Optical properties of 1D photonic crystals based on multiple-quantum-well structures

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A general approach to the analysis of optical properties of photonic crystals based on multiple-quantum-well structures is developed. The effect of the polarization state and a non-perpendicular incidence of the electromagnetic wave is taken into account by introduction of an effective excitonic susceptibility and an effective optical width of the quantum wells. This approach is applied to consideration of optical properties of structures with a pre-engineered break of the translational symmetry. It is shown, in particular, that a layer with different exciton frequency placed at the middle of an MQW structure leads to appearance of a resonance suppression of the reflection.

PACS numbers: 72.15.Rn,05.40.-a,42.25.Dd,71.55.Jv

I. INTRODUCTION

Structures with spatially modulated dielectric properties (photonic crystals) attract an ever growing interest since the first papers where they were considered.[1,2] This interest is caused by unique opportunities that such structures provide, to affect, in a controllable way, fundamental *microscopic* processes of light-matter interaction through a modification of *macroscopic* geometric characteristics of the structures. This makes such structures of obvious interest not only for fundamental physics but also for applications. Most of the works devoted to photonic crystals considered structures made of materials with a frequency independent dielectric constant.[3,4] Recently, however, a new class of structures, which can be described as resonant or optically active photonic crystals, has attracted particular attention.[5–13] In these structures periodic modulation of the dielectric constant is accompanied by the presence of internal excitations of constituent materials resonantly interacting with light within a certain frequency region, and resulting in a strong frequency dispersion of constituent dielectric constants. Extreme cases of such structures are so called optical lattices, in which well localized resonant elements are periodically distributed through the medium with a uniform dielectric constant. Originally, the concept of optical lattices referred to structures formed by cold atoms,[14] but it was also applied to a special kind of multiple quantum well structure (MQW), which were considered as a semiconductor analog of a one-dimensional optical lattice.[15]

MQW is a periodic multilayer structure built of two semiconductor materials, for instance, GaAs and $Al_xGa_{1-x}As$, in which electrons and holes are confined in narrower layers of a material with a smaller band gap (quantum wells) separated by wide layers of a semiconductor with larger band-gap (barriers). In this case, the role of dipole active resonant excitations is played by excitons confined to respective quantum wells, and if the width of the barriers is large enough, excitons from different quantum wells do not interact directly. They, however, still can interact through their common radiation field, and in this sense they are similar to atomic optical lattices. This analogy is exact only if one can neglect a difference in refractive indexes of wells and barriers. This approximation was widely used in most papers devoted to long-period MQW structures, in which the period of the structure is comparable with the wavelength of exciton radiation.[16,17] Of special interest are so called Bragg structures, in which the excitonic wavelength is in Bragg resonance with the periodicity, and which are characterized by a significantly enhanced radiative coupling between quantum well excitons. As a result of this coupling, light propagates through such a structure in the form exciton-polaritons, whose dispersion law is characterized by two branches with a band-gap between them[16,17]. The width of this stop band is significantly enhanced compared to off-Bragg structures, and this is what makes such structures of particular interest for applications.

In realistic MQW structures, however, dielectric constants of the wells and barriers are not equal to each other, and the presence of resonant optical excitations is accompanied by a periodic modulation of the background dielectric constant. These structures, therefore, represent a special case of one-dimensional resonant photonic crystals, optical properties of which are characterized by an interplay between interface reflections and resonant light-exciton interaction. The effects of the refractive index contrast on the optical properties of MQW structures have not been, of course, overlooked in previous studies. In particular, a modification of the Bragg condition and reflection spectra at normal incidence of Bragg MQWs in the presence of the contrast have been discussed in Refs. 18,19. The effects of the dielectric mismatch on optical properties of single quantum wells[20,21] or an MQW structure embedded in a dielectric environment[22] was also taken into account. **However, this problem suffers a lack of an analytical approach.** While optical spectra of any given MQW based structure can be easily obtained numerically, this is not sufficient when one needs to design a structure with predetermined optical properties, which is a key element in utilizing these structures for optoelectronic applications. The main difficulty of this task is the presence of a large number of experimental parameters such as an angle of incidence, a polarization state, indices of refraction, widths of the barriers and the quantum wells, etc, which are in a complicated way related to spectral characteristics of a structure. In order to resolve this difficulty, one needs a general effective analytical approach that would facilitate establishing relationships between material parameters and spectral properties of MQW based structures for an arbitrary angle of incidence and polarization state of incoming light. In the present paper we develop such a method and apply it to a case of MQW structure with an intentionally broken periodicity (an MQW structure with a "defect"). The method is based on a transfer matrix approach and consists of two steps. In the first step we show that a quantum well embedded in a dielectric environment can be described in exactly the same way as a quantum well in vacuum by introducing an effective excitonic susceptibility and an effective optical width of the quantum well layer. In the second step we establish relations between these effective quantum well characteristics and parameters of a total transfer matrix, describing propagation of light throughout an entire structure. The method is rather general and can be applied to a great variety of different MQW structures with light of an arbitrary polarization, incident at an arbitrary angle. In order to demonstrate the power of our approach, we consider reflection spectra of an MQW structure in which a central well is replaced with a well having a different resonant frequency. Such structures have been considered previously in a number of papers[23–25] in the optical lattice approximation. Here we show that the presence of the refractive index contrast does not destroy the remarkable reflection properties of such structures, confirming, therefore, their potential for optoelectronic applications.

II. A SINGLE QUANTUM WELL IN A DIELECTRIC ENVIRONMENT

Propagation of the electromagnetic wave in structures under discussion is governed by the Maxwell equation

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2}[\epsilon_\infty(z)\mathbf{E} + 4\pi \mathbf{P}_{exc}], \tag{1}$$

with modulated background dielectric permeability, $\epsilon_\infty(z)$, which is assumed to take values n_b^2 and n_w^2 in the barriers and the quantum wells materials respectively. For the sake of concreteness we assume hereafter that $n_w > n_b$, unless otherwise explicitly specified. P_{exc} is the excitonic contribution to the polarization and is defined by

$$\mathbf{P}_{exc} = \chi(\omega) \int \Phi(z)\Phi(z')\mathbf{E}(z')dz', \tag{2}$$

where $\Phi(z)$ is the exciton envelope function. Here we have restricted ourselves by taking into account $1s$ heavy-hole excitons only and have neglected the in-plane dispersion of the excitons. The frequency dependence of the excitonic susceptibility is described by

$$\chi(\omega) = \frac{\alpha}{\omega_0 - \omega - i\gamma}, \tag{3}$$

where ω_0 is the exciton resonance frequency, γ is the non-radiative decay rate of the exciton, $\alpha = \epsilon_b \omega_{LT} a_B^3 \omega_0^2 / 4c^2$, ω_{LT} is the exciton longitudinal-transverse splitting and a_B is the bulk exciton Bohr's radius.

