

Using Newton's Laws

What You Know

- You're a Newtonian, meaning you understand that forces cause *change* in motion.
- You recognize how Newton's second law, $\vec{F}_{\text{net}} = m\vec{a}$, embodies a vector relation between force and change in motion.
- You can work one-dimensional problems involving Newton's second law, following a clear problem-solving strategy.
- You've seen how Newton's third law describes the forces between interacting objects as always coming in pairs.

What You're Learning

- You'll learn to apply Newton's second law in two dimensions, expanding the strategy you learned in Chapter 4.
- You'll practice choosing coordinate systems and using trigonometry to break force vectors into components.
- You'll learn about the force of friction and how to incorporate friction into problems involving Newton's laws.
- You'll come out of the chapter able to analyze the richness of motion as it occurs throughout our three-dimensional universe.

How You'll Use It

- Newton's second law will help in developing energy concepts in Chapters 6 and 7.
- In Chapter 8, you'll see how Newton's laws describe motion in response to gravity as it applies to space flight and the cosmos.
- In Chapter 9, you'll learn how Newton's laws apply to systems of particles.
- In Chapters 10 and 11, rotational analogs of Newton's laws will help you understand rotational motion.
- Newton's laws will help you understand the motion of waves and fluids in Part 2, and the molecular motion that gives rise to temperature and pressure in Part 3.



Why does an airplane tip when it's turning?

Chapter 4 introduced Newton's three laws of motion and used them in one-dimensional situations. Now we apply Newton's laws in two dimensions. This material is at the heart of Newtonian physics, from textbook problems to systems that guide spacecraft to distant planets. The chapter consists largely of examples, to help you learn to apply Newton's laws and also to appreciate their wide range of applicability. We also introduce frictional forces and elaborate on circular motion. As you study the diverse examples, keep in mind that they all follow from the underlying principles embodied in Newton's laws.

5.1 Using Newton's Second Law

Newton's second law, $\vec{F}_{\text{net}} = m\vec{a}$, is the cornerstone of mechanics. We can use it to develop faster skis, engineer skyscrapers, design safer roads, compute a rocket's thrust, and solve myriad other practical problems.

We'll work Example 5.1 in great detail, applying Problem-Solving Strategy 4.1. Follow this example closely, and try to understand how our strategy is grounded in Newton's basic statement that the net force on an object determines that object's acceleration.

EXAMPLE 5.1 Newton's Law in Two Dimensions: Skiing

A skier of mass $m = 65 \text{ kg}$ glides down a slope at angle $\theta = 32^\circ$, as shown in Fig. 5.1. Find (a) the skier's acceleration and (b) the force the snow exerts on the skier. The snow is so slippery that you can neglect friction.

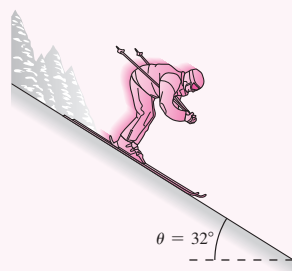


FIGURE 5.1 What's the skier's acceleration?

INTERPRET This problem is about the skier's motion, so we identify the skier as the object of interest. Next, we identify the forces acting on the object. In this case there are just two: the downward force of gravity and the normal force the ground exerts on the skier. As always, the normal force is perpendicular to the surfaces in contact—in this case perpendicular to the slope.

DEVELOP Our strategy for using Newton's second law calls for drawing a free-body diagram that shows only the object and the forces acting on it; that's Fig. 5.2. Determining the relevant equation is straightforward here: It's Newton's second law, $\vec{F}_{\text{net}} = m\vec{a}$. We write Newton's law explicitly for the forces we've identified:

$$\vec{F}_{\text{net}} = \vec{n} + \vec{F}_g = m\vec{a}$$

To apply Newton's law in two dimensions, we need to choose a coordinate system so that we can write this vector equation in components. Since the coordinate system is just a mathematical construct, you're free to choose any coordinate system you like—but a smart choice can make the problem a lot easier. In this example, the normal force is perpendicular to the slope and the skier's acceleration is along the slope. If you choose a coordinate system with axes perpendicular and parallel to the slope, then these two vectors will lie along the coordinate axes, and you'll have only one vector—the gravitational force—that you'll

need to break into components. So a tilted coordinate system makes this problem easier, and we've sketched this system on the free-body diagram in Fig. 5.2. But, again, any coordinate system will do. In Problem 34, you can rework this example in a horizontal/vertical coordinate system—getting the same answer at the expense of a lot more algebra.

EVALUATE The rest is math. First, we write the components of Newton's law in our coordinate system. That means writing a version of the equation for each coordinate direction by removing the arrows indicating vector quantities and adding subscripts for the coordinate directions:

$$x\text{-component: } n_x + F_{gx} = ma_x$$

$$y\text{-component: } n_y + F_{gy} = ma_y$$

Don't worry about signs until the next step, when we actually evaluate the individual terms in these equations. Let's begin with the x equation. With the x -axis parallel and the y -axis perpendicular to the slope, the normal force has only a y -component, so $n_x = 0$. Meanwhile, the acceleration points downslope—that's the positive x -direction—so $a_x = a$, the magnitude of the acceleration. Only gravity has two nonzero components and, as Fig. 5.2 shows, trigonometry gives $F_{gx} = F_g \sin \theta$. But F_g , the magnitude of the gravitational force, is just mg , so $F_{gx} = mg \sin \theta$. This component has a positive sign because our x -axis slopes downward. Then, with $n_x = 0$, the x equation becomes

$$x\text{-component: } mg \sin \theta = ma$$

On to the y equation. The normal force points in the positive y -direction, so $n_y = n$, the magnitude of the normal force. The acceleration has no component perpendicular to the slope, so $a_y = 0$. Figure 5.2 shows that $F_{gy} = -F_g \cos \theta = -mg \cos \theta$, so the y equation is

$$y\text{-component: } n - mg \cos \theta = 0$$

Now we can evaluate to get the answers. The x equation solves directly to give

$$a = g \sin \theta = (9.8 \text{ m/s}^2)(\sin 32^\circ) = 5.2 \text{ m/s}^2$$

which is the acceleration we were asked to find in (a). Next, we solve the y equation to get $n = mg \cos \theta$. Putting in the numbers gives $n = 540 \text{ N}$. This is the answer to (b), the force the snow exerts on the skier.

ASSESS A look at two special cases shows that these results make sense. First, suppose $\theta = 0^\circ$, so the surface is horizontal. Then the x equation gives $a = 0$, as expected. The y equation gives $n = mg$, showing that a horizontal surface exerts a force that just balances the skier's weight. At the other extreme, consider $\theta = 90^\circ$, so the slope is a vertical cliff. Then the skier falls freely with acceleration g , as expected. In this case $n = 0$ because there's no contact between skier and slope. At intermediate angles, the slope's normal force lessens the effect of gravity, resulting in a lower acceleration. As the x equation shows, that acceleration is independent of the skier's mass—just as in the case of a vertical fall. The force exerted by the snow—here $mg \cos \theta$, or 540 N —is less than the skier's weight mg because the slope has to balance only the perpendicular component of the gravitational force.

If you understand this example, you should be able to apply Newton's second law confidently in other problems involving motion with forces in two dimensions. ■

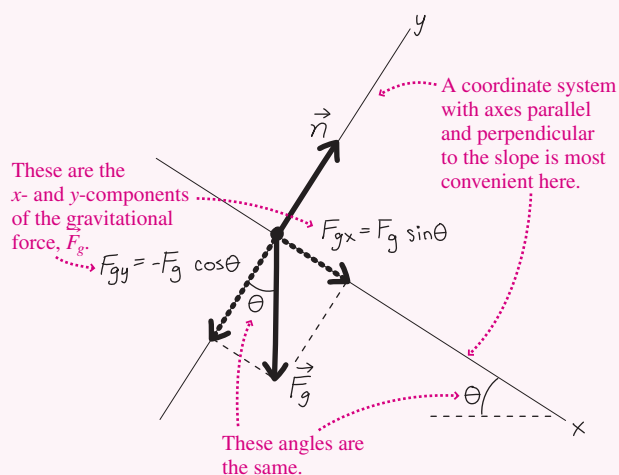


FIGURE 5.2 Our free-body diagram for the skier.

Sometimes we're interested in finding the conditions under which an object won't accelerate. Examples are engineering problems, such as ensuring that bridges and buildings don't fall down, and physiology problems involving muscles and bones. Next we give a wilder example.



PhET: The Ramp

EXAMPLE 5.2 Objects at Rest: Bear Precautions

To protect her 17-kg pack from bears, a camper hangs it from ropes between two trees (Fig. 5.3). What's the tension in each rope?

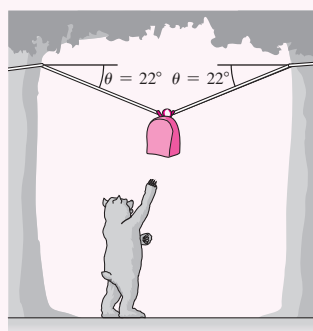


FIGURE 5.3 Bear precautions.

INTERPRET Here the pack is the object of interest. The only forces acting on it are gravity and tension forces in the two halves of the rope. To keep the pack from accelerating, they must sum to zero net force.

DEVELOP Figure 5.4 is our free-body diagram for the pack. The relevant equation is again Newton's second law, $\vec{F}_{\text{net}} = m\vec{a}$ —this time with $\vec{a} = \vec{0}$. For the three forces acting on the pack, Newton's law is then $\vec{T}_1 + \vec{T}_2 + \vec{F}_g = \vec{0}$. Next, we need a coordinate system. The two rope tensions point in different directions that aren't perpendicular, so it doesn't make sense to align a coordinate axis with either of them. Instead, a horizontal/vertical system is simplest.

EVALUATE First we need to write Newton's law in components. Formally, we have $T_{1x} + T_{2x} + F_{gx} = 0$ and $T_{1y} + T_{2y} + F_{gy} = 0$ for the component equations. Figure 5.4 shows the components of the tension forces, and we see that $F_{gx} = 0$ and $F_{gy} = -F_g = -mg$. So our component equations become

$$\text{x-component: } T_1 \cos \theta - T_2 \cos \theta = 0$$

$$\text{y-component: } T_1 \sin \theta + T_2 \sin \theta - mg = 0$$

The x equation tells us something that's apparent from the symmetry of the situation: Since the angle θ is the same for both halves

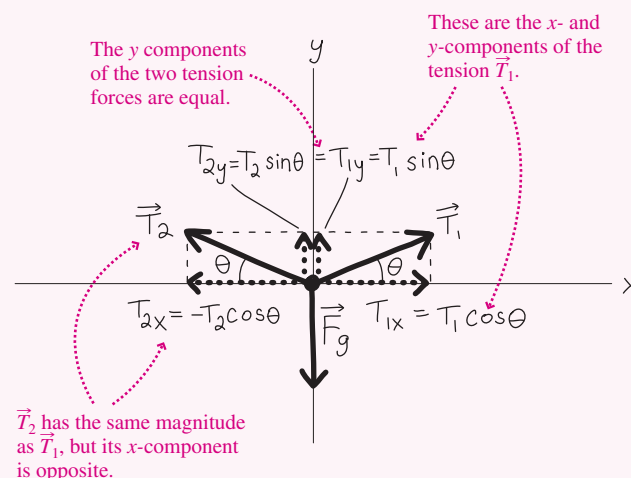


FIGURE 5.4 Our free-body diagram for the pack.

of the rope, the magnitudes T_1 and T_2 of the tension forces are the same. Let's just call the magnitude T : $T_1 = T_2 = T$. Then the terms $T_1 \sin \theta$ and $T_2 \sin \theta$ in the y equation are equal, and the equation becomes $2T \sin \theta - mg = 0$, which gives

$$T = \frac{mg}{2 \sin \theta} = \frac{(17 \text{ kg})(9.8 \text{ m/s}^2)}{2 \sin 22^\circ} = 220 \text{ N}$$

ASSESS Make sense? Let's look at some special cases. With $\theta = 90^\circ$, the rope hangs vertically, $\sin \theta = 1$, and the tension in each half of the rope is $\frac{1}{2}mg$. That makes sense, because each piece of the rope supports half the pack's weight. But as θ gets smaller, the ropes become more horizontal and the tension increases. That's because the vertical tension components together still have to support the pack's weight—but now there's a horizontal component as well, increasing the overall tension. Ropes break if the tension becomes too great, and in this example that means the rope's so-called breaking tension must be considerably greater than the pack's weight. If $\theta = 0$, in fact, the tension would become infinite—demonstrating that it's impossible to support a weight with a purely horizontal rope. ■

EXAMPLE 5.3 Objects at Rest: Restraining a Ski Racer

A starting gate acts horizontally to restrain a 62-kg ski racer on a frictionless 30° slope (Fig. 5.5). What horizontal force does the starting gate apply to the skier?

