. Remote sensing data can be analyzed in various ways using fractal techniques (Turcotte, 1997). Perhaps most relevant to our discussion, fractal algorithms are being used in military targeting to locate Euclidean man-made targets against the fractal natural background (Cooper *et al.*, 1994; Espinal *et al.*, 1998; Neil and Curtis, 1997; Priebe *et al.*, 1993). These algorithms use the fractal dimension to differentiate low-dimensional Euclidean human objects and constructions from the higher-dimensional fractal natural background. This military problem appears to be conceptually equivalent to the archaeological problem of discovering the traces of ancient features in remote sensing datasets. This, therefore, seems to be a technique that could be applied fruitfully in archaeology.

*Spatial distributions of artifacts*. Ethnoarchaeologists have mapped the spatial distribution of discarded remains at many households, camps, and activity areas. For example, Yellen (1977) mapped a number of Dobe! Kung camps in the Kalahari desert of Botswana and Namibia. He describes (1977, pp. 85–131) complex clustering of activity debris within camps. Binford (1978) also describes complex internal structure and clustering in the deposition of discarded items in a Nunamiut hunting camp. These observations suggest the possibility of fractal spatial structure in the remains. As a test, we used the FD3 program to study the spatial structure of the Mask site data.

The Mask site was a Nunamiut campsite in Alaska mapped by Lewis Binford (1978). Many researchers have used the Mask site data as a model dataset for studying intrasite spatial statistics in archaeology. Keith Kintigh (1990) has used this dataset as the principal example in his comparative study of spatial statistics in archaeology. Blankholm (1991) also used the Mask site data as the main example in his book-length study of intrasite archaeological spatial analysis. Whallon (1984), who digitized the dataset used here, has also used these data to illustrate his unconstrained clustering method and to study site structure. Thus, these data now form the standard and canonical dataset with which to study archaeological site structure. These data have a fractal dimension of 1.29, estimated using the box-counting method as implemented by the FD3 program. This implies that there are clusters of clusters of artifacts—a complex internal structure. This probably explains the long-standing difficulty of defining discrete clusters of artifacts that represent identifiable activity areas in archaeological sites.

For comparison with an archaeological case, we analyzed the data on artifact locations from the Barmose I site published by Blankholm (1991, pp. 183–206, 391–394). Barmose I is an early Mesolithic site affiliated with the Maglemosian culture and dating from ca. 7500–6000 B.C. It appears to be a homogenous deposit with a single significant occupation only slightly contaminated with a few later, Neolithic artifacts. The Mesolithic occupation appears to represent a single hut with surrounding debris. The dataset consists of the Cartesian coordinates of 473 tools from 11 typological categories: scrapers, burins, lanceolate microliths, microburins, flake axes, core axes, square knives, blade/flake knives, denticulated/notched pieces, cores, and core platforms. The fractal dimension of this dataset is 1.22, remarkably similar to that from the Mask site.

Curiously, the fractal dimensions for the artifact distributions of both the Mask site and the Barmose I are close to the fractal dimension of the "Cantor Square," $D \approx 1.26$ (Schroeder, 1991, pp. 177–181). The Cantor Square (Fig. 7) is the Cartesian product of two middle-third Cantor sets like that described earlier in this article. Imagine a Cartesian graph with a middle-third Cantor set on the $x$-axis and another one along the $y$-axis. The intersection creates a Cantor Square, which is a Cantor dust embedded in two dimensions. The mathematics of Cantor sets have been extensively studied and are well known. Therefore, the dynamics underlying the Cantor Square may provide a conceptual model for archaeological spatial distributions like those described above.

We do not know, as yet, whether other spatial archaeological datasets will exhibit fractal dimensions in this same range or whether they will present other types of variation. It would be dangerous and unwarranted to assume that all archaeological distributions will have the same fractal dimension, or even that all will be fractal. Nevertheless, it is important to consider the possibility that they may be fractal because of the statistical implications of fractality.