Due to the absence of an overlap of the exciton wave functions localized in different quantum wells and the linearity of the Maxwell equations, the propagation of the electromagnetic wave along the structure can be effectively described by a transfer-matrix. Using the usual Maxwell boundary conditions the transfer matrix through one period of the structure in the basis of incoming and outgoing plane waves can be written in the form

$$T = T_b^{1/2} T_{bw} T_w T_{wb} T_b^{1/2}, \tag{4}$$

where

$$T_b^{1/2} = \begin{pmatrix} e^{i\phi_b/2} & 0 \\ 0 & e^{-i\phi_b/2} \end{pmatrix} \tag{5}$$

is the transfer matrix through the halves of the barriers surrounding the quantum well. Here $\phi_b = \omega n_b d_b \cos\theta_b / c$ with d_b being the width of the barrier and θ_b being an angle between the wave vector $\mathbf{k}$ inside the barrier and the direction of the z-axis, $\hat{\mathbf{e}}_z$.

The scattering of the electromagnetic wave at the interface between the quantum well and the barrier caused by the mismatch of the indices of refraction of their materials is described by

$$T_{bw} = T_{wb}^{-1} = T_\rho(\rho) \equiv \frac{1}{1+\rho}\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \tag{6}$$

where ρ is the Fresnel reflection coefficient. The interface scattering depends upon both the angle of incidence of the wave and its polarization state. These effects are effectively described by Fresnel coefficients (see, e.g. Ref. 26) ρ_s and ρ_p

$$\rho_s = \frac{n_w \cos\theta_w - n_b \cos\theta_b}{n_w \cos\theta_w + n_b \cos\theta_b}, \quad \rho_p = \frac{n_w \cos\theta_b - n_b \cos\theta_w}{n_w \cos\theta_b + n_b \cos\theta_w} \tag{7}$$

for s ($\mathbf{E} \perp (\mathbf{k}, \hat{\mathbf{e}}_z)$) and p ($\mathbf{E} \parallel (\mathbf{k}, \hat{\mathbf{e}}_z)$) polarizations respectively. The angular dependence of these coefficients upon the angle of incidence measured inside the barrier is schematically shown in Fig. 1.

Finally,

$$T_w = \begin{pmatrix} e^{i\phi_w}(1 - iS) & -iS \\ iS & e^{-i\phi_w}(1+iS) \end{pmatrix} \tag{8}$$

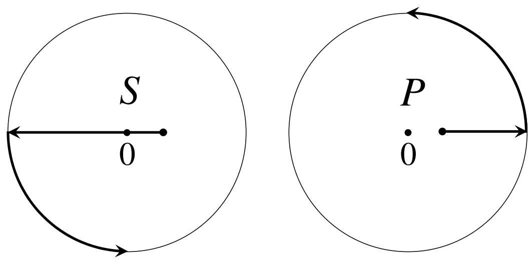

FIG. 1: A scheme of the angular dependence of the Fresnel coefficients ρ_s and ρ_p upon the angle θ_w is shown on the complex plane. At normal incidence the coefficients have values shown by small filled circles (the same for both polarizations). When the angle of incidence increases the coefficients follow the arrows on the lines. ρ_p passes through 0 at the Brewster's angle, both coefficients reach the unit circle at the angle of total internal reflection. **When the angle increases further the Fresnel coefficients become complex with the unit modulus and increasing argument.**

is the transfer matrix through the quantum well. Here $\phi_w = \omega n_w d_w \cos\theta_w/c$, where d_w is the width of the quantum well and θ_w is the angle between $\mathbf{k}$ inside the quantum well and $\hat{\mathbf{e}}_z$. The excitonic contribution to the scattering of the light is described by the function

$$S = \frac{\Gamma_0}{\omega - \omega_0 + i\gamma}, \tag{9}$$

which we will call the excitonic susceptibility in what follows. The radiative decay rate, Γ_0, at normal incidence is determined by

$$\Gamma_0 = \frac{1}{2}\pi k \omega_{LT} a_B^3 \left(\int \Phi(z) \cos kz \, dz \right)^2. \tag{10}$$

For oblique incidence the radiative decay rates are renormalized in different ways for different polarizations. For p-polarization in addition to this renormalization it is also necessary to take into account a possible splitting of Z- and L-exciton modes[29–31] that gives rise to a two-pole form of S. However, in materials with the zinc-blend structure, the Z-mode of the heavy-hole excitons is optically inactive, and one can describe angle dependencies of the radiative decay rate for s- and p-polarizations respectively by simple expressions

$$\Gamma_0^{(s)} = \Gamma_0/\cos\theta_w, \qquad \Gamma_0^{(p)} = \Gamma_0 \cos\theta_w. \tag{11}$$

Thus, propagation of light in the structures under consideration depends upon a number of natural parameters such as Fresnel coefficients, exciton frequencies and radiative decay rate, and optical widths, which (with the exception of ω_0) depend upon the angle of incidence of the wave and its polarization state.

Our next step will be to simplify the presentation of the total transfer matrix through the period of the structure in such a way that makes the relations between the elements of the transfer matrix and the natural parameters of the structure more apparent. The most complicated part of the transfer matrix is the product $T_{bw}T_wT_{wb}$, which describes the reflection of the wave from the interface and its interaction with quantum well excitons. We simplify it by noting that this product can be presented as $T_{bw}T_wT_{wb} = \widetilde{T}_w$, where $\widetilde{T}_w$ has the same form as T_w, Eq. (8), but with renormalized parameters

$$\widetilde{T}_w = T_{bw}T_wT_{wb} = \begin{pmatrix} e^{i\tilde{\phi}_w}(1 - i\tilde{S}) & -i\tilde{S} \\ i\tilde{S} & e^{-i\tilde{\phi}_w}(1 + i\tilde{S}) \end{pmatrix}, \tag{12}$$

where the effective excitonic susceptibility, $\tilde{S}$, and the phase shift, $\tilde{\phi}_w$, are defined as

$$\tilde{S} = S\frac{1 + \rho^2 - 2\rho\cos\phi_w}{1 - \rho^2} + 2\rho\frac{\sin\phi_w}{1 - \rho^2},$$
$$e^{i(\tilde{\phi}_w - \phi_w)} = \frac{1 - \rho e^{-i\phi_w}}{1 - \rho e^{i\phi_w}}. \tag{13}$$

Here ρ denotes the Fresnel coefficient for the wave of a respective polarization. Taking into account the diagonal form of the transfer matrix through the barrier T_b one can see that the total transfer matrix through the period of the structure again has the form of a single quantum well transfer matrix and is determined by Eq. (8) where the phase $\tilde{\phi}_w$ is replaced by a total phase $\phi = \phi_b + \tilde{\phi}_w$.