INTERPRET Again, we want the skier to remain unaccelerated. The skier is the object of interest, and we identify three forces acting: gravity, the normal force from the slope, and a horizontal restraining force $\vec{F}_h$ that we're asked to find.

DEVELOP Figure 5.6 is our free-body diagram. The applicable equation is Newton's second law. Again, we want $\vec{a} = \vec{0}$, so with

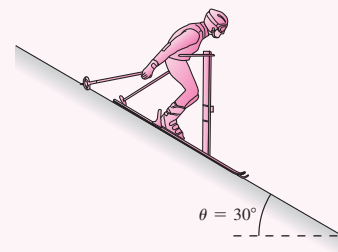


FIGURE 5.5 Restraining a skier.

(continued)

the forces we identified, $\vec{F}_{\text{net}} = m\vec{a}$ becomes $\vec{F}_h + \vec{n} + \vec{F}_g = \vec{0}$. Developing our solution strategy, we choose a coordinate system. With two forces now either horizontal or vertical, a horizontal/vertical system makes the most sense; we've shown this coordinate system in Fig. 5.6.

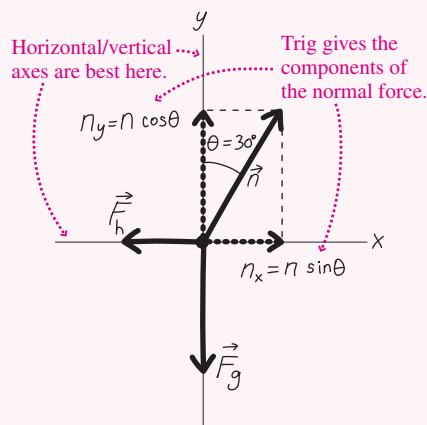


FIGURE 5.6 Our free-body diagram for the restrained skier.

EVALUATE As usual, the component equations follow directly from the vector form of Newton's law: $F_{hx} + n_x + F_{gx} = 0$ and $F_{hy} + n_y + F_{gy} = 0$. Figure 5.6 gives the components of the normal force and shows that $F_{hx} = -F_h$, $F_{gy} = -F_g = -mg$, and $F_{gx} = F_{hy} = 0$. Then the component equations become

$$x: -F_h + n \sin \theta = 0 \quad y: n \cos \theta - mg = 0$$

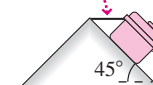
There are two unknowns here—namely, the horizontal force F_h that we're looking for and the normal force n . We can solve the y equation to get $n = mg/\cos \theta$. Using this expression in the x equation and solving for F_h then give the answer:

$$F_h = \frac{mg}{\cos \theta} \sin \theta = mg \tan \theta = (62 \text{ kg})(9.8 \text{ m/s}^2)(\tan 30^\circ) = 350 \text{ N}$$

ASSESS Again, let's look at the extreme cases. With $\theta = 0$, we have $F_h = 0$, showing that it doesn't take any force to restrain a skier on flat ground. But as the slope becomes more vertical, $\tan \theta \rightarrow \infty$, and in the vertical limit, it becomes impossible to restrain the skier with a purely horizontal force. ■

GOT IT? 5.1 A roofer's toolbox rests on an essentially frictionless metal roof with a 45° slope, secured by a horizontal rope as shown. Is the rope tension (a) greater than, (b) less than, or (c) equal to the box's weight?

How does the rope tension compare with the toolbox weight?



5.2 Multiple Objects

In the preceding examples there was a single object of interest. But often we have several objects whose motion is linked. Our Newton's law strategy still applies, with extensions to handle multiple objects.

PROBLEM-SOLVING STRATEGY 5.1 Newton's Second Law and Multiple Objects

INTERPRET Interpret the problem to be sure that you know what it's asking and that Newton's second law is the relevant concept. Identify the *multiple* objects of interest and all the individual interaction forces acting on *each* object. Finally, identify *connections* between the objects and the resulting *constraints* on their motions.

DEVELOP Draw a *separate* free-body diagram showing all the forces acting on *each* object. Develop your solution plan by writing Newton's law, $\vec{F}_{\text{net}} = m\vec{a}$, separately for each object, with $\vec{F}_{\text{net}}$ expressed as the sum of the forces acting on that object. Then choose a coordinate system appropriate to each object, so you can express each Newton's law equation in components. The coordinate systems for different objects don't need to have the same orientation.

EVALUATE At this point the physics is done, and you're ready to execute your plan by solving the equations and evaluating the numerical answer(s), if called for. Write the components of Newton's law for each object in the coordinate system you chose for each. You can then solve the resulting equations for the quantity(ies) you're interested in, using the connections you identified to relate the quantities that appear in the equations for the different objects.

ASSESS Assess your solution to see whether it makes sense. Are the numbers reasonable? Do the units work out correctly? What happens in special cases—for example, when a mass, a force, an acceleration, or an angle gets very small or very large?

EXAMPLE 5.4 Multiple Objects: Rescuing a Climber

A 73-kg climber finds himself dangling over the edge of an ice cliff, as shown in Fig. 5.7. Fortunately, he's roped to a 940-kg rock located 51 m from the edge of the cliff. Unfortunately, the ice is frictionless, and the climber accelerates downward. What's his acceleration, and how much time does he have before the rock goes over the edge? Neglect the rope's mass.

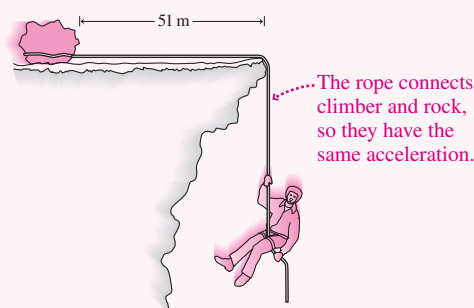


FIGURE 5.7 A climber in trouble.

INTERPRET We need to find the climber's acceleration, and from that we can get the time before the rock goes over the edge. We identify two objects of interest, the climber and the rock, and we note that the rope connects them. There are two forces on the climber: gravity and the upward rope tension. There are three forces on the rock: gravity, the normal force from the surface, and the rightward-pointing rope tension.

DEVELOP Figure 5.8 shows a free-body diagram for each object. Newton's law applies to each, so we write two vector equations:

$$\begin{aligned}\text{climber: } \vec{T}_c + \vec{F}_{gc} &= m_c \vec{a}_c \\ \text{rock: } \vec{T}_r + \vec{F}_{gr} + \vec{n} &= m_r \vec{a}_r\end{aligned}$$

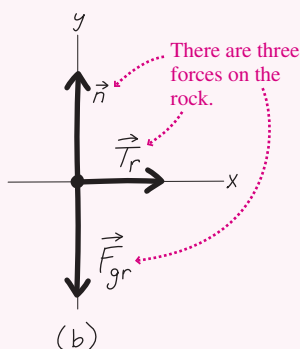
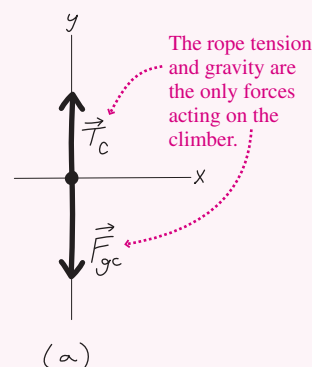


FIGURE 5.8 Our free-body diagrams for (a) the climber and (b) the rock.

where the subscripts c and r stand for climber and rock, respectively. All forces are either horizontal or vertical, so we can use the same horizontal/vertical coordinate system for both objects, as shown in Fig. 5.8.

EVALUATE Again, the component equations follow directly from the vector forms. There are no horizontal forces on the climber, so only the y equation is significant. We're skilled enough now to skip the intermediate step of writing the components without their actual expressions, and we see from Fig. 5.8a that the y-component of Newton's law for the climber becomes $T_c - m_c g = m_c a_c$. For the rock, the only horizontal force is the tension, pointing to the right or positive x-direction, so the rock's x equation is $T_r = m_r a_r$. Since it's on a horizontal surface, the rock has no vertical acceleration, so its y equation is $n - m_r g = 0$. In writing these equations, we haven't added the subscripts x and y because each vector has only a single nonzero component. Now we need to consider the connection between rock and climber. That's the rope, and its presence means that the magnitude of both accelerations is the same. Calling that magnitude a , we can see from Fig. 5.8 that $a_r = a$ and $a_c = -a$. The value for the rock is positive because $\vec{T}_r$ points to the right, which we defined as the positive x-direction; the value for the climber is negative because he's accelerating downward, which we defined as the negative y-direction. The rope, furthermore, has negligible mass, so the tension throughout it must be the same (more on this point just after the example). Therefore, the tension forces on rock and climber have equal magnitude T , so $T_c = T_r = T$. Putting this all together gives us three equations:

$$\begin{aligned}\text{climber, y: } T - m_c g &= -m_c a \\ \text{rock, x: } T &= m_r a \\ \text{rock, y: } n - m_r g &= 0\end{aligned}$$

The rock's x equation gives the tension, which we can substitute into the climber's equation to get $m_r a - m_c g = -m_c a$. Solving for a then gives the answer:

$$a = \frac{m_c g}{m_c + m_r} = \frac{(73 \text{ kg})(9.8 \text{ m/s}^2)}{(73 \text{ kg} + 940 \text{ kg})} = 0.71 \text{ m/s}^2$$

We didn't need the rock's y equation, which just says that the normal force supports the rock's weight.

ASSESS Again, let's look at special cases. Suppose the rock's mass is zero; then our expression gives $a = g$. In this case there's no rope tension and the climber plummets in free fall. Also, acceleration decreases as the rock's mass increases, so with an infinitely massive rock, the climber would dangle without accelerating. You can see physically why our expression for acceleration makes sense. The gravitational force $m_c g$ acting on the climber has to accelerate both rock and climber—whose combined mass is $m_c + m_r$. The result is an acceleration of $m_c g / (m_c + m_r)$.

