Spectral and Transport Properties of PT-Symmetric Quarter Stacks

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ABSTRACT: We study the spectral and transport properties for an array of $\mathcal{PT}$-symmetric bilayers. The model describes the propagation of an electromagnetic wave of frequency ω through a periodic array of N unit (a, b) cells embedded in a homogeneous medium. Each cell is made of two dielectric, a and b, layers (slabs) with the thicknesses d_a and d_b, respectively, where $d = d_a + d_b$ is the unit-cell size. All the a slabs contain the material absorbing electromagnetic energy, whereas all the b layers are composed of the amplifying material. The loss and gain in the a and b layers are incorporated via complex dielectric functions, while the magnetic permeabilities $\mu_{a,b}$ are assumed to be real and positive. The optic parameters (refractive indices $n_{a,b}$, impedances $Z_{a,b}$ and wave phase shifts $\varphi_{a,b}$) of two constitutive layers, a and b, read

$$n_a = n_a^{(0)}(1 + i\gamma), \qquad Z_a = Z(1 + i\gamma)^{-1}, \qquad \varphi_a = \frac{\varphi}{2}(1 + i\gamma)\,;$$

$$n_b = n_b^{(0)}(1 - i\gamma), \qquad Z_b = Z(1 - i\gamma)^{-1}, \qquad \varphi_b = \frac{\varphi}{2}(1 - i\gamma)\,.$$

Here the dimensionless key parameter γ measures the strength of loss and gain inside a and b layers. These expressions are complemented by the following relations,

$$Z = \mu_a/n_a^{(0)} = \mu_b/n_b^{(0)}, \qquad \varphi = 2\omega n_a^{(0)} d_a/c = 2\omega n_b^{(0)} d_b/c\,.$$

In the case of no loss/gain ($\gamma = 0$) the stack-structure is known as the matched quarter stack. This means that the basic a and b layers are perfectly matched (their impedances are the same) and have equal optic paths, $n_a^{(0)} d_a = n_b^{(0)} d_b$. Consequently, the phase shift in every layer equals $\varphi/2$. For $\gamma \neq 0$ the wave amplitude is attenuated or amplified by the factor $\exp(\gamma\varphi/2)$ when traveling through the a or b layer balanced loss/gain.

For our model we obtain the transfer matrix $\hat{Q}(\gamma)$ of the unit (a, b) cell that has the specific symmetry, $Q_{11}(\gamma) = Q_{22}^*(-\gamma)$ and $Q_{12}(\gamma) = Q_{21}^*(-\gamma)$. This symmetry differs from the standard one, $Q_{11} = Q_{22}^*$, $Q_{12} = Q_{21}^*$, and manifests itself in an emergence of quite exotic spectral and transport properties of the system.

We have obtained the expression for the Bloch phase φ_B in dependence on the wave frequency ω and gain/loss parameter γ. If the parameter γ is less than unity ($0 \leq \gamma < 1$), a finite number of frequency intervals (spectral bands) emerges, where the Bloch phase φ_B is real. In these bands the electromagnetic wave propagates through the bilayer stack. Outside the bands the Bloch phase φ_B is purely imaginary, thus creating spectral gaps. Here the waves are known as the evanescent Bloch states, attenuated on the scale of the order of $|\varphi_B|^{-1}$. Therefore, for a sufficiently long structure, $N|\varphi_B| > 1$, the transmission is exponentially small. The number of spectral bands is determined by the value of parameter γ: the larger the parameter, the smaller the number of the spectral bands. When γ exceeds the critical value, $\gamma > 1$, there are no spectral bands, since the Bloch phase $\varphi_B(\omega)$ becomes purely imaginary for any frequency.

We have also derived a closed analytical expression for the transmittance T_N for N bilayers connected to homogeneous leads with the impedance Z. Inside the spectral bands the transmittance T_N exhibits the Fabry-Perrot resonances with $T_N = 1$ that survive in the presence of gain/loss. We have also found a new kind of frequencies that separate, in any spectral band, the regions with $T_N > 1$ from those with $T_N < 1$. Thus, one can speak about *internal band edges* determining the frequency regions where the effects of gain are suppressed by absorption. Our results may be important in view of experimental realizations of quarter stacks with the $\mathcal{PT}$-symmetric bi-layers.