Thus, we have shown that the propagation of the wave in MQW based photonic crystals can be described in terms of properties of a respective optical lattice with renormalized parameters. The renormalization of the phase is the simplest one: the expression for $\tilde{\phi}_w$ can be rewritten as

$$\tilde{\phi}_w = \phi_w \frac{1 + \rho}{1 - \rho} \tag{14}$$

provided that the change of phase ϕ_w across the well is much smaller than 2π, which is usually true for long period MQW structures. Hence, one of the effects of the index of refraction contrast is reduced to a simple renormalization of the optical width of the quantum well.

The effective susceptibility, $\tilde{S}$, consists of two terms. One of them has a singularity at the exciton frequency while the second varies slowly in a wide frequency region. The relation between these terms essentially depends upon the frequency and near the exciton resonance the second term is negligibly small. The frequency region where the nonsingular addition is negligible can be found from Eq. (13) and is determined by

$$|\omega - \omega_0| < \omega_{\min} = \frac{\Delta_\Gamma^2}{2\Delta_{PC}} \frac{1 + \rho^2 - 2\rho\cos\phi_w}{1 - \rho^2}, \tag{15}$$

where $\Delta_\Gamma = \sqrt{2\Gamma_0\omega_0/\pi}$ is the half-width of the forbidden gap in a Bragg MQW without a mismatch of the indices of refraction, and $\Delta_{PC} = 2\omega_r\rho\sin[\phi_w(\omega_0)]/\pi(1 - \rho^2)$ **coincides with the half-width of the forbidden gap**

in a passive photonic crystal characterized by the same value of the mismatch calculated in the limit of the narrow gap in comparison with the position of its center, $\Delta_{PC} \ll \omega_r$.

It follows from Eq. (15) that the resonant part of the effective susceptibility, Eq. (13), makes the main contribution over the entire frequency region of the exciton polaritons stop band when

$$\Delta_\Gamma > 2\Delta_{PC}. \qquad (16)$$

In realistic multiple GaAs/Al$_x$Ga$_{1-x}$As long period structures the values of the Fresnel coefficient at normal incidence,[32] ρ_0, less than 0.03 and $\phi_w(\omega_0) \sim 0.1\pi$ can be considered as typical, so both quantities Δ_Γ and Δ_{PC} are of the same order of magnitude $\sim 10^{-2}$ eV. Therefore, in principle, both signs of the inequality in Eq. (16) are possible. **Also, it should be taken into account the growth of the Fresnel coefficients when the angle of incidence approaches the angle of total internal reflection, θ_c. In a typical experimental setup, however, the angle of incidence is given at the vacuum-structure rather than well-barrier interface. Therefore, because the indices of refraction of semiconductors are essentially higher than 1 the actual angles of propagation of the electromagnetic waves inside the structure are much smaller than θ_c. This makes the effect of the interfacial scattering always negligible in some vicinity of the exciton frequency. For a single quantum well light scattering this vicinity covers the whole region of the resonant interaction of excitons with light. An existence of broadenings does not change this conclusion, at least for not too high temperatures and interfacial roughnesses (as follows from Eq. (13) this remains true for $\gamma < 10$ meV).**

Of course, there are also another physical situations where negligibility of the regular term in Eq. (13) can not be justified. This includes the propagation of light in MQW structures and the exciton luminescence. Therefore, in what follows we provide exact results that are valid for an arbitrary relation between Δ_{PC} and Δ_Γ, unless otherwise is explicitly specified.

In the simplest case one can neglect the non-singular term in $\tilde{S}$ and approximate the latter by

$$\tilde{S} = S\frac{1+\rho^2-2\rho\cos\phi_w}{1-\rho^2} \approx S\frac{1-\rho}{1+\rho}. \qquad (17)$$

In this approximation, the effect of the mismatch of the indices of refraction, non-perpendicular incidence and the polarization is reduced to a renormalization of the radiative decay rate. This means that all results obtained neglecting these effects remain valid with introduction of an effective radiative decay rate (or, consequently, the

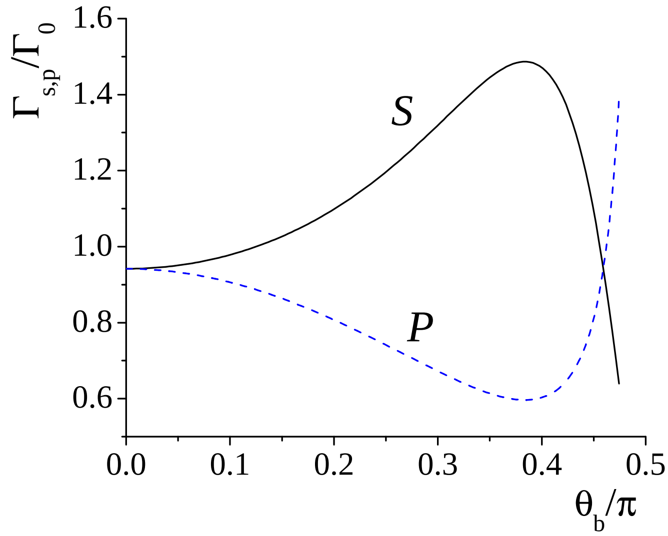

FIG. 2: Change of the effective radiative decay rates Γ_s (solid line) and Γ_p (dotted line) with the angle of incidence for a single quantum well.

modification of the oscillator strength)

$$\Gamma_{s,p} = \Gamma_0^{(s,p)}\frac{1+\rho^2-2\rho\cos\phi_w}{1-\rho^2}. \qquad (18)$$

Depending upon the value of the Fresnel coefficient one can observe either an enhancement (when $\rho < 0$) or a reduction (when $\rho > 0$) of the exciton radiative recombination. Since usually $n_w > n_b$, the Fresnel coefficient for the normal incidence is positive and therefore the oscillator strength is diminished compared to the case of the absence of the contrast. When the angle of incidence increases, in addition to different dependencies of the Fresnel coefficients corresponding to different polarization states following from Eqs. (7), it is necessary to take into account direct modification of the oscillator strength given by Eqs. (11). Fig. 2 shows the dependence of the factor modifying the radiative decay rate upon the angle of incidence.

III. OPTICAL PROPERTIES OF MQW STRUCTURES

The reduced description presented in the previous section works well within a frequency region satisfying the inequality given in Eq. (15). As has been discussed above, for a single quantum well (or even for short MQW structures) and for angles not too close to the angle of total internal reflection, the condition (15) is fulfilled for the entire frequency region of interest, which, in this case, is of the order of magnitude of Γ_0. Therefore, under these circumstances the mismatch of the indices of refraction can be treated perturbatively, so that all results neglecting the mismatch remain valid up to renormalization of the oscillator strength. However, when the number of quantum wells in an MQW structure increases, the frequency region affected by excitons widens almost linearly,[17] and, therefore, the regular contribution to the effective susceptibility becomes important, and more detailed analysis is necessary.