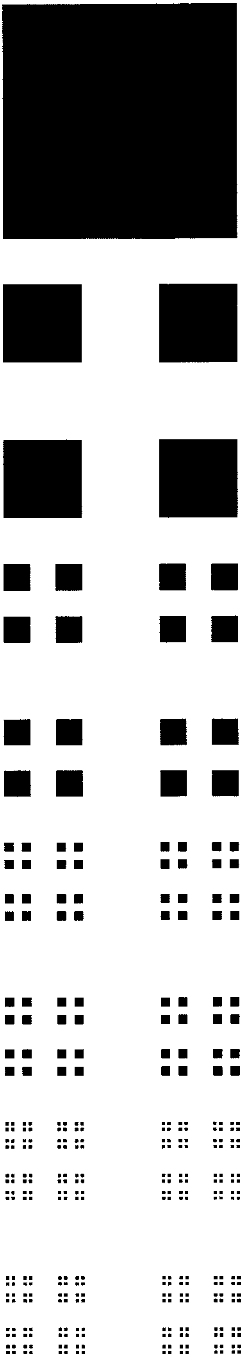

**Fig. 7.** The First Four Iterations of the Cantor Square, the cross-product of the two Cantor sets.

The idea that archaeological artifact distributions can be fractal makes empirical sense if you think about what archaeological deposits look like in the field. We have often seen dense archaeological deposits that contain many tiny artifacts that are never collected. Of course, archaeologists routinely ignore artifacts smaller than a certain arbitrary size. We do this systematically by using screens when excavating; when mapping surfaces, we do this less systematically, but we do it nonetheless. These are pragmatic choices. In one case, when trying to systematically surface collect small artifacts in a lithic workshop, Brown found it difficult to identify artifacts consistently when they were smaller than 1 or 2 mm in size. This issue does raise the question, "To what extent is the intrasite structure that we perceive statistically and analytically an artifact of the size of the remains that we observe?"

If we were to collect and record ever-smaller remains—to a theoretical limit at a molecular size—would yet smaller clusters of clusters of artifacts emerge in the spatial patterning of data? It certainly is possible. If the size frequency distribution of some artifacts follows a power-law, then we can predict that there are very many more small artifacts than large ones in a given area. If we were to measure the distances between all these artifacts (instead of only the big ones), then the mean distance between artifacts within a given area would shrink dramatically. Thus, as is typical with fractal phenomena, the result is dependent on the scale of observation.

So, to summarize, we commonly assume that the size-censored sample of artifacts that we map is a representative sample of the spatial pattern we are trying to define. If, however, the patterns are fractal, their parametric statistics will probably be dependent on the sample size. Thus, the conventional spatial statistics used by archaeologists, such as nearest-neighbor analysis and *k*-means cluster analysis, may yield inconsistent results that are sample dependent. Of course, this conclusion only applies *if* the pattern is a fractal one; again, we cannot assume that all archaeological spatial distributions are fractal, but some clearly are.

Many readers may be wondering about the relationship between fractal statistics and better-known archaeological spatial statistics, such as nearest-neighbor analysis and *k*-means cluster analysis. This is a large topic, large enough for a separate paper, so here we will limit ourselves to a couple of comments.

Fractal analysis measures characteristics different than other spatial statistics (e.g., Blankholm, 1991; Hietala, 1984; Hodder and Orton, 1976; Kintigh, 1990). So, fractal analysis does not replace other spatial statistics. Finding that a spatial distribution is fractal, however, may increase our understanding of the phenomenon and may help us select or interpret other statistics.

Some spatial statistics can yield unstable or misleading answers with fractal datasets. The mean distance between points in a fractal point pattern may not be a stable or meaningful statistic; it may depend erratically on sample size. Therefore, statistics that are based on the mean of a spatial distribution may not yield a stable result if the dataset is really fractal. For example, *k*-means cluster analysis conventionally proceeds by minimizing the sum of squared deviations in Euclidean distance from the estimated mean value (centroid) of each cluster (Kintigh and Ammerman, 1982, p. 39). If that mean value is not stable or accurate, then the clusters calculated from it will not be meaningful. Thus, the fractality of these patterns is relevant to the interpretation of other spatial statistics.