We're not quite done because we were also asked for the time until the rock goes over the cliff, putting the climber in real trouble. We interpret this as a problem in one-dimensional motion from Chapter 2, and we determine that Equation 2.10, $x = x_0 + v_0 t + \frac{1}{2} a t^2$, applies. With $x_0 = 0$ and $v_0 = 0$, we have $x = \frac{1}{2} a t^2$. We evaluate by solving for t and using the acceleration we found along with $x = 51 \text{ m}$ for the distance from the rock to the cliff edge:

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{(2)(51 \text{ m})}{0.71 \text{ m/s}^2}} = 12 \text{ s}$$

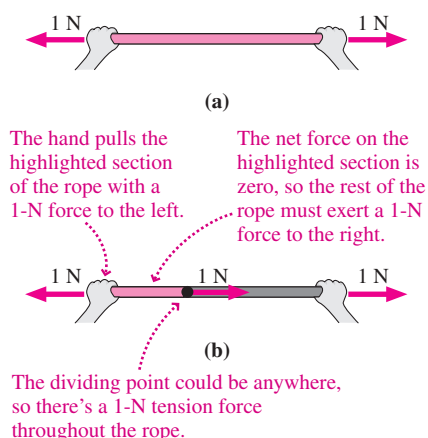


FIGURE 5.9 Understanding tension forces.



Video Tutor Demo | **Tension in String between Hanging Weights**

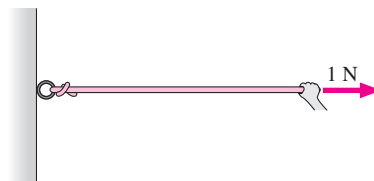
✓TIP Ropes and Tension Forces

Tension forces can be confusing. In Example 5.4, the rock pulls on one end of the rope and the climber pulls on the other. So why isn't the rope tension the sum of these forces? And why is it important to neglect the rope's mass? The answers lie in the meaning of tension.

Figure 5.9 shows a situation similar to Example 5.4, with two people pulling on opposite ends of a rope with forces of 1 N each. You might think the rope tension is then 2 N, but it's not. To see why, consider the part of the rope that's highlighted in Fig. 5.9b. To the left is the hand pulling leftward with 1 N. The rope isn't accelerating, so there must be a 1-N force pulling to the right on the highlighted piece. The remainder of the rope provides that force. We could have divided the rope anywhere, so we conclude that every part of the rope exerts a 1-N force on the adjacent rope. That 1-N force is what we mean by the rope tension.

As long as the rope isn't accelerating, the net force on it must be zero, so the forces at the two ends have the same magnitude. That conclusion would hold even if the rope were accelerating—provided it had negligible mass. That's often a good approximation in situations involving tension forces. But if a rope, cable, or chain has significant mass and is accelerating, then the tension force differs at the two ends. That difference, according to Newton's second law, is the net force that accelerates the rope.

GOT IT? 5.2 In the figure below we've replaced one of the hands from Fig. 5.9 with a hook attaching the rope to a wall. On the right, the hand still pulls with a 1-N force. How do the forces now differ from what they were in Fig. 5.9? (a) there's no difference; (b) the force exerted by the hook is zero; (c) the rope tension is now 0.5 N



5.3 Circular Motion

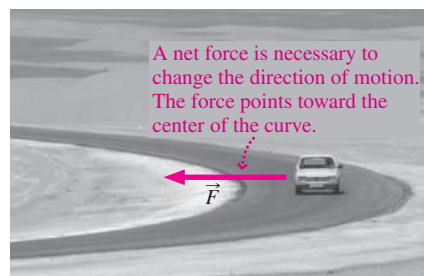


FIGURE 5.10 A car rounds a turn on the Trans-Sahara highway.

A car rounds a curve. A satellite circles Earth. A proton whirls around a giant particle accelerator. Since they're not going in straight lines, Newton tells us that a force acts on each (Fig. 5.10). We know from Section 3.6 that the acceleration of an object moving with constant speed v in a circular path of radius r has magnitude v^2/r and points toward the center of the circle. Newton's second law then tells us that the magnitude of the net force on an object of mass m in circular motion is

$$F_{\text{net}} = ma = \frac{mv^2}{r} \quad (\text{uniform circular motion}) \quad (5.1)$$

The force is in the same direction as the acceleration—toward the center of the circular path. For that reason it's sometimes called the **centripetal force**, meaning center-seeking (from the Latin *centrum*, “center,” and *petere*, “to seek”).

✓TIP Look for Real Forces

Centripetal force is *not* some new kind of force. It's just the name for *any* forces that keep an object in circular motion—which are always real, physical forces. Common examples of forces involved in circular motion include the gravitational force on a satellite, friction between tires and road, magnetic forces, tension forces, normal forces, and combinations of these and other forces.

Newton's second law describes circular motion exactly as it does any other motion: by relating net force, mass, and acceleration. Therefore, we can analyze circular motion with the same strategy we've used in other Newton's law problems.

EXAMPLE 5.5 Circular Motion: Whirling a Ball on a String

A ball of mass m whirls around in a horizontal circle at the end of a massless string of length L (Fig. 5.11). The string makes an angle θ with the horizontal. Find the ball's speed and the string tension.

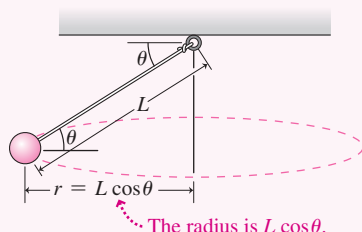


FIGURE 5.11 A ball whirling on a string.

INTERPRET This problem is similar to other Newton's law problems we've worked involving force and acceleration. The object of interest is the ball, and only two forces are acting on it: gravity and the string tension.

DEVELOP Figure 5.12 is our free-body diagram showing the two forces we've identified. The relevant equation is Newton's second law, which becomes

$$\vec{T} + \vec{F}_g = m\vec{a}$$

The ball's path is in a horizontal plane, so its acceleration is horizontal. Then two of the three vectors in our problem— $\vec{F}_g$ and $\vec{a}$ —are horizontal or vertical, so in developing our strategy, we choose a horizontal/vertical coordinate system.

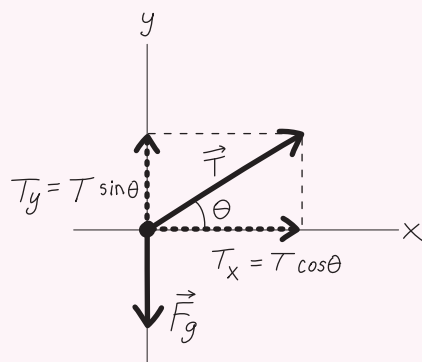


FIGURE 5.12 Our free-body diagram for the whirling ball.

✓TIP Real Forces Only!

Were you tempted to draw a third force in Fig. 5.12, perhaps pointing outward to balance the other two? *Don't!* Because the ball is accelerating, the net force is nonzero and the individual forces *do not balance*. Or maybe you were tempted to draw an inward-pointing force, mv^2/r . *Don't!* The quantity mv^2/r is *not* another force; it's just the product of mass and acceleration that appears in Newton's law (recall Fig. 4.3 and the associated tip). Students often complicate problems by introducing forces that aren't there. That makes physics seem harder than it is!

EVALUATE We now need the x - and y -components of Newton's law. Figure 5.12 shows that $F_{gy} = -F_g = -mg$ and also gives tension components in terms of trig functions. The acceleration is purely horizontal, so $a_y = 0$, and since the ball is in circular motion, $a_x = v^2/r$. But what's r ? It's the radius of the circular path and, as Fig. 5.11 shows, that's not the string length L but $L \cos \theta$. With all these expressions, the components of Newton's law become

$$x: T \cos \theta = \frac{mv^2}{L \cos \theta} \quad y: T \sin \theta - mg = 0$$

We can get the tension directly from the y equation: $T = mg/\sin \theta$. Using this result in the x equation lets us solve for the speed v :

$$v = \sqrt{\frac{TL \cos^2 \theta}{m}} = \sqrt{\frac{(mg/\sin \theta)L \cos^2 \theta}{m}} = \sqrt{\frac{gL \cos^2 \theta}{\sin \theta}}$$

ASSESS In the special case $\theta = 90^\circ$, the string hangs vertically; here $\cos \theta = 0$, so $v = 0$. There's no motion, and the string tension equals the ball's weight. But as the string becomes increasingly horizontal, both speed and tension increase. And, just as in Example 5.2, the tension becomes very great as the string approaches horizontal. Here the string tension has two jobs to do: Its vertical component supports the ball against gravity, while its horizontal component keeps the ball in its circular path. The vertical component is always equal to mg , but as the string approaches horizontal, that becomes an insignificant part of the overall tension—and thus the tension and speed grow very large. ■

EXAMPLE 5.6 Circular Motion: Engineering a Road

Roads designed for high-speed travel have banked curves to give the normal force a component toward the center of the curve. That lets cars turn without relying on friction between tires and road. At what angle should a road with 350-m curvature radius be banked for travel at 90 km/h (25 m/s)?

INTERPRET This is another example involving circular motion and Newton's second law. Although we're asked about the road, a car on the road is the object we're interested in, and we need to design the road so the car can round the curve without needing a frictional force. That means the only forces on the car are gravity and the normal force.

DEVELOP Figure 5.13 shows the physical situation, and Fig. 5.14 is our free-body diagram for the car. Newton's second law is the applicable equation, and here it becomes $\vec{n} + \vec{F}_g = m\vec{a}$. Unlike the skier of Example 5.1, the car isn't accelerating down the slope, so a horizontal/vertical coordinate system makes the most sense.

EVALUATE First we write Newton's law in components. Gravity has only a vertical component, $F_{gy} = -mg$ in our coordinate system, and Fig. 5.14 shows the two components of the normal force. The acceleration is purely horizontal and points toward the center of the curve; in our coordinate system that's the positive x -direction. Since the car is in circular motion, the magnitude of the acceleration is v^2/r . So the components of Newton's law become

$$x: n \sin \theta = \frac{mv^2}{r} \quad y: n \cos \theta - mg = 0$$

where the 0 on the right-hand side of the y equation reflects the fact that we don't want the car to accelerate in the vertical direction. Solving the y equation gives $n = mg/\cos \theta$. Then using this result in the x equation

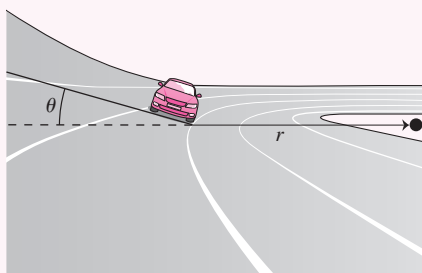


FIGURE 5.13 Car on a banked curve.

gives $mg \sin \theta / \cos \theta = mv^2/r$, or $g \tan \theta = v^2/r$. The mass canceled, which is good news because it means our banked road will work for a vehicle of any mass. Now we can solve for the banking angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right) = \tan^{-1}\left(\frac{(25 \text{ m/s})^2}{(9.8 \text{ m/s}^2)(350 \text{ m})}\right) = 10^\circ$$

ASSESS Make sense? At low speed v or large radius r , the car's motion changes gently and it doesn't take a large force to keep it on its circular path. But as v increases or r decreases, the required force increases and so does the banking angle. That's because the horizontal component of the normal force is what keeps the car in circular motion, and the steeper the angle, the greater that component. A similar thing happens when an airplane banks to turn; then the force of the air perpendicular to the wings acquires a horizontal component, and that's what turns the plane (see this chapter's opening photo and Problem 43).