Keeping in mind subsequent application to more complicated structures, it is convenient to introduce a special formal representation for a transfer matrix

$$T(\theta,\beta) = \begin{pmatrix} \cos\theta - i\sin\theta\cosh\beta & -i\sin\theta\sinh\beta \\ i\sin\theta\sinh\beta & \cos\theta + i\sin\theta\cosh\beta \end{pmatrix}, \tag{19}$$

where the parameters of the representation, θ and β, are related to the "material" parameters S and ϕ_w entering the transfer matrix by

$$\begin{aligned} \cos\theta = \mathrm{Tr}\, T/2 = \cos\phi + S\sin\phi, \\ \coth\beta = \cos\phi - S^{-1}\sin\phi. \end{aligned} \tag{20}$$

This representation is valid for an arbitrary system that possesses a mirror symmetry with respect to a plane passing through the middle of the structure. It can be easily derived taking into account the equality of the determinant of the matrix to one, and the circumstance that the mirror symmetry requires off-diagonal elements to be imaginary. Due to the general character of the representations (19), the material parameters entering Eq. (20) can be either the parameters of a single quantum well, Eq. (8) or the effective parameters $\tilde{S}$ and $\tilde{\phi}$, Eqs. (12) and (13), of a barrier-well sandwich, or even parameters characterizing the entire MQW structure as long as the latter possess the mirror symmetry.

Using this representation we can introduce the following transformation rule for transfer matrices

$$T_H(\psi)T(\theta,\beta)T_H^{-1}(\psi) = T(\theta,\beta+2\psi), \tag{21}$$

where matrix T_H describes a hyperbolic rotation with a dilation and has the form of

$$T_H(\psi) = e^{\psi}\begin{pmatrix} \cosh\psi & -\sinh\psi \\ -\sinh\psi & \cosh\psi \end{pmatrix}. \tag{22}$$

This transformation rule can be used, for instance, for diagonalization of transfer matrices, which can be achieved by choosing parameter $\psi = -\beta/2$. Matrix T_H can be turned into matrix T_{bw}, Eq. (6), which describes propagation of waves through interface between two media with different refraction coefficients by using the following relation between ψ and the Fresnel parameter ρ: $\rho = -\tanh(\psi)$ (a detailed discussion of a relation between interface scattering and the hyperbolic rotation can be found in Ref. 33). This means that the transformation, Eq. (21), can be either used to describe the interface between two different structures, or in order to present any type of non-diagonality of the transfer-matrix as resulting from some effective interface. With the help of Eq. (21), any symmetric multilayer structure can be replaced by a uniform slab with the width given by θ and the index of refraction determined by ψ. Therefore, it can be used to describe structures which are more complicated than a simple three layer barrier-well sandwich considered in the previous section. For instance, using Eq. (21) we can immediately derive an expression for the transfer matrix T_N of a sequence of identical blocks described by $T(\theta,\beta)$:

$$T_N = T(\theta,\beta)^N = T(N\theta,\beta). \tag{23}$$

Because the reflection from the structure described by the transfer matrix T given in the basis of incoming and outgoing waves is

$$r = -T_{21}/T_{22}, \tag{24}$$

for a structure containing N blocks we have

$$r_N = -\frac{i\sinh\beta}{\cot N\theta + i\cosh\beta}. \tag{25}$$

The reflection coefficient written in terms of the parameters θ and β does not depend upon the specific form of the transfer matrix and therefore has a wide range of validity. In the case of $\Gamma_0 = 0$ Eq. (25) can be easily shown to reproduce the result well-known for a passive multilayer structure.[4,27,28]

To find a relation between the quantities entering this expression and the elements of the transfer matrix through the period of the structure it is convenient to multiply both the numerator and the denominator by $\sin\theta$ and to use Eq. (20). If each block is characterized by an effective susceptibility $\tilde{S}$ and the phase $\phi = \phi_b + \tilde{\phi}_w$ then we obtain for the reflection coefficient **an exact expression**

$$r_N = \frac{i\tilde{S}}{\cot(N\theta)\sin\theta + i(\tilde{S}\cos\phi - \sin\phi)}. \tag{26}$$

We analyze the reflection coefficient for Bragg and slightly off-Bragg structures. For frequencies close to ω_0 we can use $\theta = \pi + i\lambda$, where λ is a complex number with a small modulus. Moreover, in most applications of MQW structures it is naturally to assume that structures are not too long, that is $N \ll N_c = (\mathrm{Re}\,\lambda)^{-1}$. In this approximation, $\coth(N\lambda)\sinh\lambda \approx N$ and the reflection can be written directly in terms of the material parameters

$$r_N = \frac{iN\tilde{S}}{1 + iN(\tilde{S}\cos\phi - \sin\phi)}. \tag{27}$$

The amplitude of reflection, $|r|^2$, has the typical form shown in Fig. 3. It is characterized by a strong reflection band around the exciton frequency, which is a manifestation of the strong resonant exciton-light interaction. The reflection has an asymmetric form since it is a sum of two terms: one of them is even with respect to the frequency ω_0 and the second is odd. The latter is due to nonzero

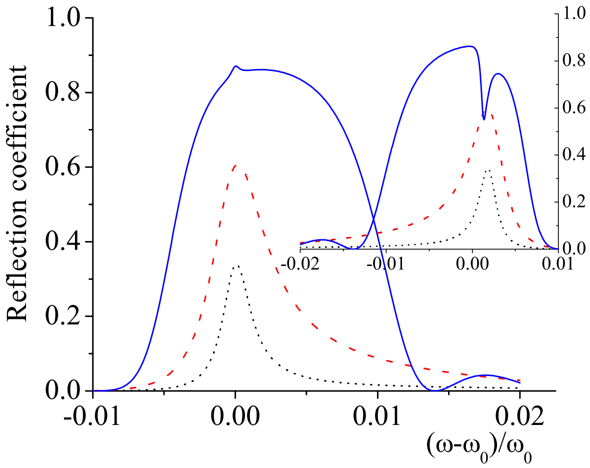

FIG. 3: Dependence of the amplitude reflection coefficient $|r_N|^2$ upon the frequency. The main plot shows the reflection of structure satisfying the modified Bragg condition with $\rho = 0.005$, the radiative decay rate $\Gamma_0 = 67$ μeV, the exciton frequency $\omega_0 = 1.491$ eV, the homogeneous broadening $\gamma = 500$ μeV and the length $N = 10, 25, 100$ (dot, dash, solid lines, respectively). On the inset reflection spectra of structures that satisfy the standard Bragg condition are shown. To make the correspondence with the results of Ref. 34 clearer we chose $\rho = -0.005$.