## FRACTALS AND DYNAMICAL SYSTEMS

Up to this point, we have focused on practical issues that we think practicing archaeologists will appreciate most intensely because of their own individual experiences with data collection and analysis. Now we wish to turn to more theoretical issues. Fractal geometry carries a variety of theoretical implications for archaeology.

Modern archaeologists view the archaeological record as a static picture of past cultural dynamics (e.g., Binford, 1981). Archaeologists use various techniques for distinguishing patterns in the archaeological record, but our ultimate goal is not the description of patterns for their own sake. We wish to use the patterns to understand the cultural dynamics that produced the patterns.

The same is true of fractal patterns. Although they are interesting in themselves, as archaeologists we are particularly interested in what fractal patterns may be able to tell us about prehistoric social dynamics. Thus, while the accurate description of these complicated patterns is not trivial, we do, nevertheless, think of description as a prelude to further explanation. We expect explanation to reveal the underlying cultural processes that lead to pattern formation.

This logic leads us inexorably to ask, "Are fractals related to any particular kinds of dynamical processes or systems?" The answer is remarkable.

Fractal geometry is the geometry of complex nonlinear systems.

Nonlinear dynamical processes of various kinds can generate fractal patterns. Iterated function systems, cellular automata, and diffusion-limited aggregation, for example, all produce fractal patterns, and those approaches can be used to simulate known types of fractals and fractal processes (Peitgen *et al.*, 1992; Zubrow, 1985). Two major classes of nonlinear dynamical systems are particularly well known for generating fractal patterns: chaotic systems and self-organized systems.

Whereas a fractal is a set, chaos is a characteristic of deterministic dynamical systems. Chaotic systems are a class or type of dynamical systems. Chaotic systems are common, perhaps more common than stable, non-chaotic ones. A deterministic dynamical system is said to be chaotic if "solutions that have initial conditions that are infinitesimally close diverge exponentially" (Turcotte, 1997, p. 219). This fundamental characteristic of chaos is also called "sensitive dependence on initial conditions." It only occurs in strongly nonlinear systems. When two systems that begin at arbitrarily close points have trajectories that diverge exponentially, they are said to be chaotic. After only a short time, the patterns they produce no longer resemble each other. This is not stochastic behavior because the systems are completely deterministic. It is neither stable nor periodic behavior, hence the use of a fresh term, *chaotic*.

The solution set of a chaotic system is called a *strange attractor*. Strange attractors are fractals. The best-known statistic that is used to measure whether a system behaves chaotically is called the Lyapunov exponent. It measures the rate of divergence of a perturbed trajectory from an unperturbed one. The Kaplan–Yorke conjecture demonstrates that the Lyapunov exponents of a chaotic system are closely related to the capacity dimension of the attractor, which in turn provides an estimate of the fractal dimension. Therefore, it is commonly said that fractals are the geometry of chaos.

So, identical systems that start at essentially the same point can diverge exponentially when they are chaotic. This fact carries a number of implications. For example, systems that have essentially the same processes and elements can produce radically different patterns merely because of an infinitesimally small difference in the starting point. One cannot make any assumptions, of course, but if prehistoric cultural systems were chaotic, then these kinds of dynamics may explain why, for instance, early civilizations that arose under similar conditions and possessed similar internal processes followed distinctive trajectories and developed different cultural patterns (Bentley, 2003, pp. 11–13, Bentley and Maschner, 2003, pp. 75–77).

Another consequence of the mathematics of chaos is that even simple systems, such as the motion of the tip of a compound pendulum (which is made up of at least two pieces and two hinges), can behave unpredictably. The behavior of such systems becomes mathematically unpredictable because any error or perturbation, no matter how small, propagates until it overwhelms the underlying pattern. This is popularly known as the "butterfly effect," wherein a tiny force (such as the beating of a butterfly's wings) can have a dramatically disproportionate (nonlinear) effect (causing a proverbial tornado in Texas). The practical significance of chaotic behavior is that it defies prediction. Naturally, this goes right to the heart of any philosophy of science that takes as its goal the discovery of predictive laws. In social science, chaos theory has been applied to political and economic systems (Brown, 1995a, 1995b; McGlade, 1995, McGlade and Van Der Leeuw, 1997; Nicolas and Prigogine, 1989, pp. 238–242).