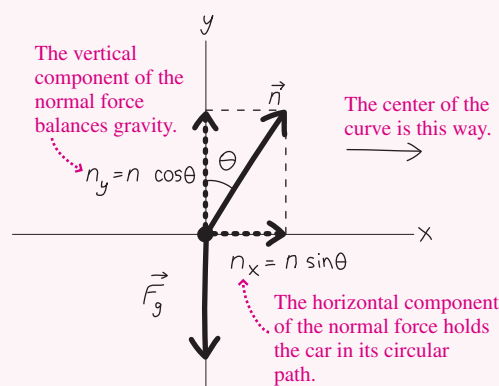


FIGURE 5.14 Our free-body diagram for the car on a banked curve.

EXAMPLE 5.7 Circular Motion: Looping the Loop

The “Great American Revolution” roller coaster at Valencia, California, includes a loop-the-loop section whose radius is 6.3 m at the top. What's the minimum speed for a roller-coaster car at the top of the loop if it's to stay on the track?

INTERPRET Again, we have circular motion described by Newton's second law. We're asked about the minimum speed for the car to stay on the track. What does it mean to stay on the track? It means there must be a normal force between car and track; otherwise, the two aren't in contact. So we can identify two forces acting on the car: gravity and the normal force from the track.

DEVELOP Figure 5.15 shows the physical situation. Things are especially simple at the top of the track, where both forces point in the same direction. We show this in our free-body diagram, Fig. 5.16 (next page). Since that common direction is downward, it makes sense to choose a coordinate system with the positive y -axis *downward*. The applicable equation is Newton's second law, and with the two forces we've identified, that becomes $\vec{n} + \vec{F}_g = m\vec{a}$.

EVALUATE With both forces in the same direction, we need only the y -component of Newton's law. With the downward direction positive, $n_y = n$ and $F_{gy} = mg$. At the top of the loop, the car is in circular

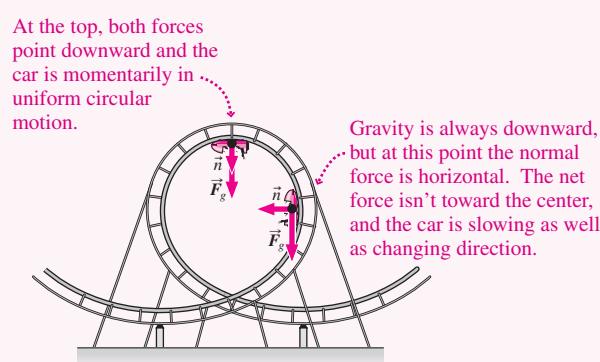


FIGURE 5.15 Forces on the roller-coaster car.

motion, so its acceleration is toward the center—downward—and has magnitude v^2/r . So $a_y = v^2/r$, and the y -component of Newton's law becomes

$$n + mg = \frac{mv^2}{r}$$

Solving for the speed gives $v = \sqrt{(nr/m) + gr}$. Now, the minimum possible speed for contact with the track occurs when n gets arbitrarily

small right at the top of the track, so we find this minimum limit by setting $n = 0$. Then the answer is

$$v_{\min} = \sqrt{gr} = \sqrt{(9.8 \text{ m/s}^2)(6.3 \text{ m})} = 7.9 \text{ m/s}$$

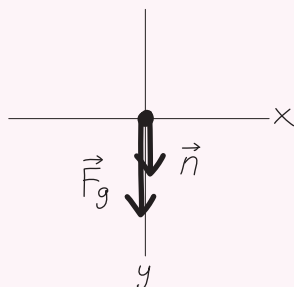


FIGURE 5.16 Our free-body diagram at the top of the loop.

✓TIP Force and Motion

We've said this before, but it's worth noting again: Force doesn't cause motion but rather *change* in motion. The direction of an object's motion need not be the direction of the force on the object. That's true in Example 5.7, where the car is moving horizontally at the top of the loop while subject to a downward force. What *is* in the same direction as the force is the *change* in motion, here embodied in the center-directed acceleration of circular motion.

ASSESS Do you see what's happening here? With the minimum speed, the normal force vanishes at the top of the loop, and gravity alone provides the force that keeps the object in its circular path. Since the motion is circular, that force must have magnitude mv^2/r . But the force of gravity alone is mg , and $v_{\min} = \sqrt{gr}$ follows directly from equating those two quantities. A car moving any slower than $v_{\min}$ would lose contact with the track and go into the parabolic trajectory of a projectile. For a car moving faster, there would be a nonzero normal force contributing to the downward acceleration at the top of the loop. In the "Great American Revolution," the actual speed at the loop's top is 9.7 m/s to provide a margin of safety. As with most problems involving gravity, the mass cancels. That's a good thing because it means the safe speed doesn't depend on the number or mass of the riders. ■

CONCEPTUAL EXAMPLE 5.1 Bad Hair Day

What's wrong with this cartoon showing riders on a loop-the-loop roller coaster (Fig. 5.17)?

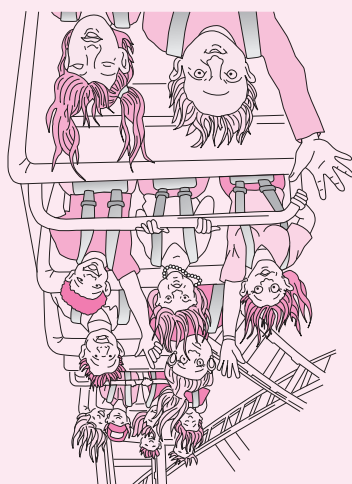


FIGURE 5.17 Conceptual Example 5.1.

EVALUATE Our objects of interest are the riders near the top of the roller coaster. We need to know the forces on them; one is obviously gravity. If the roller coaster is moving faster than Example 5.7's minimum speed—and it better be, for safety—then there are also normal forces from the seats as well as internal forces acting to accelerate parts of the riders' bodies.

Newton's law relates net force and acceleration: $\vec{F} = m\vec{a}$. This equation implies that the net force and acceleration must be in the same direction. At the top of the loop that direction is downward. Every part of the riders' bodies must therefore experience a net downward force. Again, Example 5.7 shows that the minimum force

is that of gravity alone; for safety, there must be additional downward forces.

Now focus on the riders' hair, shown hanging downward. Forces on an individual hair are gravity and tension, and our safety argument shows that they should both point in the same direction—namely, downward—to provide a downward force stronger than gravity alone. How, then, can the riders' hair hang downward? That implies an *upward* tension force, inconsistent with our argument. The artist should have drawn the hair "hanging" upward.

ASSESS Make sense? Yes: To the riders, it feels like up is down! They feel the normal force of the seat pushing down, and their hairs experience a downward-pointing tension force. Even though the riders wear seatbelts, they don't need them: If the speed exceeds Example 5.7's minimum, then they feel tightly bound to their seats. Is there some mysterious new force that pushes them against their seats and that pulls their hair up? No! Newton's second law says the net force on the riders is in the direction of their acceleration—namely, downward. And for safety, that net force must be greater than gravity. It's those additional downward forces—the normal force from the seat and the tension force in the hair—that make up feel like down.

MAKING THE CONNECTION Suppose the riders feel like they weigh 50% of what they weigh at rest on the ground. How does the roller coaster's speed compare with Example 5.7's minimum?

EVALUATE In Example 5.7, we found the speed in terms of the normal force n and other quantities: $v = \sqrt{(nr/m) + gr}$. An apparent weight 50% of normal implies that $n = mg/2$. Then $v = \sqrt{(gr/2) + gr} = \sqrt{3/2} \sqrt{gr}$. Example 5.7 shows that the minimum speed is $\sqrt{gr}$, so our result is $\sqrt{3/2} \approx 1.22$ times the minimum speed. And that 50% apparent weight the riders feel is *upward*!

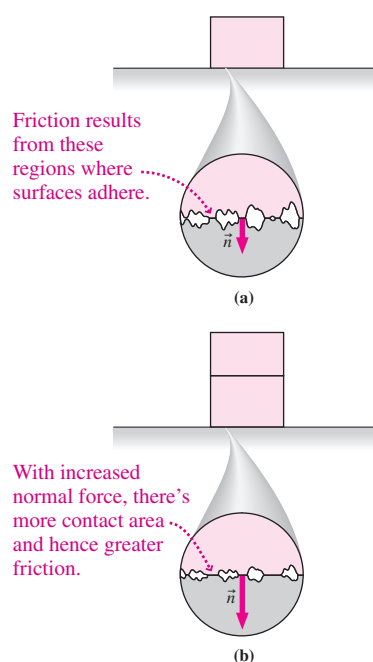


FIGURE 5.18 Friction originates in the contact between two surfaces.



PhET: Forces in 1 Dimension
PhET: Friction

GOT IT? 5.3 You whirl a bucket of water around in a vertical circle and the water doesn't fall out. A Newtonian explanation of why the water doesn't fall out is that (a) the centripetal force mv^2/r balances the gravitational force; (b) there's a centrifugal force pushing the water upward; (c) the normal force plus the gravitational force together provide the downward acceleration needed to keep the water in its circular path; or (d) an upward normal force balances gravity.

5.4 Friction

Your everyday experience of motion seems inconsistent with Newton's first law. Slide a book across the table, and it stops. Take your foot off the gas, and your car coasts to a stop. But Newton's law is correct, so these examples show that some force must be acting. That force is **friction**, a force that opposes the relative motion of two surfaces in contact.

On Earth, we can rarely ignore friction. Some 20% of the gasoline burned in your car is used to overcome friction inside the engine. Friction causes wear and tear on machinery and clothing. But friction is also useful; without it, you couldn't drive or walk.

The Nature of Friction

Friction is ultimately an electrical force between molecules in different surfaces. When two surfaces are in contact, microscopic irregularities adhere, as shown in Fig. 5.18a. At the macroscopic level, the result is a force that opposes any relative movement of the surfaces.

Experiments show that the magnitude of the frictional force depends on the normal force between surfaces in contact. Figure 5.18b shows why this makes sense: As the normal forces push the surfaces together, the actual contact area increases. There's more adherence, and this increases the frictional force.

At the microscopic level, friction is complicated. The simple equations we'll develop here provide approximate descriptions of frictional forces. Friction is important in everyday life, but it's not one of the fundamental physical interactions.

Frictional Forces

Try pushing a heavy trunk across the floor. At first nothing happens. Push harder; still nothing. Finally, as you push even harder, the trunk starts to slide—and you may notice that once it gets going, you don't have to push quite so hard. Why is that?

With the trunk at rest, microscopic contacts between trunk and floor solidify into relatively strong bonds. As you start pushing, you distort those bonds without breaking them; they respond with a force that opposes your applied force. This is the force of **static friction**, $\vec{f}_s$. As you increase the applied force, static friction increases equally, as shown in Fig. 5.19, and the trunk remains at rest. Experimentally, we find that the maximum static-friction force is proportional to the normal force between surfaces, and we write

$$f_s \leq \mu_s n \quad (\text{static friction}) \quad (5.2)$$

Here the proportionality constant μ_s (lowercase Greek mu, with the subscript s for “static”) is the **coefficient of static friction**, a quantity that depends on the two surfaces. The $\leq$ sign indicates that the force of static friction ranges from zero up to the maximum value on the right-hand side.

Eventually you push hard enough to break the bonds between trunk and floor, and the trunk begins to move; this is the point in Fig. 5.19 where the frictional force suddenly drops. Now the microscopic bonds don't have time to strengthen, so the force needed to overcome them isn't so great. In Fig. 5.19 we're assuming you then push with just enough force to overcome friction, so the trunk now moves with constant speed.