mismatch and in the approximation used above does not vanish at infinity. Both these terms have a typical width

$$\delta = \frac{N\Gamma_0(1-\rho)^2 \cos\phi_+}{(1-\rho^2)^2 + N^2(\sin\phi_+ - \rho^2\sin\phi_-)^2}, \tag{28}$$

where $\phi_\pm = \phi_b \pm \phi_w$. It should be noted that the actual optical width of the quantum well determined by ϕ_w enters the definition of $\phi_\pm$, rather than the modified $\tilde{\phi}_w$. We are interested in maximizing exciton related effects in the reflection spectra of our structures, which means designing structures with as large a width δ as possible. One can see from Eq. (28) that the width demonstrates essentially non-monotonous dependence upon the number of quantum wells in the structure: it grows linearly for small N, but starts decreasing as N^2 for larger N. If the coefficient in front of the N^2-term in the denominator of Eq. (28) vanishes, then the linear growth of δ would go unchecked as long as N does not exceed N_c. Thus, the condition for maximizing the excitonic effects in the reflection spectrum can be formulated as

$$\rho^2 = \sin\phi_+(\omega_0)/\sin\phi_-(\omega_0). \tag{29}$$

If one neglects the mismatch of the indices of refraction, this equation takes a well known form of a condition for the Bragg resonance between the period of the structure and the exciton radiation.[16] **Eq. (29) considered as an equation for the period of the structure gives in the case of small ρ approximately the same result as**

$$\rho = \cos(\phi_+/2)/\cos(\phi_-/2), \tag{30}$$

which coincides with a modified Bragg condition introduced in Ref. 19, which actually requires that the exciton frequency is equal to the low frequency boundary of the passive (without excitons) photonic crystals' stop band. **This consideration shows that the most prominent effect of the light-exciton interaction on the optical properties of long MQW structures with a mismatch of the indices of refraction occurs when the modified Bragg condition is met.**

To establish the relation between the modified Bragg condition and the reflection spectrum of the structure it is convenient to obtain the dispersion equation from the transfer matrix written in terms of $\tilde{S}$ and $\tilde{\phi}_w$, Eq. (12). Using the relation $\cos Kd = \mathrm{Tr}\,T/2$, where K is the Bloch wave-number, d is the period of the structure, one obtains the dispersion equation in a standard form

$$\cos Kd = \cos\phi + \tilde{S}\sin\phi, \tag{31}$$

where the change of the phase on the period of the structure, ϕ, is determined by the modified optical width of the quantum well, i.e. $\phi = \phi_b + \tilde{\phi}_w$. The Bragg resonance condition is written in a usual form $\phi(\omega_0) = \pi$ that can be shown to coincide with the modified Bragg condition from Ref. 19.

Thus, for Bragg MQWs the expression for the reflection coefficient, Eq. (26), is essentially simplified and can be approximated as

$$r_N = \frac{iN\tilde{S}}{1 + iN\tilde{S}}. \tag{32}$$

This expression gives a generalization of a well known result about the linear dependence of the width of the exciton-polariton reflection band on the number of quantum wells in Bragg MQW structures.[17]

When the system is detuned from the Bragg resonance, the transparency window appears in the band gap. It shows up in the form of a dip near the exciton frequency in the reflection spectrum (see inset in Fig. 3). For example, in Ref. 34 the reflection was measured for MQW structures that satisfied the Bragg condition for structures without the mismatch. In other words, the period of those structures was made to coincide with the half-wavelength at the exciton frequency calculated without taking into account photonic crystal modification of light dispersion. These effects, however, significantly modify the wavelength of light resulting in a detuning of the structures studied in Ref. 34 from actual Bragg resonance. As a result, spectra observed in that paper demonstrate features specific for slightly off-resonance structures [see Fig. (3)].

It is interesting to note that $|r_N|^2$ vanishes at $\omega = \omega_0 - \omega_{\min}$ [see Eq. (15)] where $\tilde{S} = 0$. This is the consequence of an interference of two channels of scattering of light — by the excitons and by the barrier-well interfaces. For Bragg MQW structures r_N can be written in a form that explicitly expresses the Fano-like profile of the reflection

$$r_N = r_N^{(0)} \frac{\omega - \omega_0 + \omega_{min}}{\omega - \omega_0 + \omega_{min} r_N^{(0)}}, \tag{33}$$

where

$$r_N^{(0)} = \frac{i\pi N \Delta_{PC}}{\omega_r + iN\pi\Delta_{PC}} \tag{34}$$

is the reflection of a passive (i.e. with $\Gamma_0 = 0$) not too long N-layer structure near the photonic band gap. When the mismatch of the indices of refraction vanishes ω_{min} goes infinity and the reflection restores the Wigner-like form (similar to what one has in a standard Fano resonance case when the Fano parameter, q, goes to infinity[35]) with the width $\propto N\Gamma$. On the other limiting case, when $\Gamma_0 = 0$ the reflection turns to what one has for purely passive multilayer.

Due to discussed above relation between the different contributions to the scattering of light in regular MQW structures this resonant drop of reflection occurs at the tail of the excitonic susceptibility where the smallness of the reflectivity masks this effect. In the next section we consider a situation where it is more noticeable.

IV. MQW STRUCTURES WITH DEFECTS

The results obtained in the previous sections are quite general and can be applied directly to more complicated situations. As an example, in this section we consider a reflection spectrum of a system in which one of the barrier-well-barrier elements has properties different from those of all other elements of the structure. These structures can be described as MQW structures with a "defect." The break of the translational symmetry leads to a significant modification of the reflection spectrum. This effect was studied in regular passive one-dimensional photonic crystals,[36–40] and in Bragg MQWs in the optical lattice approximation.[23,41] It was found, in particular, that the defect affects the spectrum most strongly when it is placed in the middle of the structure.[23] In this case, the system demonstrates the mirror symmetry and the results of the previous sections can be used. Indeed, for a structure ABA built of blocks A and B described by the transfer matrices (19) with parameters $\theta_{A,B}$ and $\beta_{A,B}$, one has

$$T(\theta_A, \beta_A)T(\theta_B, \beta_B)T(\theta_A, \beta_A) = T(\theta, \beta). \tag{35}$$

That is, the whole structure can also be described by the matrix (19) with

$$\begin{aligned} \cos\theta &= \cos\varphi_+ \cosh^2\delta\beta - \cos\varphi_- \sinh^2\delta\beta, \\ \coth(\beta - \beta_A) &= \cos(2\theta_A)\coth\delta\beta + \frac{\sin\varphi_-}{\sin\theta_N \sinh(2\delta\beta)}, \end{aligned} \tag{36}$$

where $\varphi_\pm = 2\theta_A \pm \theta_B$ and $\delta\beta = (\beta_B - \beta_A)/2$. Applying now relations (20) one can express the result in terms of parameters S and ϕ, and use the results for reflection described above.