The principle obstacle to the study of chaotic dynamics in prehistory is the absence of appropriate data. (This will hardly surprise most archaeologists, who can rarely collect the data they really need to test interesting hypotheses.) The empirical analysis of dynamical systems normally requires high-quality time series data, and chaotic systems are no different from other dynamical systems in this respect. For the analysis of chaotic behavior in systems one normally desires high-quality data, because small differences may be disproportionately significant. Such datasets are not generally available in archaeology, except in certain special cases, such as dendrochronology or paleoclimatology. There are widely differing opinions about the length of time series necessary to detect chaotic behavior (Liebovitch, 1998), but many believe that thousands or even millions of observations are necessary. Such data are unknown in archaeology, which, despite its focus on long-time periods, characteristically produces low-quality data and datasets with few observations.

Fortunately, fractal analysis offers a different approach to the study of chaos in prehistory. Fractal patterns in the archaeological record do imply the presence of nonlinear complex dynamics, although more study is needed to determine the exact relations between different fractal patterns and the underlying characteristics of different systems.

"Self-organized criticality" is another concept that unites dynamical systems and fractals (Bak, 1996; Bak *et al.*, 1988). The term "self-organized criticality" describes a certain class of complex nonlinear dynamical systems. "Criticality" refers to a marginally stable state toward which these systems spontaneously evolve. The classic model of this phenomenon is a sand pile to which sand is added one grain at a time. Eventually, the slope of the pile will reach a critical state—the angle of repose—after which the addition of more sand causes avalanches. Study of the avalanches reveals that they possess no natural scale, and they exhibit fractal statistics in both space and time. The avalanches allow the system to evolve back to a critical state, where the further addition of sand will cause more avalanches. Thus, after perturbation, the system evolves back to marginal stability. The fractal characteristics of the avalanches appear to explain several natural phenomena, including the fractal size–frequency distribution of geologic strata, the general fractality of erosional landscapes and hydrological systems (Bak, 1996, pp. 80–84), and the statistics of forest fires (Roberts and Turcotte, 1998). The idea of self-organized criticality has also been applied to various human social systems, particularly war and politics (Brunk, 2002; Roberts and Turcotte, 1998). Bentley and Maschner (2001, 2003) have recently applied self-organized criticality to archaeological systems. Like chaotic systems, self-organized critical ones produce fractal patterns in time and space, which implies that fractal analysis is a useful method for investigating this class of complex systems.

Some systems appear to unite all three concepts of fractals, chaos, and self-organized criticality: simulations of meandering rivers indicate that the system evolves to a critical state that oscillates between stability and chaos (Stølum, 1996). This systemic meandering along the edge of chaos, exploring alternately stable and chaotic regimes, could also be applied to human systems. There are also more general models of dynamical systems that have been proposed as the source of the power-law distributions that occur so widely in fractal analysis (Amaral *et al.*, pp. 1998).

## CONCLUSIONS

In this article, we have cataloged a wide variety of fractal patterns in archaeological data, emphasizing those that are particularly common in the archaeological record. We have also called attention to phenomena that seem likely to be fractal but which have yet to be evaluated carefully. We trust this presentation makes it clear that fractals have an important contribution to make to archaeology because some fundamental archaeological processes are fractal.

Several points merit emphasis. The classical statistical measures that we learned in school, such as mean and variance, are not capable of capturing the complexity of fractal patterns. Fractal patterns, because of their self-similarity and scale invariance, demand special statistical treatment. They need to be identified and described properly. Inappropriate observation and analysis will yield erroneous results, which will then lead to false inferences.

Fractal patterns are produced by nonlinear dynamical systems. Therefore, they hold the promise of allowing us to infer and describe the nonlinear processes in prehistory that generated the fractal patterns in the archaeological record.