The weaker frictional force between surfaces in relative motion is the force of **kinetic friction**, $\vec{f}_k$. Again, it's proportional to the normal force between the surfaces:

$$f_k = \mu_k n \quad (\text{kinetic friction}) \quad (5.3)$$

where now the proportionality constant is μ_k , the **coefficient of kinetic friction**. Because kinetic friction is weaker, the coefficient of kinetic friction for a given pair of

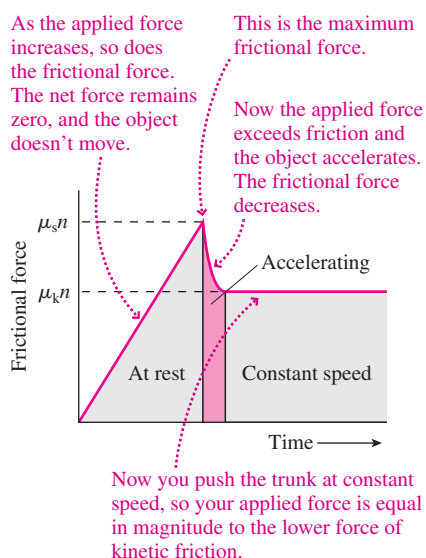


FIGURE 5.19 Behavior of frictional forces.

surfaces is less than the coefficient of static friction. Cross-country skiers exploit that fact by using waxes that provide a high coefficient of static friction for pushing against the snow and for climbing hills, while the lower kinetic friction permits effortless gliding.

Equations 5.2 and 5.3 give only the magnitudes of the frictional forces. The direction of the frictional force is parallel to the two surfaces, in the direction that opposes any applied force (Fig. 5.20a) or the surfaces' relative motion (Fig. 5.20b).

Since they describe proportionality between the magnitudes of two forces, the coefficients of friction are dimensionless. Typical values of μ_k range from less than 0.01 for smooth or lubricated surfaces to about 1.5 for very rough ones. Rubber on dry concrete—vital in driving an automobile—has μ_k about 0.8 and μ_s can exceed 1. A waxed ski on dry snow has $\mu_k \approx 0.04$, while the synovial fluid that lubricates your body's joints reduces μ_k to a low 0.003.

If you push a moving object with a force equal to the opposing force of kinetic friction, then the net force is zero and, according to Newton, the object moves at constant speed. Since friction is nearly always present, but not as obvious as the push of a hand or the pull of a rope, you can see why it's so easy to believe that force is needed to make things move—rather than, as Newton recognized, to make them accelerate.

We emphasize that the equations describing friction are empirical expressions that approximate the effects of complicated but more basic interactions at the microscopic level. Our friction equations have neither the precision nor the fundamental character of Newton's laws.

Applications of Friction

Static friction plays a vital role in everyday activities such as walking and driving. As you walk, your foot contacting the ground is momentarily at rest, pushing back against the ground. By Newton's third law, the ground pushes forward, accelerating you forward (Fig. 5.21). Both forces of the third-law pair arise from static friction between foot and ground. On a frictionless surface, walking is impossible.

Similarly, the tires of an accelerating car push back on the road. If they aren't slipping, the bottom of each tire is momentarily at rest (more on this in Chapter 10). Therefore the force is static friction. The third law then requires a frictional force of the road pushing forward on the tires; that's what accelerates the car. Braking is the opposite: The tires push forward, and the road pushes back to decelerate the car (Fig. 5.22). The brakes affect only the wheels; it's friction between tires and road that stops the car. You know this if you've applied your brakes on an icy road!

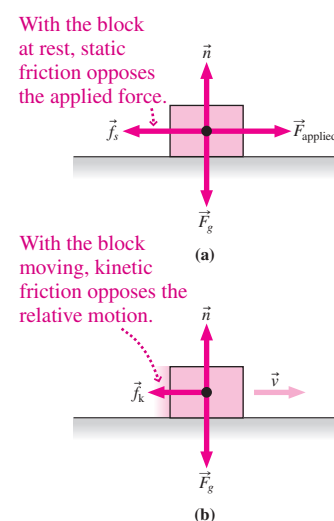


FIGURE 5.20 Direction of frictional forces.

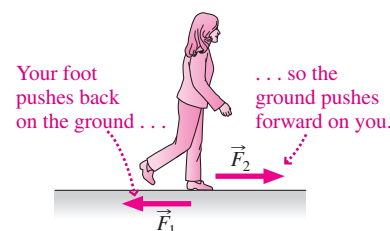


FIGURE 5.21 Walking.

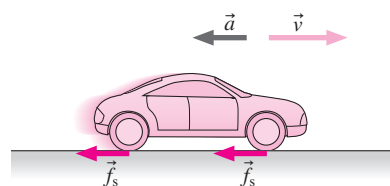


FIGURE 5.22 Friction stops the car.

EXAMPLE 5.8 Frictional Forces: Stopping a Car

The kinetic- and static-friction coefficients between a car's tires and a dry road are 0.61 and 0.89, respectively. The car is initially traveling at 90 km/h (25 m/s) on a level road. Determine (a) the minimum stopping distance, which occurs when the brakes are applied so that the wheels keep rolling as they slow and therefore static friction applies, and (b) the stopping distance with the wheels fully locked and the car skidding.

INTERPRET Since we're asked about the stopping distance, this is ultimately a question about accelerated motion in one dimension—the subject of Chapter 2. But here friction causes that acceleration, so we have a Newton's law problem. The car is the object of interest, and we identify three forces: gravity, the normal force, and friction.

DEVELOP Figure 5.23 is our free-body diagram. We have a two-part problem here: First, we need to use Newton's second law to find the acceleration, and then we can use Equation 2.11, $v^2 = v_0^2 + 2a\Delta x$, to relate distance and acceleration. With the three forces acting on the car, Newton's law becomes $\vec{F}_g + \vec{n} + \vec{f}_f = m\vec{a}$. A horizontal/vertical coordinate system is most appropriate for the components of Newton's law.

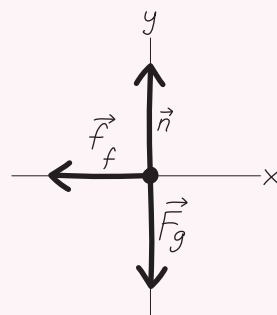


FIGURE 5.23 Our free-body diagram for the braking car.

EVALUATE The only horizontal force is friction, which points in the $-x$ -direction and has magnitude μn , where μ can be either the kinetic- or the static-friction coefficient. The normal force and gravity act in the vertical direction, so the component equations are

$$x: -\mu n = ma_x \quad y: -mg + n = 0 \quad (\text{continued})$$

Solving the y equation for n and substituting in the x equation gives the acceleration: $a_x = -\mu g$. We then use this result in Equation 2.11 and solve for the stopping distance Δx . With final speed $v = 0$, this gives

$$\Delta x = \frac{v_0^2}{-2a_x} = \frac{v_0^2}{2\mu g}$$

Using the numbers given, we get (a) $\Delta x = 36$ m for the minimum stopping distance (no skid; static friction) and (b) 52 m for the car skidding with its wheels locked (kinetic friction). The difference could well be enough to prevent an accident.

ASSESS Our result $a_x = -\mu g$ shows that a higher friction coefficient leads to a larger acceleration; this makes sense because friction is what causes the acceleration. What happened to the car's mass? A more massive car requires a larger frictional stopping force for the same acceleration—but friction depends on the normal force, and the latter is greater in proportion to the car's mass. Thus the stopping distance doesn't depend on mass.

This example shows that stopping distance increases as the *square* of the speed. That's one reason high speeds are dangerous: Doubling your speed quadruples your stopping distance! ■

EXAMPLE 5.9 Frictional Forces: Steering

A level road makes a 90° turn with radius 73 m. What's the maximum speed for a car to negotiate this turn when the road is dry ($\mu_s = 0.88$) and when the road is snow covered ($\mu_s = 0.21$)?

INTERPRET This example is similar to Example 5.8, but now the frictional force acts perpendicular to the car's motion, keeping it in a circular path. Because the car isn't moving in the direction of the force, we're dealing with *static* friction. The car is the object of interest, and again the forces are gravity, the normal force, and friction.

DEVELOP Figure 5.24 is our free-body diagram. Newton's law is the applicable equation, and we're dealing with the acceleration v^2/r

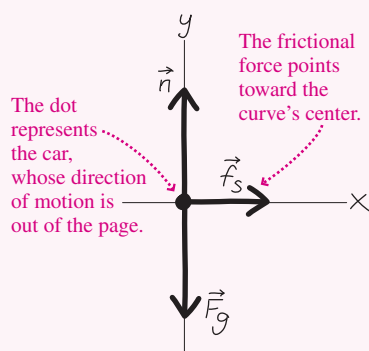


FIGURE 5.24 Our free-body diagram for the cornering car.

that occurs in circular motion. With the three forces acting on the car, Newton's law is $\vec{F}_g + \vec{n} + \vec{f}_s = m\vec{a}$. A horizontal/vertical coordinate system is most appropriate, and now it's most convenient to take the x -axis in the direction of the acceleration—namely, toward the center of the curve.

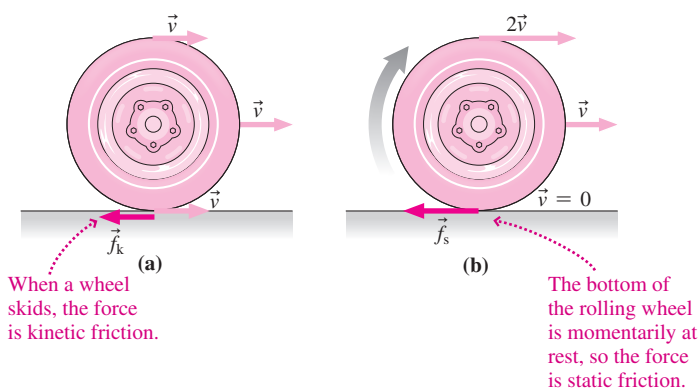
EVALUATE Again, the only horizontal force is friction, with magnitude $\mu_s n$. Here it points in the positive x -direction, as does the acceleration of magnitude v^2/r . So the x -component of Newton's law is $\mu_s n = mv^2/r$. There's no vertical acceleration, so the y -component is $-mg + n = 0$. Solving for n and using the result in the x equation give $\mu_s mg = mv^2/r$. Again the mass cancels, and we solve for v to get

$$v = \sqrt{\mu_s g r}$$

Putting in the numbers, we get $v = 25$ m/s (90 km/h) for the dry road and 12 m/s (44 km/h) for the snowy road. Exceed these speeds, and your car inevitably moves in a path with a larger radius—and that means going off the road!

ASSESS Once again, it makes sense that the car's mass doesn't matter. A more massive car needs a larger frictional force, and it gets what it needs because its larger mass results in a larger normal force. The safe speed increases with the curve radius r , and that, too, makes sense: A larger radius means a gentler turn, with less acceleration at a given speed. So less frictional force is needed. ■

APPLICATION Antilock Brakes



Today's cars have computer-controlled antilock braking systems (ABS). These systems exploit the fact that static friction is greater than kinetic friction. Slam on the brakes of a non-ABS car and the wheels lock and skid without turning. The force between tires and road is then *kinetic* friction (part a in the figure). But if you pump the brakes to keep the wheels from skidding, then it's the greater force of *static* friction (part b).

ABS improves on this brake-pumping strategy with a computer that independently controls the brakes at each wheel, keeping each just on the verge of slipping. Drivers of ABS cars should slam the brakes hard in an emergency; the ensuing clatter indicates the ABS is working.