In some particular cases, however, the problem of scattering of light can be solved without resorting to the transformation rule (36). Let us consider a situation when the block B is a single quantum well surrounded by barriers so it can be described by the matrix similar to that given by Eq. (12) with parameters S_d and ϕ_d. Let the block A be an MQW structure described by θ and β. Thus, the transfer matrix through the whole structure is

$$T = T(\theta, \beta)T_\rho(\rho)T(S_d, \phi)T_\rho^{-1}(\rho)T(\theta, \beta), \tag{37}$$

where $T_\rho(\rho)$ takes into account a possible mismatch of the indices of refraction of the defect layer and the host and, ρ is the corresponding Fresnel coefficient.

The transfer matrix (37) can be simplified in several steps. First, as has been described before, we can treat β as an addition to the mismatch noting that

$$T_H^{-1}(\beta/2)T_\rho(\rho) = T_\rho(\tilde{\rho}), \tag{38}$$

where $\tilde{\rho} = (\rho + \rho')/(1 + \rho\rho')$ and $\rho' = \tanh(\beta/2)$. Then, similar to Eq. (12) we can introduce effective quantities $\tilde{S}$ and $\tilde{\phi}$

$$\begin{aligned} \tilde{S} &= S_d \frac{1 + \tilde{\rho}^2 - 2\tilde{\rho}\cos\phi}{1 - \tilde{\rho}^2} + 2\tilde{\rho}\frac{\sin\phi}{1 - \tilde{\rho}^2}, \\ e^{i(\tilde{\phi} - \phi)} &= \frac{1 - \tilde{\rho}e^{-i\phi}}{1 - \tilde{\rho}e^{i\phi}}. \end{aligned} \tag{39}$$

The next step is a multiplication of $T(\tilde{S}, \tilde{\phi})$ by the diagonal matrices $T(\theta, 0)$ what leads to a simple shift of the phase, $T(\tilde{S}, \tilde{\phi} + 2\theta)$. Finally, the terminating matrices, $T_H(\beta/2)$ and $T_H^{-1}(\beta/2)$, are taken into account by modifying $\tilde{S}$ and $\tilde{\phi}$. Thus the resultant transfer matrix T takes the form (12), i.e. $T = T(\tilde{\tilde{S}}, \tilde{\tilde{\phi}})$ with

$$\begin{aligned} \tilde{\tilde{S}} &= \tilde{S}\frac{1 + \rho'^2 + 2\rho'\cos(\tilde{\phi} + 2\theta)}{1 - \rho'^2} - 2\rho'\frac{\sin(\tilde{\phi} + 2\theta)}{1 - \rho'^2}, \\ e^{i(\tilde{\tilde{\phi}} - \tilde{\phi} - 2\theta)} &= \frac{1 + \rho' e^{-i\tilde{\phi} - 2i\theta}}{1 + \rho' e^{i\tilde{\phi} + 2i\theta}}. \end{aligned} \tag{40}$$

These expressions together with Eq. (25) give a complete solution of the problem of propagation of light through the MQW structure with an arbitrary defect in the middle.

One can consider several particular types of defects. An example is a well with the exciton frequency different from the frequencies of all other wells, an Ω-defect. Another possible example could be a defect element with the width of the barriers different from the rest of the structure. It is interesting to note that a standard optical microcavity with a quantum well at its center can also be considered within the same formalism. For example, after substitution of $\tilde{S}$ from (39) into Eq. (40) one obtains an expression that contains a singular term (proportional to S_d) and regular terms. Choosing such widths of the barriers surrounding the quantum well so that the regular terms vanish in the vicinity of the exciton frequency one has the reflection determined by the exciton susceptibility with renormalized oscillator strength. The excitonic contribution to the scattering in such a structure will not be obscured by the interface scattering.

We demonstrate the application of the results obtained above by a detailed consideration of an Ω-defect. This type of defect was analyzed in Refs. 23–25 in the scalar model for the electromagnetic wave in MQW structures without a mismatch of the indices of refraction. It has been shown there that in the presence of homogeneous and inhomogeneous broadening of excitons, the effect of the defect is prominent when the frequency of the exciton resonance in the defect layer, ω_d, is close to the boundary of the forbidden gap in the host system, and the length of the system is not too big. The reflection spectrum in this case has the characteristic Fano-like dependence with a minimum followed by a closely located maximum. Such a spectrum makes this structure a potential candidate for novel types of devices such as optical switches or modulators.[24,25] It is interesting, therefore, to find out how the refractive index mismatch affects spectral properties of such a structure.

For the frequencies within the polariton stop-band of the host structure one has: $\theta = M(\pi + i\lambda)$, where M is the number of quantum wells in the parts of the structures surrounding the defect, and $M\lambda \ll 1$. The description becomes much simpler if we assume that the width of the defect layer is tuned to the Bragg resonance at the frequency ω_d, that is if $\phi(\omega_d) = \pi$. That makes the second term in the expression for $\tilde{S}$, Eq. (39), negligible in a wide region of frequencies and the expression for $\tilde{\tilde{S}}$ takes a very simple form

$$\tilde{\tilde{S}} = 2M\tilde{S}_h + S_d\frac{1+\rho}{1-\rho}, \tag{41}$$

where $\tilde{S}_h$ is the effective excitonic susceptibility of the host, given by an expression similar to Eq. (13) with the exciton frequency $\omega_0 = \omega_h$. In derivation of Eq. (41) we have neglected the small term $\propto S_d\tilde{S}_h$. The reflection coefficient can be obtained by substituting this expression into Eq. (25) with $N = 1$. The reflection has peculiarities near the exciton frequencies of the host and the defect, and in the absence of broadening becomes 0 at the frequency where $\tilde{\tilde{S}} = 0$. **Generally this equation is suitable for finding the resonance frequency at a relation between the widths of the photonic and the excitonic forbidden gaps in a quite wide range. Here, however, we restrict ourselves to consideration of the perturbation of the resonant drop of the reflection analyzed in Ref. 24, therefore we assume that $\Delta_{PC} < \Delta_\Gamma$ (i.e. at the defect frequency in the host the exciton scattering prevails the interfacial one). In this approximation the resonant frequency is**

$$\omega_R = \omega_d - \frac{\Delta_\omega}{2M+1} - 16\frac{M^2\Delta_\omega^2\Delta_{PC}}{\Delta_\Gamma^2(2M+1)^3}, \tag{42}$$

where $\Delta_\omega = \omega_d - \omega_h$ is the difference between the defect and the host exciton frequencies, and we assume for concreteness that $\omega_d > \omega_h$.