For archaeology, one relevant implication to be drawn from the theory of complex systems is that complex social patterns can emerge without any external stimulus; the complex patterns may be the exclusive result of the internal dynamics of the system. For example, virtually all explanations for the rise of civilization invoke one or more external factors, such as climate change or environmental circumscription (e.g., Carneiro, 1970). We know now, however, that increasingly complex patterns can arise in the absence of any exogenous force, but purely because of the endogenous process within the complex systems. Some have even proposed that "complexification" is a primordial and innate characteristic of systems (Chaisson, 2001). Knowing this, is it not more parsimonious to examine whether the increasing complexity we observe in the evolution of ancient society is the result of systems dynamics before we look for external causes?

An intriguing idea suggested by one reviewer of our manuscript, Zubrow, was that fractals might be used to test whether very different aspects of a culture are organized according to the same principles. For example, tool design, housing design, settlement pattern, ideological or religious objects, and perhaps even economic, environmental, social and ideological spheres in a single culture might all be organized according to the same fractal patterning, whereas another culture might be organized according a different fractal design. The idea that a culture may have a characteristic pattern, principle, or "genius" that pervades diverse domains of the culture has a long and distinguished history in anthropology, espoused most famously perhaps by Benedict in *Patterns of Culture* (1934), citing Spengler's *The Decline of the West* as her inspiration. If that is the case, then fractal analysis may provide a method to analytically substantiate what we all know intuitively, namely, that we recognize a culture's characteristic forms in many different aspects of a culture. This is, of course, an exciting possibility, but we must insist on two caveats. First, sometimes similar-looking fractal patterns can be produced by different underlying mechanisms, and second, the characteristic cultural patterns might not be fractal, in which case fractal analysis may not be relevant at all. Some empirical evidence of such patterns does exist, however, and it does point to fractality. Eglash's (1999) identification of fractal patterns in particular African cultures is certainly suggestive. Similarly, Vogt's (1969, pp. 571–572) description of similar patterns in different spheres of Zinacanteco life in the Chiapas highlands, which he calls "replication," is reminiscent of these ideas. Brown and Witschey (2003) have discussed the self-similar and fractal character of these replicated patterns. So, we agree that Zubrow's idea is exciting and merits additional investigation.

In addition to this idea, in this article we have suggested several avenues for future research, including more detailed investigation of fractal fragmentation of ceramic and lithic materials, studies of the fractality of ancient art styles, expanded use of fractal analysis in settlement pattern research and intrasite archaeological pattern analysis, and archaeological remote sensing. These are mostly methodological issues, and methods have been the focus of this article, but some of them have significant theoretical implications. For example, the fractality of settlement patterns carries a variety of implications about social structure and social dynamics. We believe that interested investigators would be well advised to focus on substantive empirical issues such as these rather than on grand ideas, however enticing, that can neither be verified nor falsified. Of course, big ideas that can be tested should be proposed, not eschewed.

In closing, we wish to observe that we have been careful not to claim that everything is fractal or that all dynamics are nonlinear. Exaggerated claims are sometimes made about the explanatory power of new methods, and we have diligently tried to avoid that solipsistic trap. We have not suggested that fractal thinking is a paradigm shift. It may be, but time and history will tell, and we are content to wait on their judgment. Nevertheless, we do not waver in our assertion that many archaeological patterns are fractal and that they should be described properly, using nonlinear statistics. Fractal analysis is no longer novel in most scientific fields. It is time for archaeologists to start tracing these irregular, broken patterns to see if they open a small casement upon the shores of the past.

## ACKNOWLEDGMENTS

We would like to thank John Sarraillé for assistance with the FD3 program. Many thanks also go to John Montgomery for the use of his magnificent Altar Vase drawing. We also extend our thanks to Keith Kintigh for the Mask site dataset and to Robert Whallon, who digitized it. We are also indebted to the many thoughtful comments from all the reviewers, whose suggestions have helped clarify the presentation and technical issues. Two of them, who chose not to remain anonymous, were Alex Bentley and Ezra Zubrow. We are also grateful to Richard Leventhal and Mark Aldenderfer for serving as discussants at the Society for American Archaeology symposium in which Witschey and Brown originally presented some of the ideas contained in this paper. We also thank W. James Stemp for helping organize that symposium, as well as all the others who participated.

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