Although ABS can reduce the stopping distance, its real significance is in preventing vehicles from skidding out of control as can happen when you apply the brakes with some wheels on ice and others on pavement. Increasingly, today's cars incorporate their computer-controlled brakes into sophisticated systems that enhance stability during emergency maneuvers.

EXAMPLE 5.10 Friction on a Slope: Avalanche!

A storm dumps new snow on a ski slope. The coefficient of static friction between the new snow and the older snow underneath is 0.46. What's the maximum slope angle to which the new snow can adhere?

INTERPRET The problem asks about an angle, but it's friction that holds the new snow to the old, so this is really a problem about the maximum possible static friction. We aren't given an object, but we can model the new snow as a slab of mass m resting on a slope of unknown angle θ . The forces on the slab are gravity, the normal force, and static friction $\vec{f}_s$.

DEVELOP Figure 5.25 shows the model, and Fig. 5.26 is our free-body diagram. Newton's second law is the applicable equation, here with $\vec{a} = \vec{0}$, giving $\vec{F}_g + \vec{n} + \vec{f}_s = \vec{0}$. We also need the maximum static-friction force, given in Equation 5.2, $f_{s\max} = \mu_s n$. As in Example 5.1, a tilted coordinate system is simplest and is shown in Fig. 5.26.

EVALUATE With the positive x -direction downslope, Fig. 5.26 shows that the x -component of gravity is $F_g \sin \theta = mg \sin \theta$, while the frictional force acts upslope ($-x$ -direction) and has maximum magnitude $\mu_s n$; therefore, $f_{sx} = -\mu_s n$. So the x -component of Newton's law is $mg \sin \theta - \mu_s n = 0$. We can read the y -component

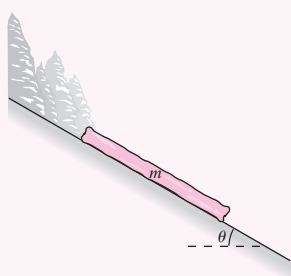


FIGURE 5.25 A layer of snow, modeled as a slab on a sloping surface.

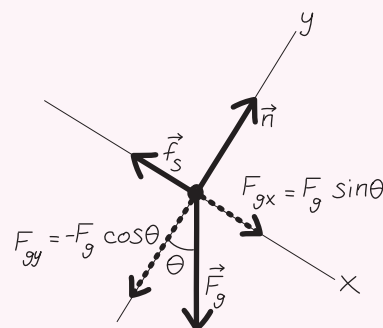


FIGURE 5.26 Our free-body diagram for the snow slab.

from Fig. 5.26: $-mg \cos \theta + n = 0$. Solving the y equation gives $n = mg \cos \theta$. Using this result in the x equation then yields $mg \sin \theta - \mu_s mg \cos \theta = 0$. Both m and g cancel, and we have $\sin \theta = \mu_s \cos \theta$ or, since $\tan \theta = \sin \theta / \cos \theta$,

$$\tan \theta = \mu_s$$

For the numbers given in this example, the result becomes $\theta = \tan^{-1} \mu_s = \tan^{-1}(0.46) = 25^\circ$.

ASSESS Make sense? Sure: The steeper the slope, the greater the friction needed to keep the snow from sliding. Two effects are at work here: First, as the slope steepens, so does the component of gravity along the slope. Second, as the slope steepens, the normal force gets smaller, and that reduces the frictional force for a given friction coefficient. Note here that the normal force is not simply the weight mg of the snow; again, that's because of the sloping surface.

The real avalanche danger comes at angles slightly smaller than our answer $\tan \theta = \mu_s$, where a thick snowpack can build up. Changes in the snow's composition with temperature may decrease the friction coefficient and unleash an avalanche.

EXAMPLE 5.11 Friction: Dragging a Trunk

You drag a trunk of mass m across a level floor using a massless rope that makes an angle θ with the horizontal (Fig. 5.27). Given a kinetic-friction coefficient μ_k , what rope tension is required to move the trunk at constant speed?

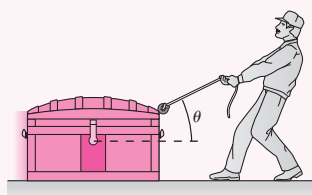


FIGURE 5.27 Dragging a trunk.

INTERPRET Even though the trunk is moving, it isn't accelerating, so here's another problem involving Newton's law with zero acceleration. The object is the trunk, and now four forces act: gravity, the normal force, friction, and the rope tension.

DEVELOP Figure 5.28 is our free-body diagram showing all four forces acting on the trunk. The relevant equation is Newton's law.

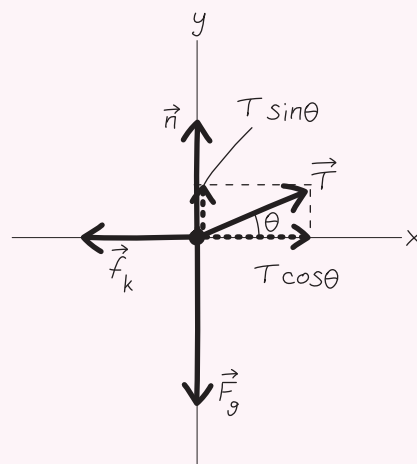


FIGURE 5.28 Our free-body diagram for the trunk.

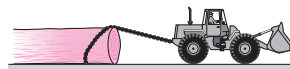
With no acceleration, it reads $\vec{F}_g + \vec{n} + \vec{f}_k + \vec{T} = \vec{0}$, with the magnitude of kinetic friction given by $f_k = \mu_k n$. All vectors except the tension force are horizontal or vertical, so the most sensible coordinate system has horizontal and vertical axes.

EVALUATE From Fig. 5.28, we can write the components of Newton's law: $T \cos \theta - \mu_k n = 0$ in the x -direction and $T \sin \theta - mg + n = 0$ in the y -direction. This time the unknown T appears in both equations. Solving the y equation for n gives $n = mg - T \sin \theta$. Putting this n in the x equation then yields $T \cos \theta - \mu_k (mg - T \sin \theta) = 0$. Factoring terms involving T and solving, we arrive at the answer:

$$T = \frac{\mu_k mg}{\cos \theta + \mu_k \sin \theta}$$

ASSESS Make sense? Without friction, we wouldn't need any force to move the trunk at constant speed, and indeed our expression gives $T = 0$ in this case. On the other hand, if there is friction but $\theta = 0$, then $\sin \theta = 0$ and we get $T = \mu_k mg$. In this case the normal force equals the weight, so the frictional force is $\mu_k mg$. Since the frictional force is horizontal and with $\theta = 0$ we're pulling horizontally, this is also the magnitude of the tension force. At intermediate angles, two effects come into play: First, the upward component of tension helps support the trunk's weight, and that means less normal force is needed. With less normal force, there's less friction—making the trunk easier to pull. But as the angle increases, less of the tension is horizontal, and that means a larger tension force is needed to overcome friction. In combination, these two effects mean there's an optimum angle at which the rope tension is a minimum. Problem 68 explores this point further. ■

GOT IT? 5.4 The figure shows a logging vehicle pulling a redwood log. Is the frictional force in this case (a) less than, (b) equal to, or (c) greater than the weight multiplied by the coefficient of friction?



5.5 Drag Forces

Friction isn't the only "hidden" force that robs objects of their motion and obscures Newton's first law. Objects moving through fluids like water or air experience **drag forces** that oppose the relative motion of object and fluid. Ultimately, drag results from collisions between fluid molecules and the object. The drag force depends on several factors, including fluid density and the object's cross-sectional area and speed.

Terminal Speed

When an object falls from rest, its speed is initially low and so is the velocity-dependent drag force. It therefore accelerates downward with nearly the gravitational acceleration g . But as the object gains speed, the drag force increases—until eventually the drag force and gravity have equal magnitudes. At that point the net force on the object is zero, and it falls with constant speed, called its **terminal speed**.

Because the drag force depends on an object's area and the gravitational force depends on its mass, the terminal speed is lower for lighter objects with large areas. A parachute, for example, is designed specifically to have a large surface area that results, typically, in a terminal speed around 5 m/s. A ping-pong ball and a golf ball have about the same size and therefore the same area, but the ping-pong ball's much lower mass leads to a terminal speed of about 10 m/s compared with the golf ball's 50 m/s. For an irregularly shaped object, the drag and thus the terminal speed depend on how large a surface area the object presents to the air. Skydivers exploit this effect to vary their rates of fall.

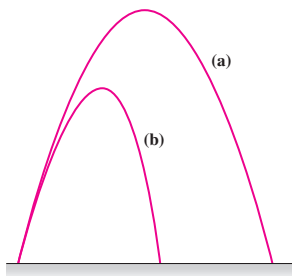


FIGURE 5.29 Projectile trajectories (a) without air resistance and (b) with substantial air resistance. Note that (b) not only achieves less height and range but that the trajectory is no longer a symmetric parabola.

Drag and Projectile Motion

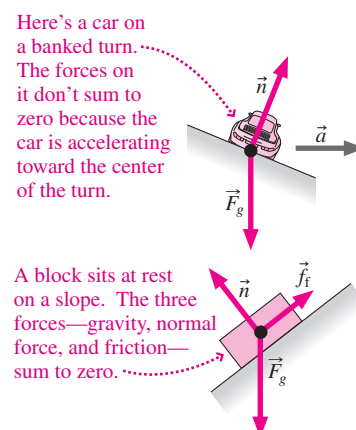
In Chapter 3, we consistently neglected air resistance—the drag force of air—in projectile motion. Determining drag effects on projectiles is not trivial and usually requires computer calculations. The net effect, though, is that air resistance decreases the range of a projectile (Fig. 5.29). Despite the physicist's need for computer calculations, others—especially athletes—have a feel for drag forces that lets them play their sports by judging correctly the trajectory of a projectile under the influence of drag forces. You can explore drag forces further in Problems 70 and 71.

CHAPTER 5 SUMMARY

Big Idea

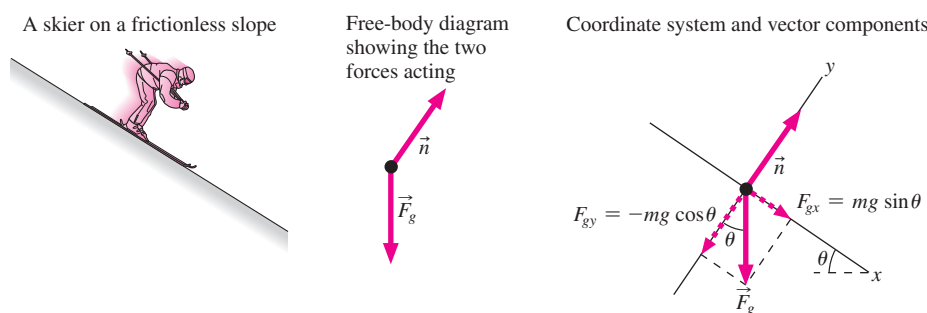
The big idea here is the same as in Chapter 4—namely, that Newton’s laws are a universal description of motion, in which force causes not motion itself but change in motion. Here we focus on Newton’s second law, extended to the richer and more complex examples of motion in two dimensions. To use Newton’s law, we now sum forces that may point in different directions, but the result is the same: The net force determines an object’s acceleration.

Common forces include gravity, the normal force from surfaces, tension forces, and a force introduced here: friction. Important examples are those where an object is accelerating, including in circular motion, and those where there’s no acceleration and therefore the net force is zero.