Setting the mismatch of the indices of refraction in this expression to zero we reproduce the expression for ω_R obtained in Ref. 24. The fact that the contrast does not preclude the reflection coefficient from going to zero at a certain point is not at all obvious because it might have been expected that the interface reflection would set a limit on the decrease of the reflection. However, as seen from Eq. (42), the mismatch leads only to an additional shift of the zero point away from the defect exciton frequency. We would also like to comment on the dependence of the resonance frequency on the angle of incidence and the polarization of the electromagnetic wave. These characteristics enter Eq. (42) through the Fresnel coefficients, Eq. (7), that determine the photonic band gap, $\Delta_{PC} \propto \rho/(1-\rho^2)$. Therefore angular and polarization dependencies of the zero point of reflection follow the behavior of Δ_{PC}. The obtained expression for the reflection coefficient also allows for analyzing the position of the maximum of reflection, and its maximum magnitude, but the resulting expressions turn out to be too cumbersome and we do not present them here.

The formalism presented in this paper allows one to take into account effects of homogeneous and inhomogeneous broadenings because the main results obtained do not use a particular form of the excitonic susceptibility. For example, the reflection coefficient of a structure with an inhomogeneous broadening can be obtained in the effective medium approximation by using Eq. (25) with $\tilde{\tilde{S}}$ instead of S and with inhomogeneously broadened $S_{h,d}$ in Eq. (42):

$$S_{h,d} = \int d\omega_0 f_{h,d}(\omega_0)\frac{\Gamma_0}{\omega - \omega_0 + i\gamma}. \tag{43}$$

where $f_{h,d}$ are the distribution functions of the exciton frequencies in the host and defect layers. respectively[24,42,43] However, as has been discussed in Ref. 24, if ω_R is far enough from ω_d, i.e. if $|\omega_R - \omega_d| \gg \Delta$, where Δ is the inhomogeneous broadening, the effect of the latter is negligible and the magnitude of the reflection

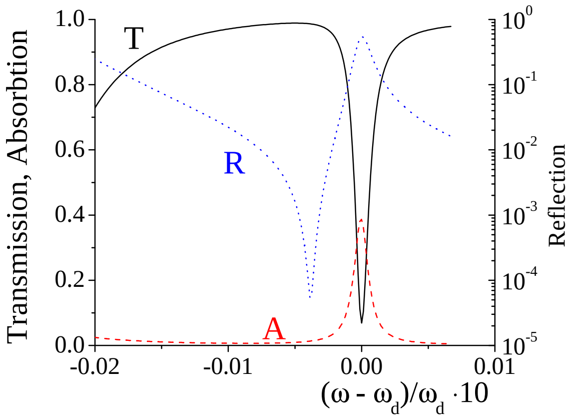

FIG. 4: The reflection (dotted line, right scale), transmission and absorbtion (solid and dashed lines respectively, left scale) are shown in a vicinity of ω_R for a Bragg $5-1-5$ structure. The parameters of the quantum wells are the same as those used in Fig. 3 except: $\gamma = 25$ μeV, $\omega_h = 1.491$ eV, $\omega_d = 1.495$ eV. To emphasize the resonant character of the change of the reflection it is plotted in the log-scale. It should be noted that due to large (in comparison with γ) separation between ω_R and ω_d the drop of the reflection is *not* accompanied with a resonant absorbtion.

is determined by

$$|\tilde{\tilde{S}}|_{min} \approx \frac{\pi\gamma(2M+1)}{4M\omega_d\Delta_\omega^2}\left[\Delta_\Gamma^2(2M+1)^2 + 16M(2M-1)\Delta_{PC}\Delta_\omega\right] \tag{44}$$

and is small provided the smallness of the homogeneous broadening, $\gamma \ll \omega_d$, and $\Delta_\omega \gtrsim \Delta_\Gamma \gtrsim \Delta_{PC}$.

There is a certain analogy with vanishing the reflection from a single quantum well in an environment with a different index of refraction considered in the previous section. Here in a defect MQW structure when one considers a narrow vicinity of ω_d the Fano-like profile of the reflection can be understood as an interference of the scattering of light by the host and by the defect. Expanding $\tilde{\tilde{S}}$ near ω_R the reflection can be represented in a form similar to Eq. (33). There is an essential quantitative difference between these two cases, however. Because in a defect MQW structure the drop of the reflection occurs not so far away from the exciton frequency this effect becomes more noticeable. A typical form of the reflection, transmission and absorbtion near ω_R is shown in Fig. 4.

It should be noted, that taking into account a regular term in the effective susceptibility of the defect layer the expression (41) for $\tilde{\tilde{S}}$ becomes valid in much wider region, including for example ω_h, than just an immediate vicinity of ω_R. In this region the reflection can be seen to resemble the Fano profile in the case of two metastable states interacting with a continuum[44]. As the result the reflection has two resonances — near ω_h and ω_d, with the first $2N$ times wider — and reaches zero between them at ω_R.

To conclude this consideration we would like to note that for multiple defect structures composed of several blocks ABA considered above the reflection can be qualitatively described by Eq. (32) where N is understood as the number of such blocks. This expression shows that increasing N can be interpreted as increasing an effective radiative decay rate. This qualitatively explains the results of numerical calculations in Ref. 25 where it was found that the maximal value of the reflection increases with N while the value of the ratio of the maximal and minimal values remains about the same.

V. CONCLUSION

In the present paper, the general problem of light propagation in an MQW based photonic crystal characterized by both spatial modulation of the dielectric constant, and dipole active exciton states in quantum wells, is considered. It is shown that the mismatch of indices of refraction between barriers and wells can be taken into account by introduction of an effective excitonic susceptibility and an effective optical width of the quantum wells. The effective susceptibility has two terms, one of which is almost independent of frequency, while the second is resonant in nature. For short enough MQW structures (or in the vicinity of the exciton frequency) the regular term can be neglected and the effect of the mismatch of the indices of refraction reduces to a modification of the excitonic oscillator strength. In a general case, the reflection spectrum becomes essentially asymmetric and non-trivially dependent upon the number of quantum wells in the structure. It is shown that in order to obtain the strongest exciton induced reflection band the structure must satisfy a certain resonance condition. This is a Bragg resonance condition between the period of the MQW structures and the wavelength of the electromagnetic wave. The latter has to be calculated from a dispersion law for a structure with the spatially modulated refraction index.