Solving Problems with Newton’s Laws

The problem-solving strategy in this chapter is exactly the same as in Chapter 4, except that in two dimensions the choice of coordinate system and the division of forces into components become crucial steps. You usually need both component equations to solve a problem.



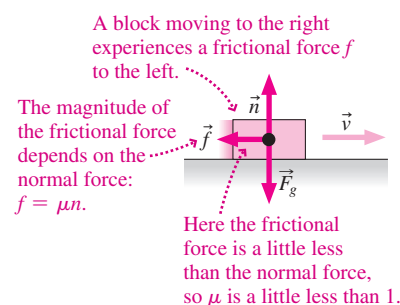
$$\vec{F} = m\vec{a} \rightarrow \vec{n} + \vec{F}_g = m\vec{a} \rightarrow \begin{cases} n_x + F_{gx} = ma_x \\ n_y + F_{gy} = ma_y \end{cases} \rightarrow \begin{cases} mg \sin \theta = ma_x \\ n - mg \cos \theta = 0 \end{cases}$$

Key Concepts and Equations

Newton’s second law, $\vec{F}_{\text{net}} = m\vec{a}$, is the key equation in this chapter. It’s crucial to remember that it’s a *vector* equation, representing a pair of scalar equations for its two components in two dimensions.

Applications

Friction acts between surfaces to oppose their relative motion, and its strength depends on the normal force $\vec{n}$ acting perpendicular to them. When surfaces aren’t actually in relative motion, the force is **static friction**, whose value ranges from zero to a maximum value $\mu_s n$ as needed to oppose any applied force: $f_s \leq \mu_s n$. Here μ_s is the **coefficient of static friction**, which depends on the nature of the two surfaces. For surfaces in relative motion, the force is **kinetic friction**, given by $f_k = \mu_k n$, where the **coefficient of kinetic friction** is less than the coefficient of static friction.





For homework assigned on MasteringPhysics, go to www.masteringphysics.com

BIO Biology and/or medicine-related problems **DATA** Data problems **ENV** Environmental problems **CH** Challenge problems **COMP** Computer problems

For Thought and Discussion

- Compare the net force on a heavy trunk when it's (a) at rest on the floor; (b) being slid across the floor at constant speed; (c) being pulled upward in an elevator whose cable tension equals the combined weight of the elevator and trunk; and (d) sliding down a frictionless ramp.
- The force of static friction acts only between surfaces at rest. Yet that force is essential in walking and in accelerating or braking a car. Explain.
- A jet plane flies at constant speed in a vertical circular loop. At what point in the loop does the seat exert the greatest force on the pilot? The least force?
- In cross-country skiing, skis should easily glide forward but should remain at rest when the skier pushes back against the snow. What frictional properties should the ski wax have to achieve this goal?
- Why do airplanes bank when turning?
- Why is it easier for a child to stand nearer the inside of a rotating merry-go-round?
- Gravity pulls a satellite toward Earth's center. So why doesn't the satellite actually fall to Earth?
- Explain why a car with ABS brakes can have a shorter stopping distance.
- A fishing line has a 20-lb breaking strength. Is it possible to break the line while reeling in a 15-lb fish? Explain.
- Two blocks rest on slopes of unequal angles, connected by a rope passing over a pulley (Fig. 5.30). If the blocks have equal masses, will they remain at rest? Why? Neglect friction.

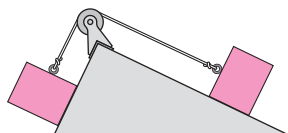


FIGURE 5.30 For Thought and Discussion 10; Exercises 21 and 22

- You're on a plane undergoing a banked turn, so steep that out the window you see the ground below. Yet your pretzels stay put on the seatback tray, rather than sliding downward. Why?
- A backcountry skier weighing 700 N skis down a steep slope, unknowingly crossing a snow bridge that spans a deep, hidden crevasse. If the bridge can support 580 N—meaning that's the maximum normal force it can sustain without collapsing—is there any chance the mountaineer can cross safely? Explain.

Exercises and Problems

Exercises

Section 5.1 Using Newton's Second Law

- Two forces, both in the x - y plane, act on a 3.25-kg mass that accelerates at 5.48 m/s^2 in a direction 38.0° counterclockwise from the x -axis. One force has magnitude 8.63 N and points in the $+x$ -direction. Find the other force.
- Two forces act on a 3.1-kg mass that undergoes acceleration $\vec{a} = 0.91\hat{i} - 0.27\hat{j} \text{ m/s}^2$. If one force is $-1.2\hat{i} - 2.5\hat{j} \text{ N}$, what's the other?

- At what angle should you tilt an air table to simulate free fall at the surface of Mars, where $g = 3.71 \text{ m/s}^2$?
- A skier starts from rest at the top of a 24° slope 1.3 km long. Neglecting friction, how long does it take to reach the bottom?
- A tow truck is connected to a 1400-kg car by a cable that makes a 25° angle to the horizontal. If the truck accelerates at 0.57 m/s^2 , what's the magnitude of the cable tension? Neglect friction and the cable's mass.
- BIO** Studies of gymnasts show that their high rate of injuries to the Achilles tendon is due to tensions in the tendon that typically reach 10 times body weight. That force is provided by a pair of muscles, each exerting a force at 25° to the vertical, with their horizontal components opposite. For a 55-kg gymnast, find the force in each of these muscles.

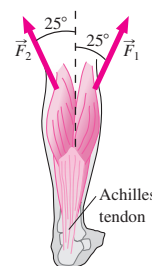


FIGURE 5.31

- Find the minimum slope angle for which the skier in Question 12 can safely traverse the snow bridge.

Section 5.2 Multiple Objects

- Your 12-kg baby sister pulls on the bottom of the tablecloth with all her weight. On the table, 60 cm from the edge, is a 6.8-kg roast turkey. (a) What's the turkey's acceleration? (b) From the time your sister starts pulling, how long do you have to intervene before the turkey goes over the edge? Neglect friction.
- If the left-hand slope in Fig. 5.30 makes a 60° angle with the horizontal, and the right-hand slope makes a 20° angle, how should the masses compare if the objects are not to slide along the frictionless slopes?
- Suppose the angles shown in Fig. 5.30 are 60° and 20° . If the left-hand mass is 2.1 kg, what should the right-hand mass be so that it accelerates (a) downslope at 0.64 m/s^2 and (b) upslope at 0.76 m/s^2 ?
- Two unfortunate climbers, roped together, are sliding freely down an icy mountainside. The upper climber (mass 75 kg) is on a slope at 12° to the horizontal, but the lower climber (mass 63 kg) has gone over the edge to a steeper slope at 38° . (a) Assuming frictionless ice and a massless rope, what's the acceleration of the pair? (b) The upper climber manages to stop the slide with an ice ax. After the climbers have come to a complete stop, what force must the ax exert against the ice?

Section 5.3 Circular Motion

- Suppose the Moon were held in its orbit not by gravity but by tension in a massless cable. Estimate the magnitude of the cable tension. (*Hint:* See Appendix E.)
- Show that the force needed to keep a mass m in a circular path of radius r with period T is $4\pi^2 mr/T^2$.

26. A 940-g rock is whirled in a horizontal circle at the end of a 1.30-m-long string. (a) If the breaking strength of the string is 120 N, what's the minimum angle the string can make with the horizontal? (b) At this minimum angle, what's the rock's speed?
27. You're investigating a subway accident in which a train derailed while rounding an unbanked curve of radius 150 m, and you're asked to estimate whether the train exceeded the 35-km/h speed limit for this curve. You interview a passenger who had been standing and holding onto a strap; she noticed that an unused strap was hanging at about a 15° angle to the vertical just before the accident. What do you conclude?
28. A tetherball on a 1.55-m rope is struck so that it goes into circular motion in a horizontal plane, with the rope making a 12.0° angle to the horizontal. What's the ball's speed?
29. An airplane goes into a turn 3.6 km in radius. If the banking angle required is 28° from the horizontal, what's the plane's speed?

Section 5.4 Friction

30. Movers slide a 73-kg file cabinet along a floor where the coefficient of kinetic friction is 0.81. What's the frictional force on the cabinet?
31. A hockey puck is given an initial speed of 14 m/s. If it comes to rest in 56 m, what's the coefficient of kinetic friction?
32. Starting from rest, a skier slides 100 m down a 28° slope. How much longer does the run take if the coefficient of kinetic friction is 0.17 instead of 0?
33. A car moving at 40 km/h negotiates a 130-m-radius banked turn designed for 60 km/h. What coefficient of friction is needed to keep the car on the road?

Problems

34. Repeat Example 5.1, this time using a horizontal/vertical coordinate system.
35. A block is launched with initial speed 2.2 m/s up a 35° frictionless ramp. How far up the ramp does it slide?
36. In the process of mitosis (cell division), two motor proteins pull on a spindle pole, each with a 7.3-pN force. The two force vectors make a 65° angle. What's the magnitude of the force the two motor proteins exert on the spindle pole?
37. A 14.6-kg monkey hangs from the middle of a massless rope, each half of which makes an 11.0° angle with the horizontal. What's the rope tension? Compare with the monkey's weight.
38. A camper hangs a 26-kg pack between two trees using separate ropes of different lengths, as shown in Fig. 5.32. Find the tension in each rope.

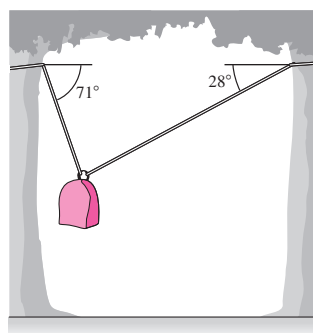


FIGURE 5.32 Problem 38

39. A mass m_1 undergoes circular motion of radius R on a horizontal frictionless table, connected by a massless string through a hole in the table to a second mass m_2 (Fig. 5.33). If m_2 is stationary,

find expressions for (a) the string tension and (b) the period of the circular motion.

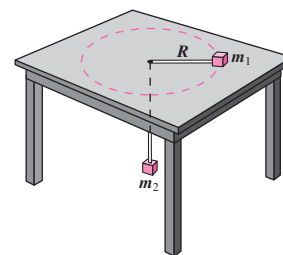


FIGURE 5.33 Problem 39

40. Patients with severe leg breaks are often placed in *traction*, with an external force countering muscles that would pull too hard on the broken bones. In the arrangement shown in Fig. 5.34, the mass m is 4.8 kg, and the pulleys can be considered massless and frictionless. Find the horizontal traction force applied to the leg.

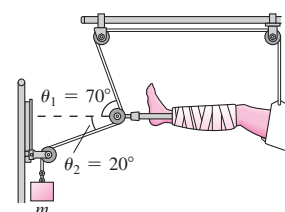


FIGURE 5.34 Problem 40

41. Riders on the "Great American Revolution" loop-the-loop roller coaster of Example 5.7 wear seatbelts as the roller coaster negotiates its 6.3-m-radius loop at 9.7 m/s. At the top of the loop, what are the magnitude and direction of the force exerted on a 60-kg rider (a) by the roller-coaster seat and (b) by the seatbelt? (c) What would happen if the rider unbuckled at this point?
42. A 45-kg skater rounds a 5.0-m-radius turn at 6.3 m/s. (a) What are the horizontal and vertical components of the force the ice exerts on her skate blades? (b) At what angle can she lean without falling over?
43. When a plane turns, it banks as shown in Fig. 5.35 to give the wings' lifting force $\vec{F}_w$ a horizontal component that turns the plane. If a plane is flying level at 950 km/h and the banking angle θ is not to exceed 40° , what's the minimum curvature radius for the turn?