The developed approach is applied to analysis of the reflection spectrum of a structure with an intentionally introduced defect element, which breaks the translational symmetry of a system. More detailed analysis is carried out for a special kind of defect, characterized by the presence of a layer in the middle of the structure with a different frequency of the exciton resonance. It is shown that the main characteristics of the reflection spectrum of such structures obtained in the absence of the refraction index contrast survive in the presence of the additional interface reflections. In particular, a significant decrease of the reflection takes place even in the presence of the contrast, which is important for possible applications of such structures.

[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).

[2] S. John, Phys. Rev. Lett. **58**, 2486 (1987).

[3] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light* (Princeton Univ. Press, 1995).

[4] A. Yariv, and P. Yeh, *Optical waves in crystals*, (Wiley, New Jersey, 2003).

[5] 9. V. Kuzmiak, A.A. Maradudin, and A.R. McGurn, Phys. Rev. B **55**, 4298 (1997).

[6] L.I. Deych, D. Livdan, and A.A. Lisyansky, Phys. Rev. E. **57**, 7254 (1998).

[7] S. Nojima, Phys. Rev. B **61**, 9940 (2000).

[8] L. Pilozzi, and A. D'Andrea, Phys. Rev. B **61**, 4771 (2000).

[9] A. Yu. Sivachenko, M. E. Raikh, and Z. V. Vardeny, Phys. Rev. A **64**, 013809 (2001).

[10] M. Lonăr, T. Yoshie, A. Scherer, P. Gogna, and Y. Qiu, Appl. Phys. Lett. **81**, 2680 (2002); M. Lonăr, T. Yoshie, K. Okamoto, Y. Qiu, J. Vučković, and A. Scherer, IEICE Trans. Electron. **E87-C**, 291 (2004).

[11] A. Christ, S. G. Tikhodeev, N. A. Gippius, J. Kuhl, and H. Giessen, Phys. Rev. Lett. **91**, 183901 (2003).

[12] K.C. Huang, P. Bienstman, J. D. Joannopoulos, K. A. Nelson, and S. Fan, Phys. Rev. B **68**, 075209 (2003).

[13] K.C. Huang, P. Bienstman, J. D. Joannopoulos, K. A. Nelson, and S. Fan, Phys. Rev. Lett. **90**, 196402 (2003).

[14] I. H. Deutsch, R. J. C. Spreeuw, S. L. Rolston, and W. D. Phillips, Phys. Rev. A **52**, 1394 (1995).

[15] E.L. Ivchenko, and G.E. Pikus, *Superlattices and other heterostructures: symmetry and optical phenomena* (Springer, New York, 1997).

[16] E. L. Ivchenko, Fiz. Tverd. Tela (St.-Peterburg) **33**, 2388 (1991) [Phys. Solid State **33**, 1344 (1991)].

[17] E. L. Ivchenko, A. I. Nesvizhinskii, and S. Jorda, Fiz. Tverd. Tela (St.-Peterburg) **36**, 2118 (1994) [Phys. Solid State **36**, 1156 (1994)].

[18] E. L. Ivchenko, V. P. Kochereshko, A. V. Platonov, D. R. Yakovlev, A. Waag, W. Ossau, and G. Landwehr, Fiz. Tverd. Tela (St.-Peterburg) **39**, 2072 (1997) [Phys. Solid State **39**, 1852 (1997)].

[19] E. L. Ivchenko, M. M. Voronov, M. V. Erementchouk, L. I. Deych, and A. A. Lisyansky, to appear in Phys. Rev. B **70**, (2004).

[20] A. V. Kavokin and M. A. Kaliteevski, Solid State Commun. **95** 859 (1995).

[21] F. Tassone, F. Bassani, and L. C. Andreani, Phys. Rev. B **45**, 6023 (1992).

[22] D. Ammerlahn, B. Grote, S. W. Koch, J. Kuhl, M. Hübner, R. Hey, and K. Ploog, Phys. Rev. B, **61**, 4801 (2000).

[23] L. I. Deych, A. Yamilov, and A. A. Lisyansky, Phys. Rev. B **64**, 075321 (2001).

[24] L. I. Deych, M. V. Erementchouk, A. A. Lisyansky, Phys. Rev. B **69**, 75308 (2004).

[25] L. I. Deych, M. V. Erementchouk, and A. A. Lisyansky, Appl. Phys. Lett. **83**, 4562 (2003).

[26] M. Born and E. Wolf, *Principles of Optics*, 7th edition (Cambridge:Cambridge University Press), 1999.

[27] A. Yariv, P. Yeh, and C.-S. Hong, J. Opt. Soc. Am. **67**, 423 (1977).

[28] J. M. Bendickson, J. P. Dowling, and M. Scalora, Phys. Rev. E **53**, 4107 (1996).

[29] L. C. Andreani, F. Tassone, and F. Bassani, Solid State Commun. **77**, 641 (1991).

[30] D. S. Citrin, Phys. Rev. B **47**, 3832 (1993).

[31] D. S. Citrin, Phys. Rev. B **50**, 5497 (1994).

[32] S. Adachi, J. Appl. Phys. **58**, R1 (1985).

[33] J. J. Monzon, L. L. Sanchez-Soto, Eur. J. Phys **23**, 1 (2002).

[34] M. Hübner, J. P. Prineas, C. Ell, P. Brick, E. S. Lee, G. Khitrova, H. M. Gibbs, S. W. Koch, Phys. Rev. Lett. **83**, 2841 (1999); J. P. Prineas, C. Ell, E. S. Lee, G. Khitrova, H. M. Gibbs, S. W. Koch, Phys. Rev. B **61**, 13863 (2000).

[35] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions. Basic processes and applications*, (Wiley, New York), 1992.

[36] R. P. Stanley, R. Houdré, U. Oesterle, M. Ilegems, and C. Weisbuch, Phys. Rev. A **48**, 2246 (1993).

[37] N.H. Liu, Phys. Rev. B **55**, 4097 (1997).

[38] A. Figotin, and V. Gorentsveig, Phys. Rev. B. **58**, 180 (1998).

[39] D. Felbacq, J. Phys. A: Math. Gen. **33**, 7137 (2000).

[40] R. Ozaki, Y. Matsuhisa, M. Ozaki, and K. Yoshino, Appl. Phys. Lett. **84**, 1844 (2004).

[41] L. I. Deych, and A. A. Lisyansky, Phys. Lett. A **243**, 156 (1998).

[42] G. Malpuech, A. Kavokin, G. Panzarini, Phys. Rev B **60**, 16788 (1999).

[43] L. C. Andreani, G. Panzarini, A. V. Kavokin, and M. R. Vladimirova, Phys. Rev. B **57**, 4670 (1998).

[44] Ph. Durand, I. Paidarová, and F. X. Gadéa, J. Phys. B: At. Mol. Opt. Phys. **34**, 1953 (2001).