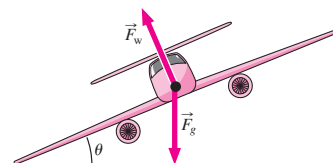


FIGURE 5.35 Problem 43

44. You whirl a bucket of water in a vertical circle of radius 85 cm. What's the minimum speed that will keep the water from falling out?
45. A child sleds down an 8.5° slope at constant speed. What's the frictional coefficient between slope and sled?
46. The handle of a 22-kg lawnmower makes a 35° angle with the horizontal. If the coefficient of friction between lawnmower and ground is 0.68, what magnitude of force, applied in the direction of the handle, is required to push the mower at constant velocity? Compare with the mower's weight.
47. Repeat Example 5.4, now assuming that the coefficient of kinetic friction between rock and ice is 0.057.

48. A bat crashes into the vertical front of an accelerating subway train. If the frictional coefficient between bat and train is 0.86, what's the minimum acceleration of the train that will allow the bat to remain in place?
49. The coefficient of static friction between steel train wheels and steel rails is 0.58. The engineer of a train moving at 140 km/h spots a stalled car on the tracks 150 m ahead. If he applies the brakes so the wheels don't slip, will the train stop in time?
50. A bug crawls outward from the center of a CD spinning at 200 revolutions per minute. The coefficient of static friction between the bug's sticky feet and the disc surface is 1.2. How far does the bug get from the center before slipping?
51. **CH** A 310-g paperback book rests on a 1.2-kg textbook. A force is applied to the textbook, and the two books accelerate together from rest to 96 cm/s in 0.42 s. The textbook is then brought to a stop in 0.33 s, during which time the paperback slides off. Within what range does the coefficient of static friction between the two books lie?
52. Children sled down a 41-m-long hill inclined at 25° . At the bottom, the slope levels out. If the coefficient of friction is 0.12, how far do the children slide on the level ground?
53. In a typical front-wheel-drive car, 70% of the car's weight rides on the front wheels. If the coefficient of friction between tires and road is 0.61, what's the car's maximum acceleration?
54. A police officer investigating an accident estimates that a moving car hit a stationary car at 25 km/h. Before the collision, the car left 47-m-long skid marks as it braked. The officer determines that the coefficient of kinetic friction was 0.71. What was the initial speed of the moving car?
55. A slide inclined at 35° takes bathers into a swimming pool. With water sprayed onto the slide to make it essentially frictionless, a bather spends only one-third as much time on the slide as when it's dry. What's the coefficient of friction on the dry slide?
56. You try to move a heavy trunk, pushing down and forward at an angle of 50° below the horizontal. Show that, no matter how hard you push, it's impossible to budge the trunk if the coefficient of static friction exceeds 0.84.
57. A block is shoved up a 22° slope with an initial speed of 1.4 m/s. The coefficient of kinetic friction is 0.70. (a) How far up the slope will the block get? (b) Once stopped, will it slide back down?
58. At the end of a factory production line, boxes start from rest and slide down a 30° ramp 5.4 m long. If the slide can take no more than 3.3 s, what's the maximum allowed frictional coefficient?
59. You're in traffic court, arguing against a speeding citation. You entered a 210-m-radius banked turn designed for 80 km/h, which was also the posted speed limit. The road was icy, yet you stayed in your lane, so you argue that you must have been going at the design speed. But police measurements show there was a frictional coefficient $\mu = 0.15$ between tires and road. Is it possible you were speeding, and if so by how much?
60. A space station is in the shape of a hollow ring, 450 m in diameter (Fig. 5.36). At how many revolutions per minute should it rotate in order to simulate Earth's gravity—that is, so the normal force on an astronaut at the outer edge would equal the astronaut's weight on Earth?

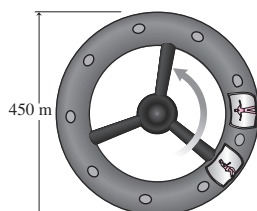


FIGURE 5.36 Problem 60

61. In a loop-the-loop roller coaster, show that a car moving too slowly would leave the track at an angle ϕ given by $\cos \phi = v^2/rg$, where ϕ is the angle made by a vertical line through the center of the circular track and a line from the center to the point where the car leaves the track.
62. **CH** Find an expression for the minimum frictional coefficient needed to keep a car with speed v on a banked turn of radius R designed for speed v_0 .
63. An astronaut is training in an earthbound centrifuge that consists of a small chamber whirled horizontally at the end of a 5.1-m-long shaft. The astronaut places a notebook on the vertical wall of the chamber and it stays in place. If the coefficient of static friction is 0.62, what's the minimum rate at which the centrifuge must be revolving?
64. You stand on a spring scale at the north pole and again at the equator. Which scale reading will be lower, and by what percentage will it be lower than the higher reading? Assume g has the same value at pole and equator.
65. Driving in thick fog on a horizontal road, you spot a tractor-trailer truck jackknifed across the road. To avert a collision, you could brake to a stop or swerve in a circular arc, as suggested in Fig. 5.37. Which option offers the greater margin of safety? Assume that there is the same coefficient of static friction in both cases, and that you maintain constant speed if you swerve.

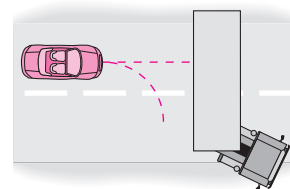


FIGURE 5.37 Problem 65

66. **CH** A block is projected up an incline at angle θ . It returns to its initial position with half its initial speed. Show that the coefficient of kinetic friction is $\mu_k = \frac{3}{5} \tan \theta$.
67. A 2.1-kg mass is connected to a spring with spring constant $k = 150 \text{ N/m}$ and unstretched length 18 cm. The two are mounted on a frictionless air table, with the free end of the spring attached to a frictionless pivot. The mass is set into circular motion at 1.4 m/s. Find the radius of its path.
68. Take $\mu_k = 0.75$ in Example 5.11, and plot the tension force in units of the trunk's weight, as a function of the rope angle θ (that is, plot T/mg versus θ). Use your plot to determine (a) the minimum tension necessary to move the trunk and (b) the angle at which this minimum tension should be applied.
69. **CH** Repeat the preceding problem for an arbitrary value of μ_k , by using calculus to find the minimum force needed to move the trunk with constant speed.
70. **CH** Moving through a liquid, an object of mass m experiences a resistive drag force proportional to its velocity, $F_{\text{drag}} = -bv$, where b is a constant. (a) Find an expression for the object's speed as a function of time, when it starts from rest and falls vertically through the liquid. (b) Show that it reaches a terminal velocity mg/b .
71. **CH** Suppose the object in Problem 70 had an initial velocity in the horizontal direction equal to the terminal speed, $v_{x0} = mg/b$. Show that the horizontal distance it can go is limited to $x_{\text{max}} = mv_{x0}/b$, and find an expression for its trajectory (y as a function of x).
72. **CH** A block is launched with speed v_0 up a slope making an angle θ with the horizontal; the coefficient of kinetic friction is μ_k . (a) Find an expression for the distance d the block travels along the slope. (b) Use calculus to determine the angle that minimizes d .

73. A florist asks you to make a window display with two hanging pots as shown in Fig. 5.38. The florist is adamant that the strings be as invisible as possible, so you decide to use fishing line but want to use the thinnest line you can. Will fishing line that can withstand 100 N of tension work?

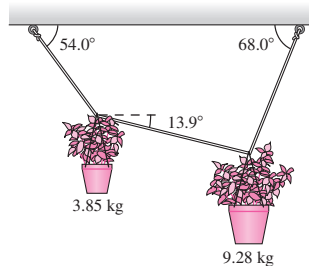


FIGURE 5.38 Problem 73

74. You're at the state fair. A sideshow barker claims that the star of the show can throw a 7.3-kg Olympic-style hammer "faster than a speeding bullet." You recall that bullets travel at several hundreds of meters per second. The burly hammer thrower whirls the hammer in a circle that you estimate to be 2.4 m in diameter. You guess the chain holding the hammer makes an angle of 10° with the horizontal. When the hammer flies off, is it really moving faster than a bullet?
75. One of the limiting factors in high-performance aircraft is the acceleration to which the pilot can be subjected without blacking out; it's measured in "gees," or multiples of the gravitational acceleration. The F-22 Raptor fighter can achieve Mach 1.8 (1.8 times the speed of sound, which is about 340 m/s). Suppose a pilot dives in a circle and pulls up. If the pilot can't exceed $6g$, what's the tightest circle (smallest radius) in which the plane can turn?
76. Figure 5.39 shows an apparatus used to verify Newton's second law. A "pulling mass" m_1 hangs vertically from a string of negligible mass that passes over a pulley, also of negligible mass and with nearly frictionless bearings. The other end of the string is attached to a glider of mass m_2 riding on an essentially frictionless, horizontal air track. Both m_1 and m_2 may be varied by placing additional masses on the pulling mass and glider. The experiment consists of starting the glider from rest and letting the pulling mass accelerate it down the track. Three photogates are used to time the glider over two distance intervals, and an experimental value for its acceleration is determined from these data, using constant-acceleration equations from Chapter 2. The table in the next column lists the measured acceleration for a number of mass combinations. (a) Determine a quantity that, when plotted on the horizontal axis of a graph, should result in a straight line of slope g when acceleration is plotted on the vertical axis. (b) Make your plot, fit a line to the plotted data, and report the experimentally determined value of g .

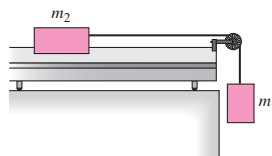


FIGURE 5.39 Problem 76

m_1 (g)	m_2 (g)	a (m/s ²)
10.0	170	0.521
10.0	270	0.376
10.0	370	0.274
20.0	170	1.06
20.0	270	0.652
20.0	370	0.534

Passage Problems

A *spiral* is an ice-skating position in which the skater glides on one foot with the other foot held above hip level. It's a required element in women's singles figure skating competition and is related to the arabesque performed in ballet. Figure 5.40 shows skater Sarah Hughes executing a spiral during her gold-medal performance at the Winter Olympics in Salt Lake City.



FIGURE 5.40 Passage Problems 77–80

77. From the photo, you can conclude that the skater is
- executing a turn to her left.
 - executing a turn to her right.
 - moving in a straight line out of the page.
78. The net force on the skater
- points to her left.
 - points to her right.
 - is zero.
79. If the skater were to execute the same maneuver but at higher speed, the tilt evident in the photo would be
- less.
 - greater.
 - unchanged.
80. The tilt angle θ that the skater's body makes with the vertical is given approximately by $\theta = \tan^{-1}(0.5)$. From this you can conclude that the skater's centripetal acceleration has approximate magnitude
- 0.
 - 0.5 m/s^2 .
 - 5 m/s^2 .
 - can't be determined without knowing the skater's speed

Answers to Chapter Questions

Answer to Chapter Opening Question

The airplane tips, or *banks*, so there's a horizontal component of the aerodynamic force on the wings. That component provides the mv^2/r force that keeps the plane in its circular path. The vertical component of the aerodynamic force is what balances the gravitational force, keeping the plane aloft.

Answers to GOT IT? Questions

- 5.1 (c) Equal—but only because of the 45° slope. At larger angles, the tension would be greater than the weight; at smaller angles, less.
- 5.2 (a) The left hand in Fig. 5.9 and the hook in this figure play exactly the same role, balancing the 1-N tension force in the rope.
- 5.3 (c)
- 5.4 (c) Greater because the chain is pulling downward, making the normal force greater than the log's weight.